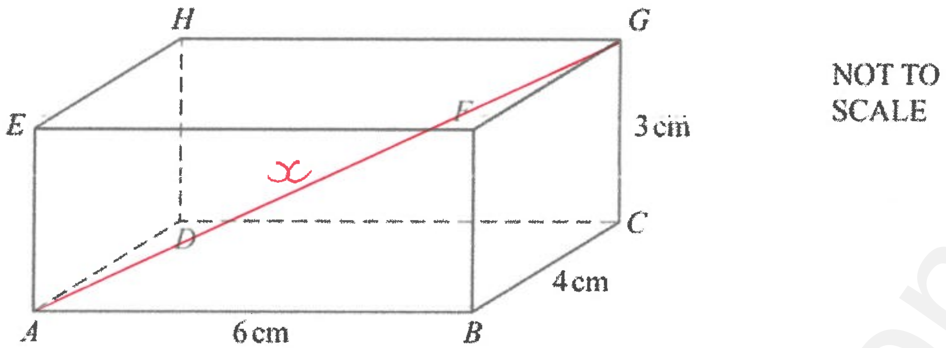
11



ABCDEFGH is a cuboid.

Find the length of *AG*.
Give your answer in surd form.

$x^2 = 6^2 + 4^2 + 3^2$

$x = \sqrt{6^2 + 4^2 + 3^2}$

$= \sqrt{36 + 16 + 9}$

$= \sqrt{61}$

$\sqrt{61}$ cm [3]

24 A cuboid measures 24 cm by 12 cm by 8 cm.

Calculate the length of a diagonal of the cuboid.

$x^2 = 24^2 + 12^2 + 8^2$

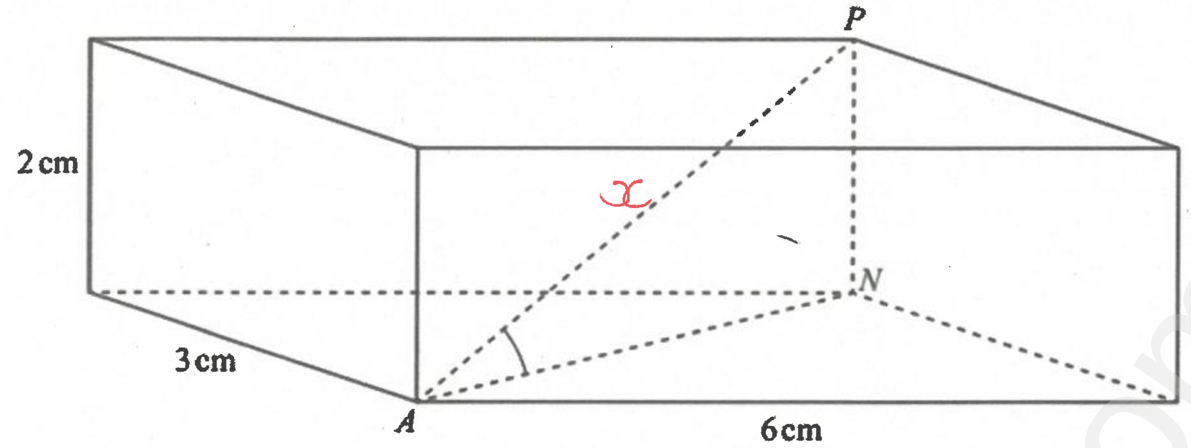
$x = \sqrt{24^2 + 12^2 + 8^2}$

$= \sqrt{784}$

$= 28$

28 cm [3]

17



The diagram shows a cuboid measuring 6 cm by 3 cm by 2 cm.

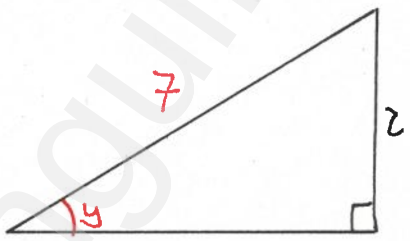
Find the sine of the angle *PAN*, the angle between the diagonal *PA* and the base of the cuboid.

