6 The variables x and θ satisfy the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}\theta} = \left(\frac{1}{5}x + 1\right)\sin^2 2\theta,$$

and $x = 5$ when $\theta = 0$.

Solve the differential equation and obtain an expression for x in terms of θ. [7]

7 The variables x and y satisfy the differential equation

$$e^{2x}\frac{dy}{dx} = 4xy^2,$$

and it is given that $y = 1$ when $x = 0$.

Solve the differential equation, obtaining an expression for y in terms of x. [7]

7 The variables x and θ satisfy the differential equation

$$\frac{x}{\tan\theta}\frac{\mathrm{d}x}{\mathrm{d}\theta} = x^2 + 3.$$

It is given that $x = 1$ when $\theta = 0$.

Solve the differential equation, obtaining an expression for x^2 in terms of θ. [7]

7 The variables x and y satisfy the differential equation

$$\cos 2x \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4 \tan 2x}{\sin^2 3y},$$

where $0 \leqslant x < \frac{1}{4}\pi$. It is given that $y = 0$ when $x = \frac{1}{6}\pi$.

Solve the differential equation to obtain the value of x when $y = \frac{1}{6}\pi$. Give your answer correct to 3 decimal places. [8]

6 The variables x and y satisfy the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = x\mathrm{e}^{y-x},$$

and $y = 0$ when $x = 0$.

(a) Solve the differential equation, obtaining an expression for y in terms of x. [7]

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(b) Find the value of y when $x = 1$, giving your answer in the form $a - \ln b$, where a and b are integers. [1]

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6 The variables x and y satisfy the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1 + 4y^2}{\mathrm{e}^x}.$$

It is given that $y = 0$ when $x = 1$.

(a) Solve the differential equation, obtaining an expression for y in terms of x. [7]

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(b) State what happens to the value of y as x tends to infinity. [1]

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8 The variables x and y satisfy the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y^2 + 4}{x(y + 4)}$$

for $x > 0$. It is given that $x = 4$ when $y = 2\sqrt{3}$.

Solve the differential equation to obtain the value of x when $y = 2$. [8]

9 The variables x and y satisfy the differential equation

$$(x + 1)(3x + 1)\frac{dy}{dx} = y,$$

and it is given that $y = 1$ when $x = 1$.

Solve the differential equation and find the exact value of y when $x = 3$, giving your answer in a simplified form. [9]

7 The variables x and t satisfy the differential equation

$$e^{3t}\frac{dx}{dt} = \cos^2 2x,$$

for $t \geqslant 0$. It is given that $x = 0$ when $t = 0$.

(a) Solve the differential equation and obtain an expression for x in terms of t. [7]

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(b) State what happens to the value of x when t tends to infinity. [1]

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10 The variables x and t satisfy the differential equation $\frac{\mathrm{d}x}{\mathrm{d}t} = x^2(1 + 2x)$, and $x = 1$ when $t = 0$.

Using partial fractions, solve the differential equation, obtaining an expression for t in terms of x. [11]

8 At time t days after the start of observations, the number of insects in a population is N. The variation in the number of insects is modelled by a differential equation of the form $\frac{\mathrm{d}N}{\mathrm{d}t} = kN^{\frac{3}{2}} \cos 0.02t$, where k is a constant and N is a continuous variable. It is given that when $t = 0$, $N = 100$.

(a) Solve the differential equation, obtaining a relation between N, k and t. [5]

(b) Given also that $N = 625$ when $t = 50$, find the value of k. [2]

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(c) Obtain an expression for N in terms of t, and find the greatest value of N predicted by this model. [2]

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11 The variables y and θ satisfy the differential equation

$$(1+y)(1+\cos 2\theta)\frac{\mathrm{d}y}{\mathrm{d}\theta} = \mathrm{e}^{3y}.$$

It is given that $y = 0$ when $\theta = \frac{1}{4}\pi$.

Solve the differential equation and find the exact value of $\tan\theta$ when $y = 1$. [9]