12 The position vector of A is $\begin{pmatrix}5\\3\end{pmatrix}$ and $\overrightarrow{BA} = \begin{pmatrix}4\\8\end{pmatrix}$.

Show that $|\overrightarrow{OB}| = 5.1$, correct to 1 decimal place.

$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$

$\overrightarrow{AB} = -\overrightarrow{BA}$

$= \begin{pmatrix}-4\\-8\end{pmatrix}$

$\overrightarrow{OB} = \begin{pmatrix}5\\3\end{pmatrix} + \begin{pmatrix}-4\\-8\end{pmatrix}$

$= \begin{pmatrix}1\\-5\end{pmatrix}$

$|\overrightarrow{OB}| = \sqrt{1^2 + (-5)^2}$

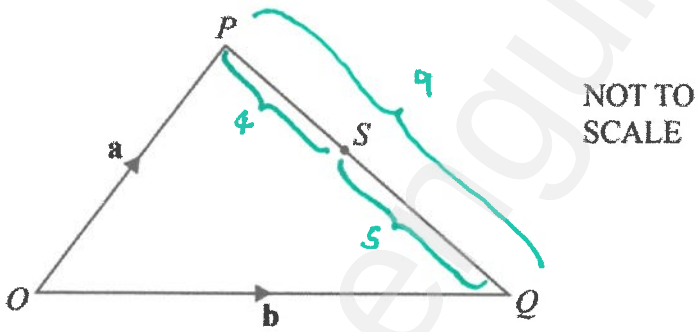
$= \sqrt{1 + 25}$

$= \sqrt{26}$

$= \underline{5.1}$

[3]

18



S is a point on PQ such that $PS : SQ = 4 : 5$.

Find $\overrightarrow{OS}$, in terms of **a** and **b**, in its simplest form.

$\overrightarrow{OS} = \overrightarrow{OP} + \overrightarrow{PS}$

$\overrightarrow{PS} = \frac{4}{9}\overrightarrow{PQ}$

$\overrightarrow{PQ} = -\underline{a} + \underline{b}$

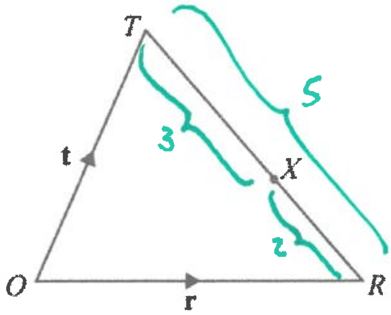
$\overrightarrow{PS} = \frac{4}{9}(-\underline{a} + \underline{b})$

$= -\frac{4}{9}\underline{a} + \frac{4}{9}\underline{b}$

$\overrightarrow{OS} = \underline{a} + -\frac{4}{9}\underline{a} + \frac{4}{9}\underline{b}$

$= \frac{5}{9}\underline{a} + \frac{4}{9}\underline{b}$

$\overrightarrow{OS} = \frac{5}{9}\underline{a} + \frac{4}{9}\underline{b}$ [2]



NOT TO SCALE

ORT is a triangle.
X is a point on *TR* so that $TX : XR = 3 : 2$.
O is the origin, $\overrightarrow{OR} = \mathbf{r}$ and $\overrightarrow{OT} = \mathbf{t}$.

Find the position vector of *X*.
Give your answer in terms of **r** and **t** in its simplest form.

$\overrightarrow{OX} = \overrightarrow{OT} + \overrightarrow{TX}$

$\overrightarrow{TX} = \frac{3}{5}\overrightarrow{TR}$

$\overrightarrow{TR} = -\underline{t} + \underline{r}$

$\overrightarrow{TX} = \frac{3}{5}(-\underline{t} + \underline{r})$

$\overrightarrow{TX} = -\frac{3}{5}\underline{t} + \frac{3}{5}\underline{r}$

$\overrightarrow{OX} = \underline{t} + \frac{-3}{5}\underline{t} + \frac{3}{5}\underline{r}$

$= \frac{2}{5}\underline{t} + \frac{3}{5}\underline{r}$

$\frac{2}{5}\underline{t} + \frac{3}{5}\underline{r}$ [3]

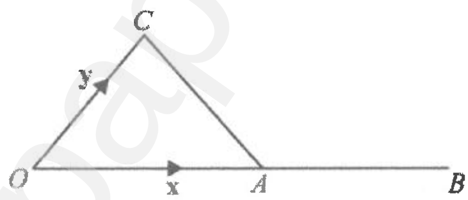
16 **(a)** Find the magnitude of the vector $\begin{pmatrix} -4 \\ 5 \end{pmatrix}$.