Use 3D Pythag to find side AP:

$x^2 = 6^2 + 3^2 + 2^2$

$x^2 = 49$

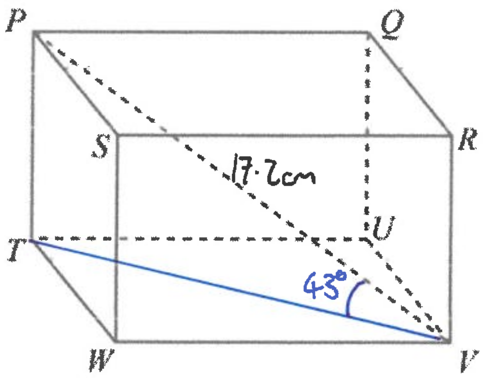
$x = 7$ cm



$\sin y = \frac{2}{7}$

Sine of angle *PAN* = $\frac{2}{7}$ [4]

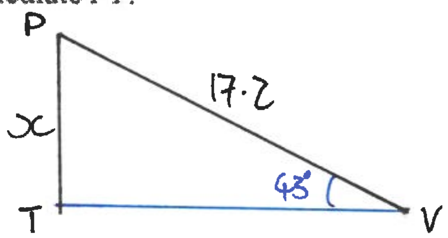
21



NOT TO SCALE

The diagram shows a cuboid *PQRSTUVW*.
PV = 17.2 cm
The angle between the line *PV* and the base *TUVW* of the cuboid is 43°.

Calculate *PT*.



$\sin 43 = \dfrac{x}{17.2}$

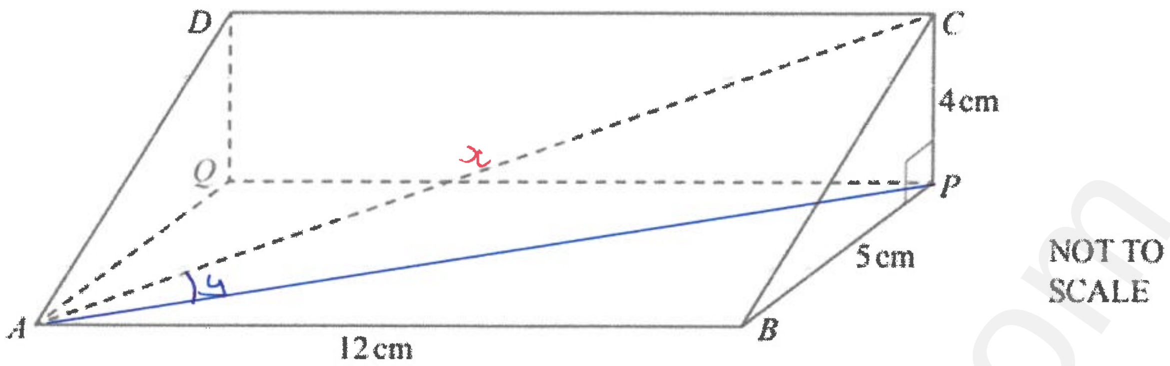
$\times 17.2 \qquad \times 17.2$

$17.2 \sin 43 = x$

$\underline{11.7} = x$

PT = ...11.7.......... cm [3]

23



The diagram shows a triangular prism.
Angle $BPC = 90°$.

(a) Calculate AC.

$x^2 = 12^2 + 5^2 + 4^2$

$x^2 = 144 + 25 + 16$

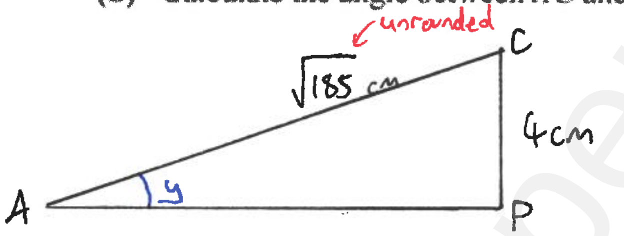
$x = \sqrt{185}$

or:

13.6 cm

$AC =$ ......13.6...... cm [3]

(b) Calculate the angle between AC and the base $ABPQ$.



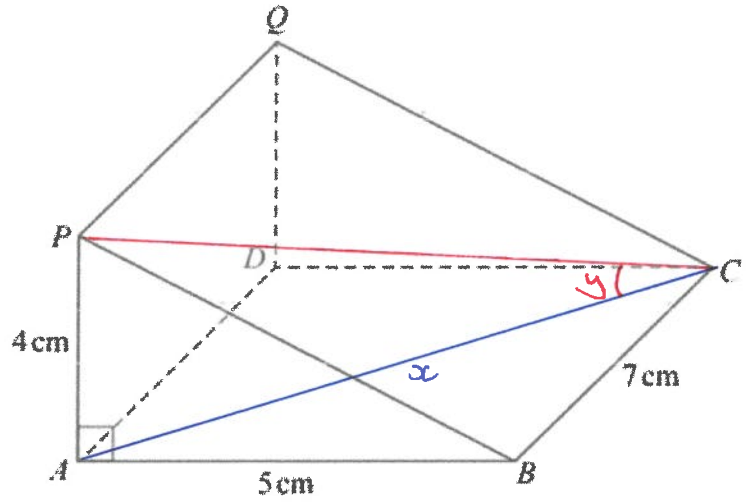
$\sin y = \frac{4}{\sqrt{185}}$

$y = \sin^{-1}\left(\frac{4}{\sqrt{185}}\right)$

$= 17.1°$

......17.1°...... [3]

22

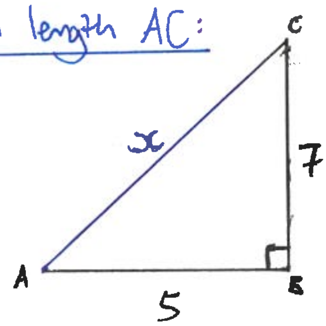


NOT TO SCALE

The diagram shows a triangular prism *ABCDQP* of length 7 cm.
The cross-section is triangle *PAB* with $PA = 4$ cm, $AB = 5$ cm and angle $PAB = 90°$.

