8 The nth term of a sequence is $n^2 + 12$.

Find the first three terms of this sequence.

n^2: 1 , 4 , 9

n^2+12: 13 , 16 , 21

13 , 16 , 21 [2]

5 The nth term of a sequence is $n^2 - 1$.

Find the first three terms of this sequence.

n^2: 1 , 4 , 9

n^2-1: 0 , 3 , 8

0 , 3 , 8 [2]

6 **(a)** The nth term of a sequence is $n^2 + 3n$.

Find the first three terms of this sequence.

n^2: 1 , 4 , 9

$3n$: 3 , 6 , 9

n^2+3n: 4 , 10 , 18

4 , 10 , 18 [2]

8 **(a)** The nth term of a sequence is $10 - n^2$.

Write down the first three terms of this sequence.

n^2: 1 , 4 , 9

$10-n^2$: 9 , 6 , 1

9 , 6 , 1 [2]

10 **(a)** The nth term of a sequence is $n^2 + 7$.

Find the first three terms of this sequence.

n^2: 1 , 4 , 9

$n^2 + 7$: 8 , 11 , 16

8 , 11 , 16 [2]

(b) These are the first five terms of another sequence.

4 7 12 19 28

+3 +5 +7 +9

+2 +2 +2

Find the nth term.

n^2: 1 4 9 16 25

4 7 12 19 28 (+3)

$n^2 + 3$

Half second difference: $2 \div 2 = 1$

Use this as coefficient of n^2: $1n^2$

..... [2]

8 Find the next term and the nth term for this sequence.

1, 7, 17, 31, 49, 71

+6 +10 +14 +18 +22

+4 +4 +4

$2n^2$: 2 8 18 32 50

1 7 17 31 49 (−1)

$2n^2 - 1$

Half second difference: $4 \div 2 = 2$

Coeff of n^2: $2n^2$

next term 71

nth term $2n^2 - 1$ [3]

14 Find the nth term of this sequence.

8, 17, 32, 53, 80, …

+9 +15 +21 +27

+6 +6 +6

Half second difference: $6 \div 2 = 3$

Coeff of n^2: $3n^2$

$3n^2$: 3 12 27 48 75

8 17 32 53 80 +5

$3n^2 + 5$

..... [2]

11 Find the next term and an expression for the nth term of this sequence.

35, 29, 19, 5, −13

−6 −10 −14 −18

−4 −4 −4

Half second difference: $-4 \div 2 = -2$

Coeff of n^2: $-2n^2$

$-2n^2$: −2 −8 −18 −32

35 29 19 5 +37

$-2n^2 + 37$

next term = −13

nth term = $-2n^2 + 37$ [3]

(c) Find the nth term of each sequence.

(i) 4 2 0 −2 −4 …

−2 −4 −6 −8 −10

−2 −2 −2 −2

$-2n + 6$ [2]

(ii) 1 7 17 31 49 …

+6 +10 +14 +18

+4 +4 +4

Half second difference: $4 \div 2 = 2$

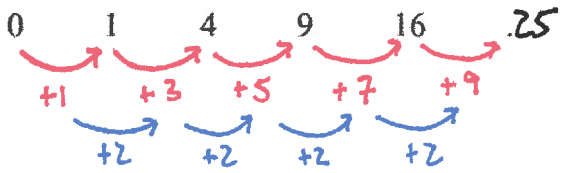
Coeff of n^2: $2n^2$

$2n^2$: 2 8 18 32 50

1 7 17 31 49 −1

$2n^2 - 1$ [2]

6 Find the next term and the nth term of this sequence.



Half second difference: $2 \div 2 = 1$

Coeff of n^2: $1n^2$

n^2: 1 (-1) 4 (-3) 9 (-5) 16 (-7) 25 (-9)
0 1 4 9 16

Find n^{th} term of differences (blue numbers):

-1 (-2) -3 (-4) -5 (-6) -7 (-8) -9 (-10)
(-2) (-2) (-2) (-2)

$-2n + 1$

next term = 25

nth term = $n^2 - 2n + 1$ [3]

or $(n-1)^2$

$n^2 - 2n + 1$

8 **(a)** 3, 9, 27, 81, ..243

×3 ×3 ×3 ×3

Write down the next term in this sequence.

243 [1]

8 **(a)** The nth term of a sequence is $n^2 - 3$.

Find the first three terms of this sequence.

n^2: 1 , 4 , 9

$n^2 - 3$: -2 , 1 , 6

-2, 1, 6 [2]

(b) These are the first five terms of a different sequence.