$\sqrt{(-4)^2 + 5^2}$

$= \sqrt{16 + 25}$ $= \sqrt{41}$

$\sqrt{41}$ [2]

(b)



NOT TO SCALE

The diagram shows a triangle *OAC*.
A is the midpoint of the straight line *OB*.
$\overrightarrow{OA} = \mathbf{x}$ and $\overrightarrow{OC} = \mathbf{y}$.

Find $\overrightarrow{CB}$ in terms of **x** and **y**.

$\overrightarrow{CB} = \overrightarrow{CO} + \overrightarrow{OB}$

$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$

$= \underline{x} + \underline{x}$

$= 2\underline{x}$

$\overrightarrow{CB} = -\underline{y} + 2\underline{x}$

$\overrightarrow{CB} =$ $2\underline{x} - \underline{y}$ [1]

17 **(a)** **(i)** $\mathbf{m} = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$

Find $3\mathbf{m}$.

$3\begin{pmatrix} 5 \\ 7 \end{pmatrix} = \begin{pmatrix} 15 \\ 21 \end{pmatrix}$

$\begin{pmatrix} 15 \\ 21 \end{pmatrix}$ [1]

(ii) $\overrightarrow{VW} = \begin{pmatrix} 10 \\ -24 \end{pmatrix}$

Find $|\overrightarrow{VW}|$.

$\sqrt{10^2 + (-24)^2}$

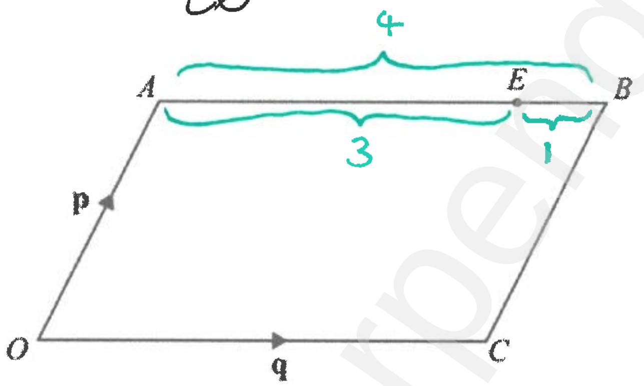
$= \sqrt{100 + 576}$

$= \sqrt{676}$

$= 26$

26 [2]

(b)



NOT TO SCALE

$OABC$ is a parallelogram.
$\overrightarrow{OA} = \mathbf{p}$ and $\overrightarrow{OC} = \mathbf{q}$.
E is the point on AB such that $AE : EB = 3 : 1$.

Find $\overrightarrow{OE}$, in terms of $\mathbf{p}$ and $\mathbf{q}$, in its simplest form.

$\overrightarrow{OE} = \overrightarrow{OA} + \overrightarrow{AE}$

$\overrightarrow{AE} = \frac{3}{4}\overrightarrow{AB}$

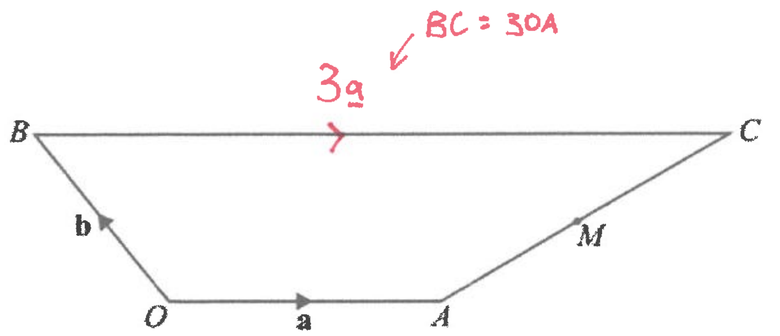
$\overrightarrow{AB} = q$ (parallelogram)

$\overrightarrow{AE} = \frac{3}{4}q$

$\overrightarrow{OE} = p + \frac{3}{4}q$

$\overrightarrow{OE} =$ $p + \frac{3}{4}q$ [2]

23



NOT TO SCALE

In the diagram, OA is parallel to BC.
$BC = 3OA$
M is the midpoint of AC.
The position vector of A is **a** and the position vector of B is **b**.

Find the position vector of M.
Give your answer in terms of **a** and **b**, in its simplest form.