Calculate the angle between the line *PC* and the base *ABCD*.

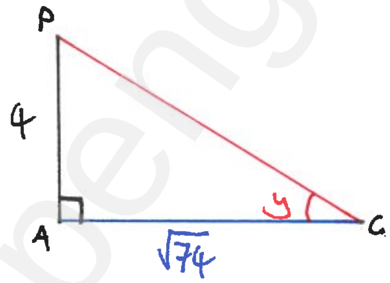
Find length AC:



$x^2 = 5^2 + 7^2$

$x^2 = 74$

$x = \sqrt{74}$



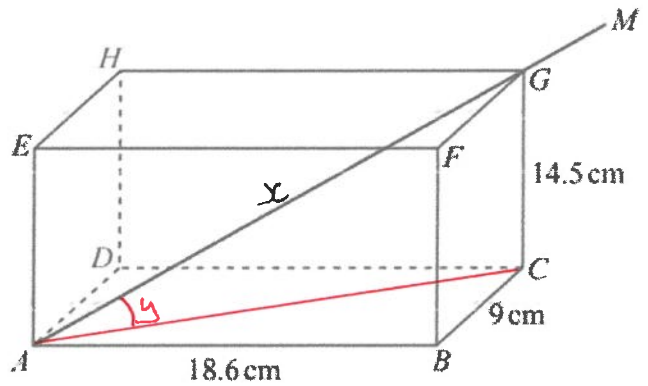
$\tan y = \frac{4}{\sqrt{74}}$

$y = \tan^{-1}\left(\frac{4}{\sqrt{74}}\right)$

$= 24.9°$

........$24.9°$........ [4]

21



NOT TO SCALE

The diagram shows an open rectangular box *ABCDEFGH*.
$AB = 18.6$ cm, $BC = 9$ cm and $CG = 14.5$ cm.
A straight stick *AGM* rests against *A* and *G* and extends outside the box to *M*.

(a) Calculate the angle between the stick and the base of the box.

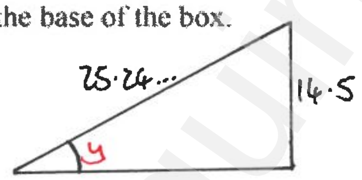
Find length AG:

$x^2 = 18.6^2 + 9^2 + 14.5^2$

$x = \sqrt{18.6^2 + 9^2 + 14.5^2}$

$x = 25.24...$

STO



$\sin y = \frac{14.5}{25.24..}$

$y = \sin^{-1}\left(\frac{14.5}{25.24...}\right)$

$= 35.1°$

35.1° .... [4]

(b) $AM = 30$ cm.

Show that $GM = 4.8$ cm, correct to 1 decimal place.

$GM = AM - AG$

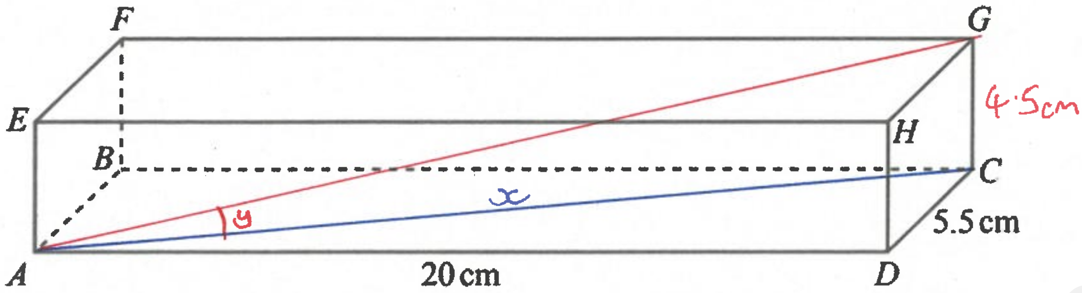
$= 30 - 25.2430...$

$= 4.7569$

$= 4.8$ to 1dp

[3]

19



NOT TO SCALE

The diagram shows cuboid *ABCDEFGH* of length 20 cm and width 5.5 cm.
The volume of the cuboid is 495 cm^3.