1 (×3) 3 (×3) 9 (×3) 27 (×3) 81

Find the nth term of this sequence.

First term: 1 (a)

Ratio : 3 (r)

use formula: $ar^{n-1} = 1 \times 3^{n-1}$

3^{n-1} [2]

11 Here are the first five terms of a sequence.

$\frac{1}{4}$ (×4) 1 (×4) 4 (×4) 16 (×4) 64 (×4) 256

(a) Find the next term.

256 [1]

(b) Find the nth term.

First term: $\frac{1}{4}$

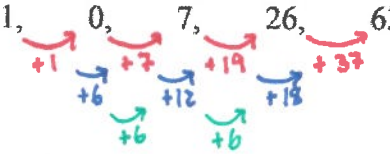
Ratio : 4

ar^{n-1} : $\frac{1}{4} \times 4^{n-1}$

$\frac{1}{4} \times 4^{n-1}$ [2]

20 Find the nth term of each sequence.

(a) $-1,\ 0,\ 7,\ 26,\ 63,\ \ldots$



Same third difference means it's a cubic sequence:

n^3: 1, 8, 27, 64, 125

These are close to the sequence, but shifted along one:

n^3: 1 → 8 → 27 → 64 → 125 →

-1 → 0 → 7 → 26 → 63

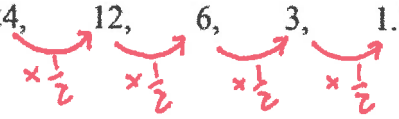
So instead of n^3, do $(n-1)^3$:

$(n-1)^3$: 0 1 8 27 64 ↓ −1

−1 0 7 26 63

$= (n-1)^3 - 1$

(b) $24,\ 12,\ 6,\ 3,\ 1.5,\ \ldots$



First term: 24

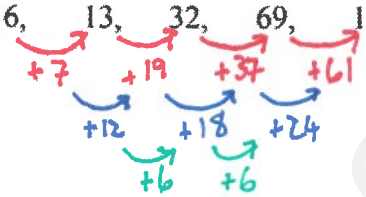
Ratio : $\frac{1}{2}$

ar^{n-1} : $24 \times \left(\frac{1}{2}\right)^{n-1}$

$24 \times \left(\frac{1}{2}\right)^{n-1}$ ..... [2]

(b) Find the nth term of each sequence.

(i) $6,\ 13,\ 32,\ 69,\ 130,\ \ldots$



→ Cubic sequence

coeff of n^3 = 6 ÷ 6

= 1

n^3: 1 8 27 64 125 ↓ +5

6 13 32 69 130

$n^3 + 5$

..... [2]

(ii) $100,\ 50,\ 25,\ 12.5,\ 6.25,\ \ldots$

a: 100

r: $\frac{1}{2}$

ar^{n-1}: $100 \times \left(\frac{1}{2}\right)^{n-1}$

$100 \times \left(\frac{1}{2}\right)^{n-1}$ ..... [2]

16 Find the *n*th term of each sequence.

(a) 8, 15, 34, 71, 132, ...

+7, +19, +37, +61

+12, +18, +24

+6, +6

→ Cubic sequence

coeff of n^3: $6 \div 6 = 1$

n^3:	1	8	27	64	125
	8	15	34	71	132

(+7)

$n^3 + 7$

..... [2]

(b) $\frac{2}{1}$, $\frac{3}{4}$, $\frac{4}{16}$, $\frac{5}{64}$, $\frac{6}{256}$, ...

Numerator: 2, 3, 4, 5, 6 (1, 2, 3, 4, 5)

+1, +1, +1, +1

$n+1$

Denominator: 1, 4, 16, 64, 256

×4, ×4, ×4, ×4

a: 1

r: 4

ar^{n-1}: $1 \times 4^{n-1} = 4^{n-1}$

$\frac{n+1}{4^{n-1}}$

..... [3]

(c) Find the *n*th term for each of these sequences.

(i) 13, 16, 19, 22, 25, ... (3, 6, 9, 12, 15)

+3, +3, +3, +3

$3n + 10$

$3n + 10$ ..... [2]

(ii) 3, 17, 55, 129, 251, ...

