

- 3 The function f is defined as follows:

$$f(x) = \frac{x+3}{x-1} \text{ for } x > 1.$$

- (a) Find the value of $ff(5)$.

[2]

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- (b) Find an expression for $f^{-1}(x)$.

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6 Functions f and g are defined for $x \in \mathbb{R}$ by

$$f : x \mapsto \frac{1}{2}x - a,$$

$$g : x \mapsto 3x + b,$$

where a and b are constants.

- (a) Given that $gg(2) = 10$ and $f^{-1}(2) = 14$, find the values of a and b . [4]

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- (b) Using these values of a and b , find an expression for $gf(x)$ in the form $cx + d$, where c and d are constants. [2]

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6 The function f is defined by $f(x) = 2x^2 - 16x + 23$ for $x < 3$.

(a) Express $f(x)$ in the form $2(x + a)^2 + b$.

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(b) Find the range of f .

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- (c) Find an expression for $f^{-1}(x)$.

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The function g is defined by $g(x) = 2x + 4$ for $x < -1$.

- (d) Find and simplify an expression for $fg(x)$.

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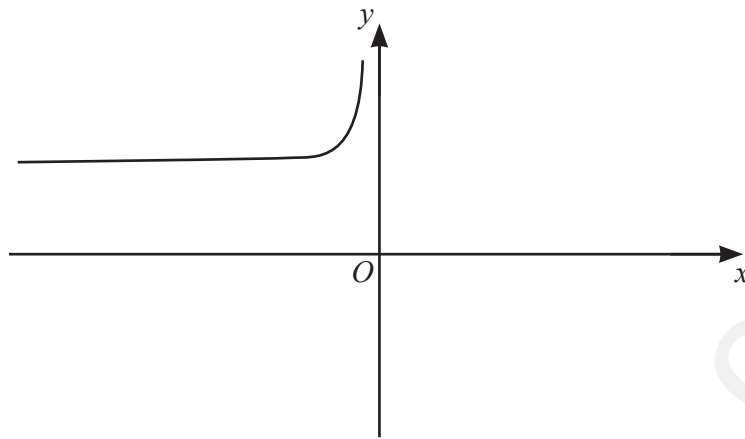
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The function f is defined by $f(x) = \frac{2}{x^2} + 4$ for $x < 0$. The diagram shows the graph of $y = f(x)$.

- (a) On this diagram, sketch the graph of $y = f^{-1}(x)$. Show any relevant mirror line. [2]
- (b) Find an expression for $f^{-1}(x)$. [3]

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- (c) Solve the equation $f(x) = 4.5$. [1]

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- (d) Explain why the equation $f^{-1}(x) = f(x)$ has no solution. [1]

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7 The function f is defined by $f(x) = 1 + \frac{3}{x-2}$ for $x > 2$.

(a) State the range of f .

[1]

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(b) Obtain an expression for $f^{-1}(x)$ and state the domain of f^{-1} .

[4]

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The function g is defined by $g(x) = 2x - 2$ for $x > 0$.

(c) Obtain a simplified expression for $gf(x)$.

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(a) Find the value of $fg(7)$.

[5]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. A large, light gray watermark is oriented diagonally from the bottom-left towards the top-right, reading "pastpaperpenz".

- 9 (a) Express $4x^2 - 12x + 13$ in the form $(2x + a)^2 + b$, where a and b are constants. [2]

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The function f is defined by $f(x) = 4x^2 - 12x + 13$ for $p < x < q$, where p and q are constants. The function g is defined by $g(x) = 3x + 1$ for $x < 8$.

- (b) Given that it is possible to form the composite function gf , find the least possible value of p and the greatest possible value of q . [3]

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- [1]

[illegible]

The function h is defined by $h(x) = 4x^2 - 12x + 13$ for $x < 0$.

- [3]

[illegible]



8 (a) Express $3x^2 - 12x + 14$ in the form $3(x+a)^2 + b$, where a and b are constants to be found. [2]

Handwriting practice lines with a large, faint watermark reading "dynamilis.com" diagonally across the page.

The function $f(x) = 3x^2 - 12x + 14$ is defined for $x \geq k$, where k is a constant.

(b) Find the least value of k for which the function f^{-1} exists. [1]

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For the rest of this question, you should assume that k has the value found in part (b).