Find the angle between the line *AG* and the base of the cuboid *ABCD*.

Find height:

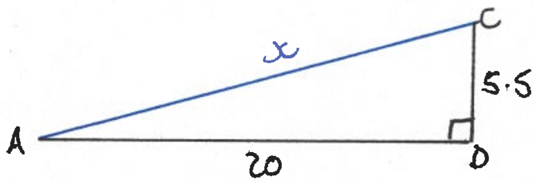
$20 \times 5.5 \times h = 495$

$110 \times h = 495$

$\div 110 \qquad \div 110$

$h = 4.5\text{cm}$

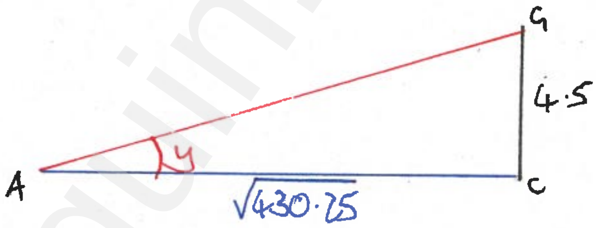
Find AC:



$x^2 = 20^2 + 5.5^2$

$x^2 = 430.25$

$x = \sqrt{430.25}$



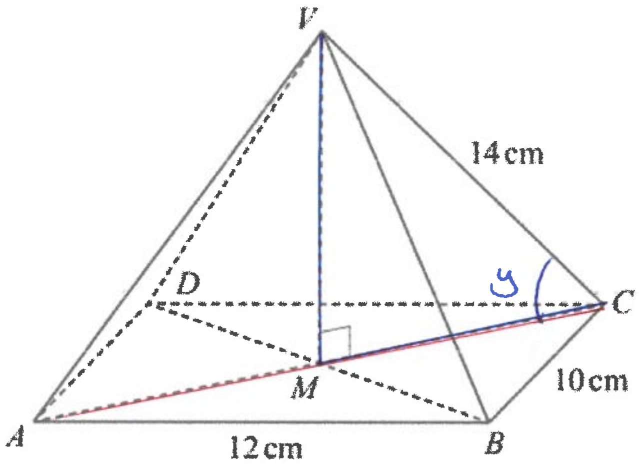
$\tan y = \dfrac{4.5}{\sqrt{430.25}}$

$y = \tan^{-1}\left(\dfrac{4.5}{\sqrt{430.25}}\right)$

$= 12.2°$

......12.2°...... [5]

23

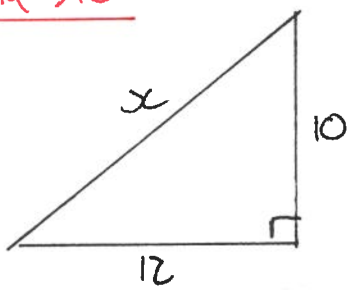


NOT TO SCALE

The diagram shows a pyramid $VABCD$ with a rectangular base.
V is vertically above M, the intersection of the diagonals AC and BD.
$AB = 12$ cm, $BC = 10$ cm and $VC = 14$ cm.

Calculate the angle that VC makes with the base $ABCD$.

Find AC:



$x^2 = 10^2 + 12^2$

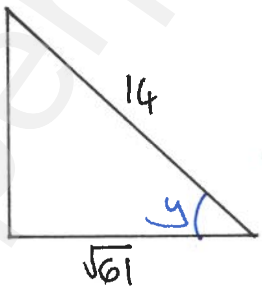
$x = \sqrt{244}$

$= 2\sqrt{61}$

$MC = \frac{1}{2} AC$

$= \frac{1}{2} \times 2\sqrt{61}$

$= \sqrt{61}$



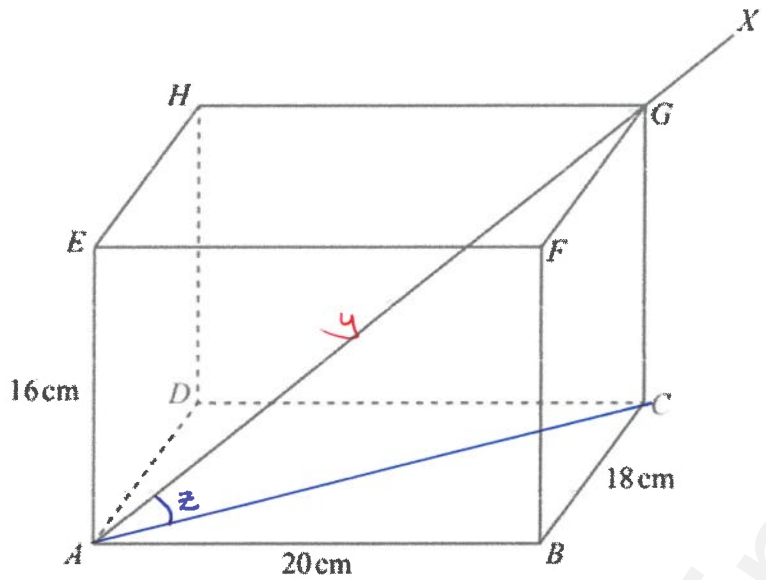
$\cos y = \frac{\sqrt{61}}{14}$

$y = \cos^{-1}\left(\frac{\sqrt{61}}{14}\right)$

$= 56.1°$

$56.1°$ .... [4]