+14, +38, +74, +122

+24, +36, +48

+12, +12

→ Cubic sequence

coeff of n^3: $12 \div 6 = 2$

$2n^3$:	2	16	54	128	250
	3	17	55	129	251

(+1)

$2n^3 + 1$ ..... [2]

8 Find an expression for the nth term of each sequence.

(b) 1, −2, 3, −4, 5, …

Without negatives:

1, 2, 3, 4, 5

→ n

To alternate between positive and negative:

$(-1)^n$

problem: this is negative when $n=1$, positive when $n=2$, etc. We want it to be positive for $n=1$, then negative

→ $(-1)^{n+1}$

$= n \times (-1)^{n+1}$

(b) 1, 5, 25, 125, 625, …

×5 ×5 ×5 ×5

a: 1

r: 5

ar^{n-1}: $1 \times 5^{n-1}$ → 5^{n-1}

(c) The **sum of the first n terms** of another sequence is $\frac{n}{2}(5n-1)$.

(i) Use $n = 2$ to find the sum of the first two terms in this sequence.

$\frac{2}{2}(5(2)-1) = 1 \times 9$

$= 9$

9 ..... [1]

(ii) Find the 3rd term of this sequence.

Find sum of 3 terms:

$\frac{3}{2}(5(3)-1) = \frac{3}{2} \times 14$

$= 21$

Sum of 3 terms − Sum of 2 terms

$21 - 9 = 12$ ← third term

12 ..... [2]

6 (a) These are the first five terms of a sequence.

27 , 26 , 23 , 18 , 11 , 2 , -9

-1 -3 -5 -7 -9 -11

Find the next two terms in the sequence.

2 , -9 [2]

(b) The table shows information about two different sequences.

	First five terms of sequence					*n*th term
Sequence A	3	10	17	24	31	$7n-4$
Sequence B	2	11	26	47	74	$3n^2-1$

Complete the table.

A: 3 10 17 24 31

7 14 21 28 35

+7 +7 +7 +7

→ $7n-4$

[4]

B: 2 11 26 47 74

+9 +15 +21 +27

+6 +6 +6

Half 2nd diff is coeff of n^2:

$3n^2$

$3n^2$: 3 12 27 48 75

2 11 26 47 74 (-1)

→ $3n^2-1$

12 The table shows the first five terms of sequences A, B and C.

	1st term	2nd term	3rd term	4th term	5th term	nth term
Sequence A	8	3	-2	-7	-12	$-5n + 13$
Sequence B	2	$\frac{3}{2}$	$\frac{4}{3}$	$\frac{5}{4}$	$\frac{6}{5}$	$\frac{n+1}{n}$
Sequence C	$\frac{1}{2}$	1	2	4	8	$\frac{1}{2} \times 2^{n-1}$

Complete the table to show the nth term of each sequence.

A: 8, 3, −2, −7, −12 (−5, −10, −15, −20, −25; +13)

-5 -5 -5 -5

→ $-5n + 13$

[5]

B: $\frac{2}{1}$ $\frac{3}{2}$ $\frac{4}{3}$ $\frac{5}{4}$ $\frac{6}{5}$

Numerator: 2, 3, 4, 5, 6 (1, 2, 3, 4, 5; +1)

+1 +1 +1 +1

→ $n+1$

Denominator: 1 2 3 4 5

→ n

$\frac{n+1}{n}$

C: $\frac{1}{2}$, 1, 2, 4, 8

×2 ×2 ×2 ×2

a: $\frac{1}{2}$

r: 2

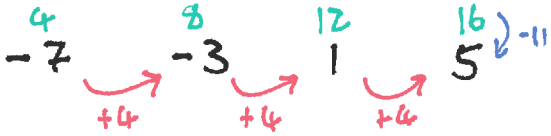
ar^{n-1}: $\frac{1}{2} \times 2^{n-1}$

6 (a)

Sequence	1st term	2nd term	3rd term	4th term	5th term		*n*th term
A	−7	−3	1	5	9		$4n-11$
B	7	13	23	37	55		$2n^2+5$
C	$\frac{2}{27}$	$\frac{3}{81}$	$\frac{4}{243}$	$\frac{5}{729}$	$\frac{6}{2187}$		$\frac{n+1}{27\times3^{n-1}}$

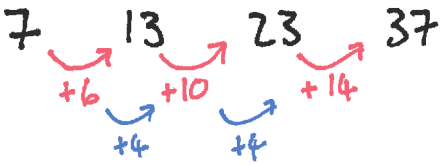
Complete the table for the three sequences.

