

4 The complex number u is given by $u = -1 - i\sqrt{3}$.

- (a) Express u in the form $r(\cos \theta + i \sin \theta)$, where $r > 0$ and $-\pi < \theta \leq \pi$. Give the exact values of r and θ . [2]

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The complex number v is given by $v = 5\left(\cos \frac{1}{6}\pi + i \sin \frac{1}{6}\pi\right)$.

- (b) Express the complex number $\frac{v}{u}$ in the form $re^{i\theta}$ where $r > 0$ and $-\pi < \theta \leq \pi$. [2]

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$$\frac{z-3i}{z+3i} = \frac{2-9i}{5}.$$

[5]

[illegible]

- 5 The complex number $2 + yi$ is denoted by a , where y is a real number and $y < 0$. It is given that $f(a) = a^3 - a^2 - 2a$.

(a) Find a simplified expression for $f(a)$ in terms of y . [3]

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(b) Given that $\operatorname{Re}(f(a)) = -20$, find $\arg a$. [3]

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- 4 The complex number u is defined by $u = \frac{3 + 2i}{a - 5i}$, where a is real.

(a) Express u in the Cartesian form $x + iy$, where x and y are in terms of a . [3]

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(b) Given that $\arg u = \frac{1}{4}\pi$, find the value of a . [2]

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4 Solve the equation

$$\frac{5z}{1+2i} - zz^* + 30 + 10i = 0,$$

giving your answers in the form $x + iy$, where x and y are real.

[5]

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- 5 (a) Solve the equation $z^2 - 6iz - 12 = 0$, giving the answers in the form $x + iy$, where x and y are real and exact. [3]

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- (b) On a sketch of an Argand diagram with origin O , show points A and B representing the roots of the equation in part (a). [1]

(c) Find the exact modulus and argument of each root.

[3]

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(d) Hence show that the triangle OAB is equilateral.

[1]

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3 It is given that $z = -\sqrt{3} + i$.

(a) Express z^2 in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$. [3]

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(b) The complex number ω is such that $z^2\omega$ is real and $\left|\frac{z^2}{\omega}\right| = 12$.

Find the two possible values of ω , giving your answers in the form $Re^{i\alpha}$, where $R > 0$ and $-\pi < \alpha \leq \pi$. [3]

[illegible]

- 6 The complex number u is defined by

$$u = \frac{7+i}{1-i}.$$

- (a) Express u in the form $x + iy$, where x and y are real. [3]

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- (b) Show on a sketch of an Argand diagram the points A , B and C representing u , $7 + i$ and $1 - i$ respectively. [2]

$$\tan^{-1}\left(\frac{4}{3}\right) = \tan^{-1}\left(\frac{1}{7}\right) + \frac{1}{4}\pi. \quad [3]$$

[illegible]

- By first forming a quartic equation in x or y , find the square roots of $-4+6\sqrt{5}i$ in exact Cartesian form. [5]

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5 The complex number $3 - i$ is denoted by u .

- (a) Show, on an Argand diagram with origin O , the points A , B and C representing the complex numbers u , u^* and $u^* - u$ respectively.

State the type of quadrilateral formed by the points O , A , B and C .

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- (b) Express $\frac{u^*}{u}$ in the form $x + iy$, where x and y are real.

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[illegible]

8 The complex numbers u and v are defined by $u = -4 + 2i$ and $v = 3 + i$.

- (a) Find $\frac{u}{v}$ in the form $x + iy$, where x and y are real. [3]

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- (b) Hence express $\frac{u}{v}$ in the form $re^{i\theta}$, where r and θ are exact. [2]

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In an Argand diagram, with origin O , the points A , B and C represent the complex numbers u , v and $2u + v$ respectively.

- (c) State fully the geometrical relationship between OA and BC . [2]

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- (d) Prove that angle $AOB = \frac{3}{4}\pi$. [2]

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- 5 (a) The complex number u is given by

$$u = \frac{(\cos \frac{1}{7}\pi + i \sin \frac{1}{7}\pi)^4}{\cos \frac{1}{7}\pi - i \sin \frac{1}{7}\pi}.$$

Find the exact value of $\arg u$.

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- (b) The complex numbers u and u^* are plotted on an Argand diagram.

Describe the single geometrical transformation that maps u onto u^* and state the exact value of $\arg u^*$.

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9 The complex numbers z and ω are defined by $z = 1 - i$ and $\omega = -3 + 3\sqrt{3}i$.

- (a) Express $z\omega$ in the form $a + bi$, where a and b are real and in exact surd form. [1]

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- (b) Express z and ω in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$. Give the exact values of r and θ in each case. [4]

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- (c) On an Argand diagram, the points representing ω and $z\omega$ are A and B respectively.

Prove that OAB is an isosceles right-angled triangle, where O is the origin. [2]

- (d) Using your answers to part (b), prove that $\tan \frac{5}{12}\pi = \frac{\sqrt{3}+1}{\sqrt{3}-1}$. [3]

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10 The polynomial $x^3 + 5x^2 + 31x + 75$ is denoted by $p(x)$.

- (a) Show that $(x + 3)$ is a factor of $p(x)$. [2]

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- (b) Show that $z = -1 + 2\sqrt{6}i$ is a root of $p(z) = 0$. [3]

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