(b)



NOT TO SCALE

The diagram shows an open box *ABCDEFGH* in the shape of a cuboid.
$AB = 20$ cm, $BC = 18$ cm and $AE = 16$ cm.
A thin rod *AGX* rests partly in the box as shown.
The rod is 40 cm long.

(i) Calculate *GX*, the length of the rod which is outside the box.

Find length AG:

$y^2 = 20^2 + 18^2 + 16^2$

$y = \sqrt{20^2 + 18^2 + 16^2}$

$= \underline{14\sqrt{5}}$

$GX = AX - AG$

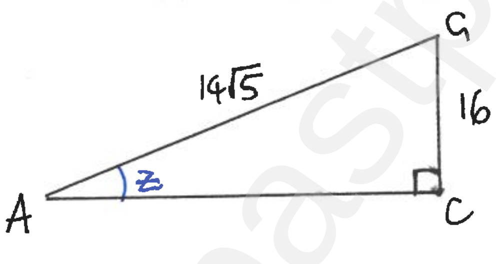
$= 40 - 14\sqrt{5}$

$= 8.695\ldots$

$= \underline{8.70}$ (3sf)

$GX =$ ....8.70........ cm [4]

(ii) Calculate the angle the rod makes with the base of the box.



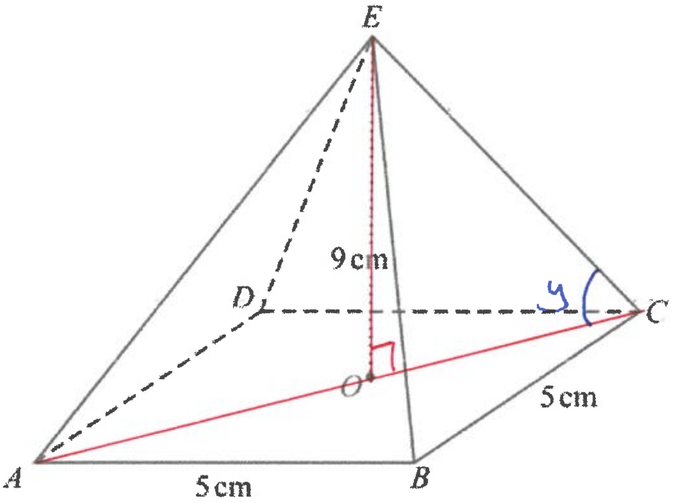
$\sin z = \dfrac{16}{14\sqrt{5}}$

$z = \sin^{-1}\left(\dfrac{16}{14\sqrt{5}}\right)$

$= \underline{30.7^\circ}$

........30.7°........ [3]

21

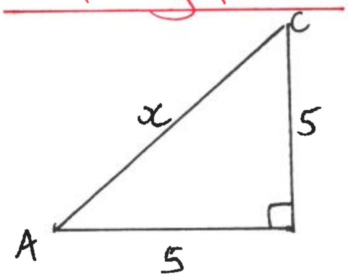


NOT TO SCALE

The diagram shows a pyramid *ABCDE*.
The pyramid has a square horizontal base *ABCD* with side 5 cm.
The vertex *E* is vertically above the centre *O* of the base.
The height *OE* of the pyramid is 9 cm.

Calculate the angle that *EC* makes with the base *ABCD*.

Find length AC:



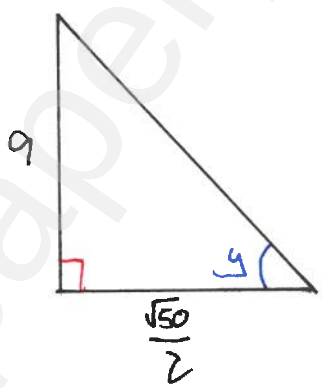
$x^2 = 5^2 + 5^2$

$x = \sqrt{25+25}$

$= \sqrt{50}$

$OC = \frac{1}{2}AC$

$= \frac{\sqrt{50}}{2}$

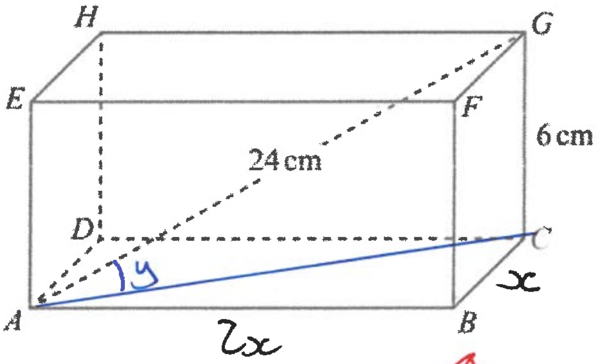


$\tan y = \frac{9}{\sqrt{50}/2}$

$y = \tan^{-1}\left(\frac{9}{\sqrt{50}/2}\right)$

$= 68.6°$

68.6° .... [4]



NOT TO SCALE

The diagram shows a cuboid *ABCDEFGH*.
$CG = 6$ cm, $AG = 24$ cm and $AB = 2BC$.

(a) Calculate *AB*.

3D Pythagoras:

$$x^2 + (2x)^2 + 6^2 = 24^2$$

$$x^2 + 4x^2 + 36 = 576$$

$$-36 \qquad -36$$

$$5x^2 = 540$$

$$\div 5 \qquad \div 5$$

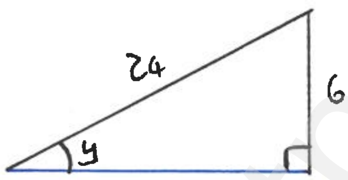
$$x^2 = 108$$

$$x = \sqrt{108}$$

$$= \underline{6\sqrt{3}} \text{ or } \underline{10.4} \text{ (3sf)}$$

$AB =$ ......10.4...... cm [4]

(b) Calculate the angle between *AG* and the base *ABCD*.



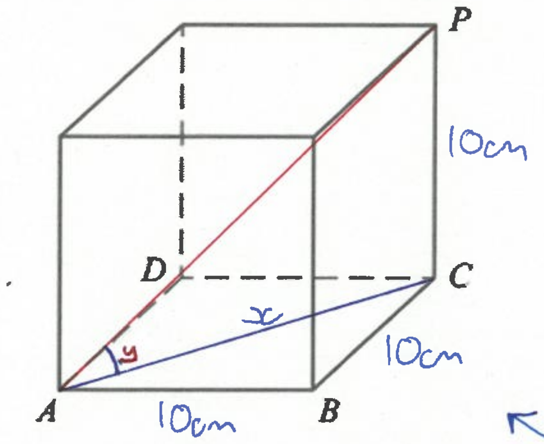
$$\sin y = \frac{6}{24}$$

$$y = \sin^{-1}\left(\frac{6}{24}\right)$$

$$= \underline{14.5°}$$

......14.5°...... [3]

27

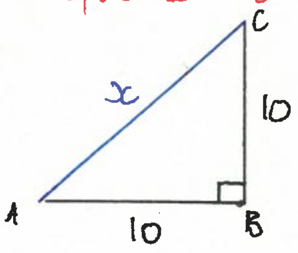


NOT TO SCALE

The diagram shows a cube.

Calculate the angle between the diagonal *AP* and the base *ABCD*.

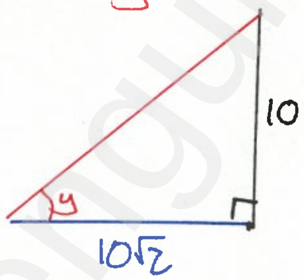
In questions like this, you can use any side length for the cube, e.g. 10:



$x^2 = 10^2 + 10^2$

$x^2 = 200$

$x = 10\sqrt{2}$



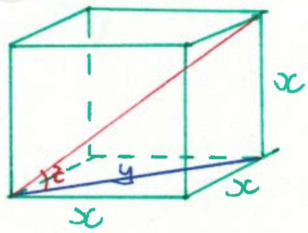
$\tan y = \frac{10}{10\sqrt{2}}$

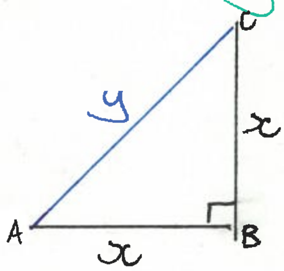
$y = \tan^{-1}\left(\frac{10}{10\sqrt{2}}\right)$

$= \underline{35.3^\circ}$

35.3° .... [4]

Alternative, algebraic method:



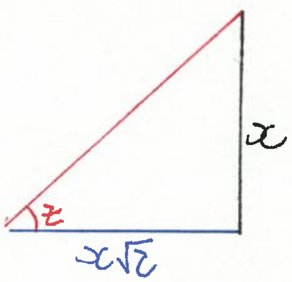


$y^2 = x^2 + x^2$

$y^2 = 2x^2$

$y = \sqrt{2x^2}$

$= \underline{x\sqrt{2}}$



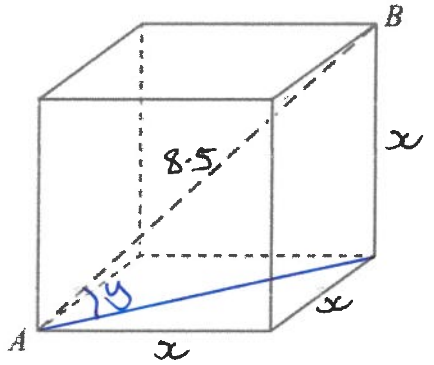
$\tan z = \frac{\cancel{x}}{\cancel{x}\sqrt{2}}$

$\tan z = \frac{1}{\sqrt{2}}$

$z = \tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$= \underline{\underline{35.3^\circ}}$

(b)



NOT TO SCALE

The diagram shows a cube.
The length of the diagonal AB is 8.5 cm.

(i) Calculate the length of an edge of the cube.

3D Pythagoras:

$x^2 + x^2 + x^2 = 8.5^2$

$3x^2 = 72.25$

$\div 3 \qquad \div 3$

$x^2 = \frac{289}{12}$

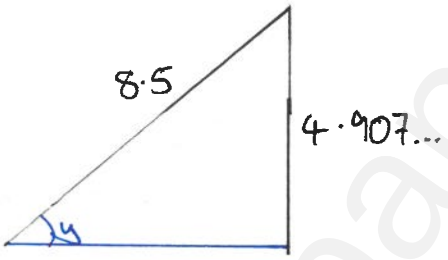
$\sqrt{} \qquad \sqrt{}$

$x = 4.91\text{cm} \ (3sf)$

↖ Store unrounded answer

.... 4.91 .... cm [3]

(ii) Calculate the angle between AB and the base of the cube.



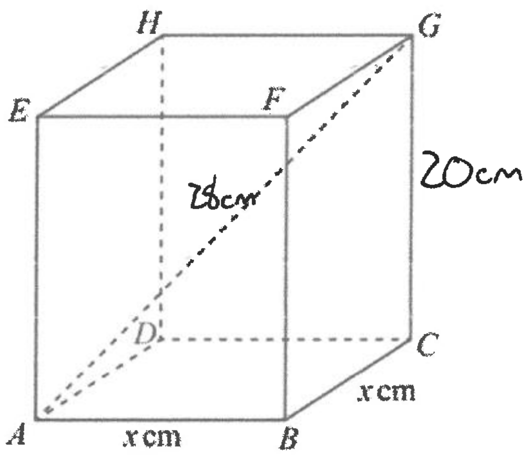
$\sin y = \frac{4.907\ldots}{8.5}$

$y = \sin^{-1}\left(\frac{4.907\ldots}{8.5}\right)$

$= 35.3°$

.... 35.3° .... [3]

10 (a)



NOT TO SCALE

ABCDEFGH is a cuboid with a square base of side x cm.
$CG = 20$ cm and $AG = 28$ cm.

Calculate the value of x.

3D Pythagoras:

$x^2 + x^2 + 20^2 = 28^2$

$2x^2 + 400 = 784$

$-400 \quad -400$

$2x^2 = 384$

$\div 2 \quad \div 2$

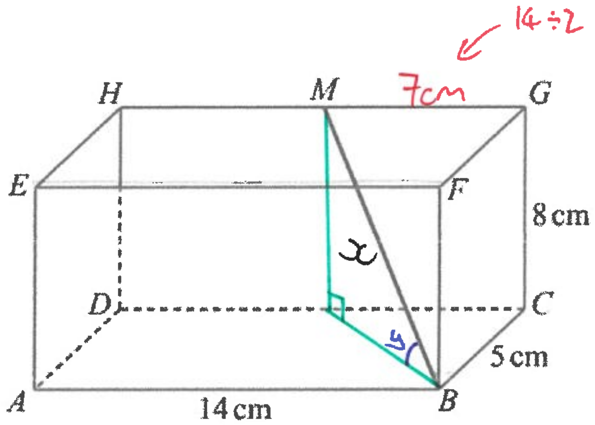
$x^2 = 192$

$\sqrt{\ } \quad \sqrt{\ }$

$x = 8\sqrt{3}$ or 13.9cm

$x =$ 13.9 cm [4]

21



NOT TO SCALE

The diagram shows a cuboid *ABCDEFGH*.
$AB = 14$ cm, $BC = 5$ cm and $CG = 8$ cm.
M is the midpoint of *HG*.

(a) Calculate *BM*.