A:



→ $4n-11$

B:



$2n^2$:

2	8	18	32
7	13	23	37

↓ +5

Half 2nd diff as coeff of n^2:

$2n^2$

→ $2n^2+5$

[10]

C:

$\frac{2}{27}$ $\frac{3}{81}$ $\frac{4}{243}$ $\frac{5}{729}$

Numerator:

→ $n+1$

Denominator:

a: 27

r: 3

ar^{n-1}: $27\times3^{n-1}$

13 The table shows the first 5 terms of sequences A, B and C.

	1st term	2nd term	3rd term	4th term	5th term	nth term
Sequence A	5	12	31	68	129	$n^3 + 4$
Sequence B	$\frac{10}{3}$	$\frac{9}{4}$	$\frac{8}{5}$	$\frac{7}{6}$	$\frac{6}{7}$	$\frac{-n+11}{n+2}$
Sequence C	4	8	16	32	64	$4 \times 2^{n-1}$

Complete the table to show the nth term of each sequence.

A: 5, 12, 31, 68, 129

+7, +19, +37, +61

+12, +18, +24

+6, +6

→ Cubic sequence.

coeff of n^3 : $6 \div 6 = 1$

n^3: 1, 8, 27, 64, 125 (+4)

5, 12, 31, 68, 129

→ $n^3 + 4$

B: $\frac{10}{3}$ $\frac{9}{4}$ $\frac{8}{5}$ $\frac{7}{6}$ $\frac{6}{7}$

[6]

Numerator: 10, 9, 8, 7, 6 (−1, −2, −3, −4, −5; +11; −1 each)

$-n + 11$

Denominator: 3, 4, 5, 6, 7 (1, 2, 3, 4, 5; +2; +1 each)

$n + 2$

$\frac{-n+11}{n+2}$

C: 4, 8, 16, 32, 64

×2, ×2, ×2, ×2

a: 4

r: 2

ar^{n-1}: $4 \times 2^{n-1}$

11 The table shows three sequences.

	1st term	2nd term	3rd term	4th term	5th term	6th term	*n*th term
Sequence *A*	28	22	16	10	4	−2	$-6n+34$
Sequence *B*	$\frac{1}{6}$	$\frac{2}{7}$	$\frac{3}{8}$	$\frac{4}{9}$	$\frac{1}{2}$	$\frac{6}{11}$	$\frac{n}{n+5}$
Sequence *C*	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8	$\frac{1}{4}\times 2^{n-1}$

Complete the table. [8]

A: 28 (−6) 22 (−12) 16 (−18) 10 (−24) 4 (−30) ↘ +34

(−6, −6, −6, −6)

$-6n+34$

B: $\frac{1}{6}$ $\frac{2}{7}$ $\frac{3}{8}$ $\frac{4}{9}$ $\frac{5}{10}$

Numerator: 1 2 3 4 5

n

Denominator: 6 (1) 7 (2) 8 (3) 9 (4) 10 (5) ↘ +5

(+1, +1, +1, +1)

$n+5$

$\frac{n}{n+5}$

C: $\frac{1}{4}$ $\frac{1}{2}$ 1 2 4

(×2, ×2, ×2, ×2)

a : $\frac{1}{4}$

r : 2

ar^{n-1} : $\frac{1}{4}\times 2^{n-1}$

11

Sequence	1st term	2nd term	3rd term	4th term	5th term	*n*th term
A	13	9	5	1	−3	$-4n+17$
B	0	7	26	63	124	n^3-1
C	$\frac{7}{8}$	$\frac{8}{16}$	$\frac{9}{32}$	$\frac{10}{64}$	$\frac{11}{128}$	$\frac{n+6}{8\times 2^{n-1}}$

(a) Complete the table for the three sequences.

A: 13 (−4) 9 (−4) 5 (−4) 1 (−16) ↓ +17

$-4n+17$

B: 0 (+7) 7 (+19) 26 (+37) 63 (+61) 124

(+12) (+18) (+24)

(+6) (+6)

Cubic Sequence

coeff of n^3 is $6 \div 6 = 1$

n^3: 1 8 27 64 ↓ −1

0 7 26 63 [10]

n^3-1

C: $\frac{7}{8}$ $\frac{8}{16}$ $\frac{9}{32}$ $\frac{10}{64}$

Numerator: 1 7, 2 8, 3 9, 4 10 ↓ +6

(+1) (+1) (+1)

$n+6$

Denominator: 8 (×2) 16 (×2) 32 (×2) 64

a: 8

r: 2

ar^{n-1}: $8\times 2^{n-1}$ → $\frac{n+6}{8\times 2^{n-1}}$

(b) One term in Sequence C is $\frac{p}{q}$.

Write down the next term in Sequence C in terms of p and q.

$\frac{7}{8}$ (+1) $\frac{8}{16}$ (+1) $\frac{9}{32}$ (+1) $\frac{10}{64}$

(×2) (×2) (×2)

$\frac{p}{q}$ (+1) $\frac{p+1}{2q}$

(×2)

$\frac{p+1}{2q}$ [2]