

- 4 The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = \frac{xy}{1+x^2},$$

and $y = 2$ when $x = 0$.

Solve the differential equation, obtaining a simplified expression for y in terms of x . [7]

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- 8 The variables x and y satisfy the differential equation

$$e^{4x} \frac{dy}{dx} = \cos^2 3y.$$

It is given that $y = 0$ when $x = 2$.

Solve the differential equation, obtaining an expression for y in terms of x .

[7]

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- 4** The variables x and y satisfy the differential equation

$$(1 - \cos x) \frac{dy}{dx} = y \sin x.$$

It is given that $y = 4$ when $x = \pi$.

- (a) Solve the differential equation, obtaining an expression for y in terms of x . [6]

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(b) Sketch the graph of y against x for $0 < x < 2\pi$.

[1]

- 9 The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = e^{3y} \sin^2 2x.$$

It is given that $y = 0$ when $x = 0$.

Solve the differential equation and find the value of y when $x = \frac{1}{2}$.

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- 8** A certain curve is such that its gradient at a point (x, y) is proportional to $\frac{y}{x\sqrt{x}}$. The curve passes through the points with coordinates $(1, 1)$ and $(4, e)$.

- (a) By setting up and solving a differential equation, find the equation of the curve, expressing y in terms of x . [8]

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(b) Describe what happens to y as x tends to infinity.

[1]

7 (a) Given that $y = \ln(\ln x)$, show that

$$\frac{dy}{dx} = \frac{1}{x \ln x}. \quad [1]$$

[illegible]

The variables x and t satisfy the differential equation

$$x \ln x + t \frac{dx}{dt} = 0.$$

It is given that $x = e$ when $t = 2$.

(b) Solve the differential equation obtaining an expression for x in terms of t , simplifying your answer. [7]

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- (c) Hence state what happens to the value of x as t tends to infinity.

[1]

7 The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = \frac{y-1}{(x+1)(x+3)}.$$

It is given that $y = 2$ when $x = 0$.

Solve the differential equation, obtaining an expression for y in terms of x .

[9]

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- 7 The variables x and θ satisfy the differential equation

$$x \sin^2 \theta \frac{dx}{d\theta} = \tan^2 \theta - 2 \cot \theta,$$

for $0 < \theta < \frac{1}{2}\pi$ and $x > 0$. It is given that $x = 2$ when $\theta = \frac{1}{4}\pi$.

- (a) Show that $\frac{d}{d\theta}(\cot^2 \theta) = -\frac{2 \cot \theta}{\sin^2 \theta}$.

(You may assume without proof that the derivative of $\cot \theta$ with respect to θ is $-\operatorname{cosec}^2 \theta$.) [1]

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- (b) Solve the differential equation and find the value of x when $\theta = \frac{1}{6}\pi$. [7]

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- The coordinates (x, y) of a general point of a curve satisfy the differential equation

$$x \frac{dy}{dx} = (1 - 2x^2)y,$$

for $x > 0$. It is given that $y = 1$ when $x = 1$.

Solve the differential equation, obtaining an expression for y in terms of x .

[6]

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It is given that $y = 0$ when $x = 1$.

$$\frac{dy}{dx} = \frac{4 + 9y^2}{e^{2x+1}}.$$

Solve the differential equation, obtaining an expression for y in terms of x .

[7]

- (b) State what happens to the value of y as x tends to infinity. Give your answer in an exact form.

[1]

- 11 The variables x and y satisfy the differential equation

$$x^2 \frac{dy}{dx} + y^2 + y = 0.$$

It is given that $x = 1$ when $y = 1$.

- (a) Solve the differential equation to obtain an expression for y in terms of x . [8]

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- (b) State what happens to the value of y when x tends to infinity. Give your answer in an exact form. [1]

- (a) Solve the differential equation, obtaining a relation between x , t and k . [5]

$$\frac{dx}{dt} = kxe^{-0.1t},$$

where k is a positive constant. It is given that $x = 20$ at the start of the reaction.

(a) Solve the differential equation, obtaining a relation between x , t and k . [5]

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- (b) Given that $x = 40$ when $t = 10$, find the value of k and find the value approached by x as t becomes large. [3]

- 10 (a) By writing $y = \sec^3 \theta$ as $\frac{1}{\cos^3 \theta}$, show that $\frac{dy}{d\theta} = 3 \sin \theta \sec^4 \theta$. [2]

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- (b) The variables x and θ satisfy the differential equation

$$(x^2 + 9) \sin \theta \frac{d\theta}{dx} = (x + 3) \cos^4 \theta.$$

It is given that $x = 3$ when $\theta = \frac{1}{3}\pi$.

Solve the differential equation to find the value of $\cos \theta$ when $x = 0$. Give your answer correct to 3 significant figures. [8]

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