(c) Find an expression for $f^{-1}(x)$. [3]

[illegible]



(d) Hence or otherwise solve the equation $ff(x) = 29$. [3]



(a) Solve the equation $f(x) = 0$, giving your solutions in the form $x = a + b\sqrt{c}$, where a , b and c are integers. [4]

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[illegible]

(a) Find an expression for $f^{-1}(x)$. [3]

$$f(x) = \frac{x^2 - 4}{x^2 + 4} \quad \text{for } x > 2.$$

(a) Find an expression for $f^{-1}(x)$. [3]

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- (b) Show that $1 - \frac{8}{x^2 + 4}$ can be expressed as $\frac{x^2 - 4}{x^2 + 4}$ and hence state the range of f . [4]

[illegible]

- (c) Explain why the composite function ff cannot be formed. [1]

Handwriting practice lines with a large diagonal watermark reading "Pastor".

- 8 The function f is defined by $f(x) = 2 - \frac{3}{4x-p}$ for $x > \frac{p}{4}$, where p is a constant.

(b) Express $f^{-1}(x)$ in the form $\frac{p}{a} - \frac{b}{cx-d}$, where a, b, c and d are integers.

[4]

(c) Hence state the value of p for which $f^{-1}(x) \equiv f(x)$.

[1]

11 The functions f and g are defined by

$$\begin{aligned} f(x) &= x^2 + 3 \quad \text{for } x > 0, \\ g(x) &= 2x + 1 \quad \text{for } x > -\frac{1}{2}. \end{aligned}$$

(a) Find an expression for $\text{fg}(x)$. [1]

[illegible]

(b) Find an expression for $(fg)^{-1}(x)$ and state the domain of $(fg)^{-1}$. [4]

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page. A faint watermark reading "Pastpaperpenz" is visible diagonally across the center.

This image shows a blank sheet of white paper designed for handwriting practice. It features multiple sets of horizontal dotted lines spaced evenly down the page. A large, light gray watermark is oriented diagonally from the bottom-left towards the top-right, displaying the text "pastpaperpenquinn.com". The watermark is semi-transparent, allowing the ruled lines to remain visible beneath it. There are no other markings, text, or illustrations on the page.

- 9 Functions f and g are defined by

$$f(x) = x + \frac{1}{x} \quad \text{for } x > 0,$$

$$g(x) = ax + 1 \quad \text{for } x \in \mathbb{R},$$

where a is a constant.

- (a) Find an expression for $gf(x)$. [1]

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- (b) Given that $gf(2) = 11$, find the value of a . [2]

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- (c) Given that the graph of $y = f(x)$ has a minimum point when $x = 1$, explain whether or not f has an inverse. [1]

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It is given instead that $a = 5$.

- (d) Find and simplify an expression for $g^{-1}f(x)$. [3]

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- (e) Explain why the composite function fg cannot be formed. [1]

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- 11** The function f is defined by $f(x) = 10 + 6x - x^2$ for $x \in \mathbb{R}$.

(a) By completing the square, find the range of f .

[3]

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- Determine the coordinates of P .

[6]

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- 9 (a) Express $2x^2 + 12x + 11$ in the form $2(x + a)^2 + b$, where a and b are constants. [2]

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The function f is defined by $f(x) = 2x^2 + 12x + 11$ for $x \leq -4$.

- (b) Find an expression for $f^{-1}(x)$ and state the domain of f^{-1} . [3]

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- This image shows a full page of primary-ruled paper. It features multiple sets of horizontal lines designed to guide handwriting. Each set consists of a solid top line, a dashed middle line, and a solid bottom line. The entire page is covered by these repeating patterns, providing a template for practicing letter formation and alignment. A faint watermark reading "paperpenquin.com" is visible diagonally across the page.

- Handwriting practice lines (dotted lines on a solid background).

- 9** The function f is defined by $f(x) = -3x^2 + 2$ for $x \leq -1$.

(a) State the range of f .

[1]

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(b) Find an expression for $f^{-1}(x)$.

[3]

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[5]

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- 9 The functions f and g are defined by

$$f(x) = x^2 - 4x + 3 \quad \text{for } x > c, \text{ where } c \text{ is a constant,}$$

$$g(x) = \frac{1}{x+1} \quad \text{for } x > -1.$$

- (a) Express $f(x)$ in the form $(x - a)^2 + b$. [2]

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It is given that f is a one-one function.

- (b) State the smallest possible value of c . [1]

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It is now given that $c = 5$.

- (c) Find an expression for $f^{-1}(x)$ and state the domain of f^{-1} . [3]

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- (d) Find an expression for $gf(x)$ and state the range of gf . [3]

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