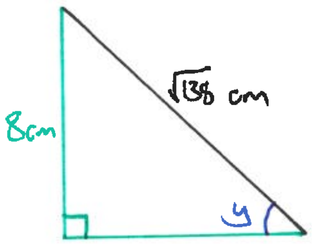
$x^2 = 5^2 + 8^2 + 7^2$

$x^2 = 138$

$x = \sqrt{138}$ or 11.7 cm

.......11.7....... cm [3]

(b) Calculate the angle that *BM* makes with the base *ABCD*.



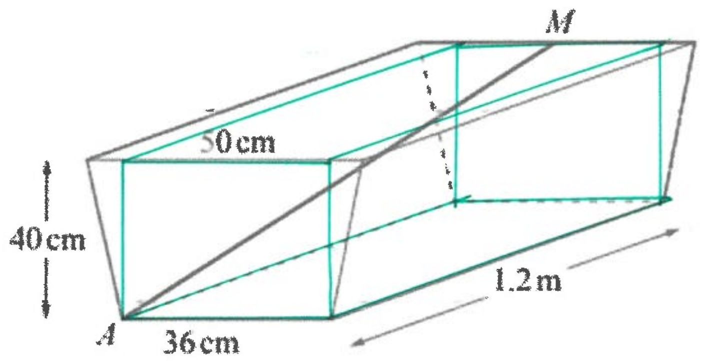
$\sin y = \frac{8}{\sqrt{138}}$

$y = \sin^{-1}\left(\frac{8}{\sqrt{138}}\right)$

$= 42.9°$

.......42.9°....... [3]

(d)

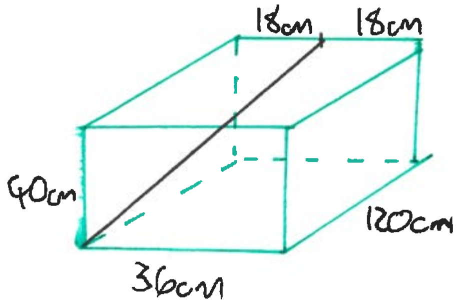


NOT TO SCALE

A steel rod AM is placed inside the empty water trough as shown in the diagram.
A is a vertex at the base of the isosceles trapezium and M is the midpoint of the top edge on the opposite face.

Calculate the length of the steel rod, AM.

Imagine chopping off the "extra" bits from the trapezium to leave a cuboid:



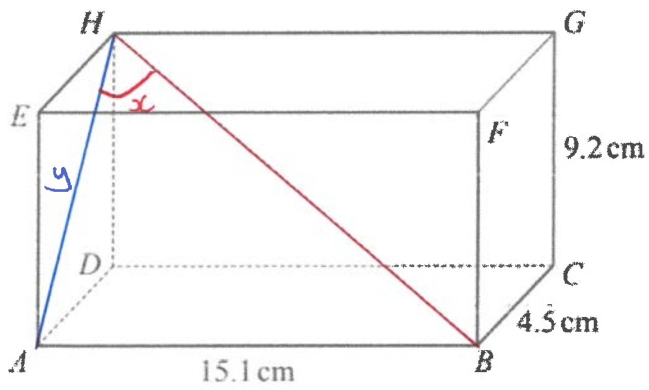
3D Pythag: $x^2 = 18^2 + 40^2 + 120^2$

$x = \sqrt{18^2 + 40^2 + 120^2}$

$= \underline{128\text{cm}}$ (3sf)

$AM =$ ......128...... cm [4]

21

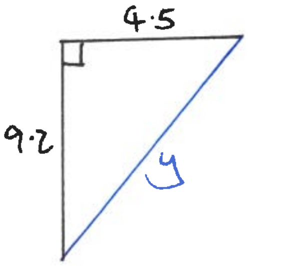


NOT TO SCALE

The diagram shows a cuboid *ABCDEFGH*.
$AB = 15.1$ cm, $BC = 4.5$ cm and $CG = 9.2$ cm.

Calculate the angle that the diagonal *BH* makes with the face *ADHE*. ← angle x on diagram.

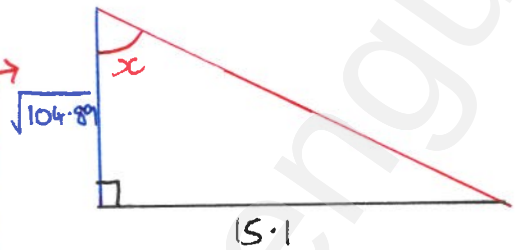
Find length AH:



$y^2 = 4.5^2 + 9.2^2$

$y^2 = 104.89$

$y = \sqrt{104.89}$



$\tan x = \dfrac{15.1}{\sqrt{104.89}}$

$x = \tan^{-1}\left(\dfrac{15.1}{\sqrt{104.89}}\right)$

$= 55.9°$ [4]