$\vec{OM} = \vec{OA} + \vec{AM}$

$\vec{AM} = \frac{1}{2}\vec{AC}$

$\vec{AC} = \vec{AO} + \vec{OB} + \vec{BC}$

$= -\underline{a} + \underline{b} + 3\underline{a}$

$= 2\underline{a} + \underline{b}$

$\vec{AM} = \frac{1}{2}(2\underline{a} + \underline{b})$

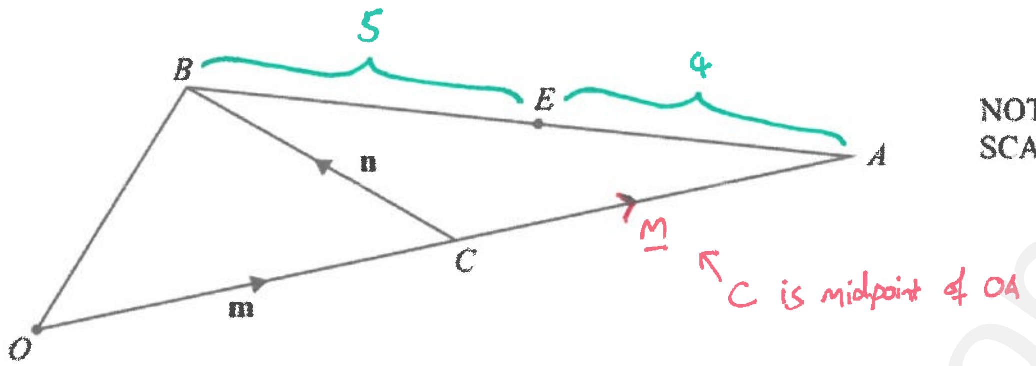
$\vec{AM} = \underline{a} + \frac{1}{2}\underline{b}$

$\vec{OM} = \underline{a} + \underline{a} + \frac{1}{2}\underline{b}$

$= 2\underline{a} + \frac{1}{2}\underline{b}$

$2\underline{a} + \frac{1}{2}\underline{b}$ .... [3]

21



NOT TO SCALE

OAB is a triangle.
C is the midpoint of *OA*.
$\overrightarrow{OC} = \mathbf{m}$ and $\overrightarrow{CB} = \mathbf{n}$.
E lies on *AB* and $AE : EB = 4 : 5$.

Find, in terms of **m** and **n**, the position vector of *E*.
Give your answer in its simplest form.

$\overrightarrow{OE} = \overrightarrow{OB} + \overrightarrow{BE}$

$\overrightarrow{OB} = \underline{m} + \underline{n}$

$\overrightarrow{BE} = \frac{5}{9}\overrightarrow{BA}$

$\overrightarrow{BA} = \overrightarrow{BC} + \overrightarrow{CA}$

$= -\underline{n} + \underline{m}$

$= \underline{m} - \underline{n}$

$\overrightarrow{BE} = \frac{5}{9}(\underline{m} - \underline{n})$

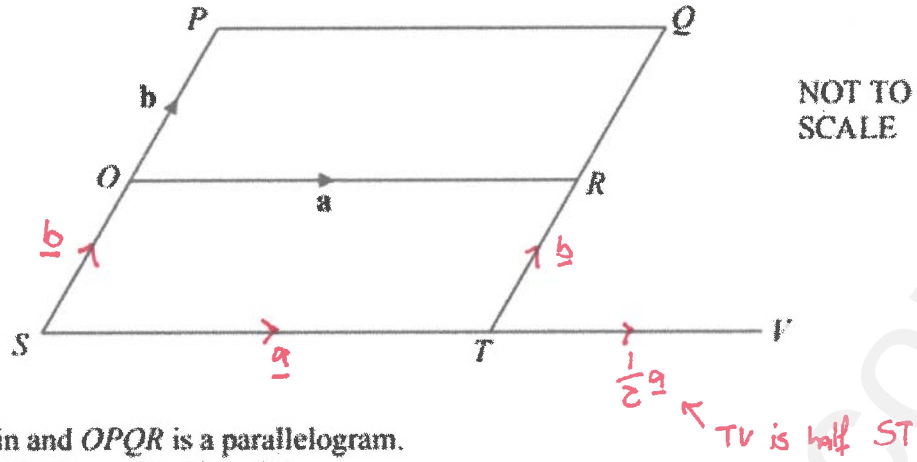
$= \frac{5}{9}\underline{m} - \frac{5}{9}\underline{n}$

$\overrightarrow{OE} = \underline{m} + \underline{n} + \frac{5}{9}\underline{m} - \frac{5}{9}\underline{n}$

$= \frac{14}{9}\underline{m} + \frac{4}{9}\underline{n}$

$\frac{14}{9}\underline{m} + \frac{4}{9}\underline{n}$ [4]

21



NOT TO SCALE

O is the origin and $OPQR$ is a parallelogram.
SOP is a straight line with $SO = OP$.
TRQ is a straight line with $TR = RQ$.
STV is a straight line and $ST : TV = 2 : 1$.
$\overrightarrow{OR} = \mathbf{a}$ and $\overrightarrow{OP} = \mathbf{b}$.

(a) Find, in terms of **a** and **b**, in its simplest form,

(i) the position vector of T,

$$\overrightarrow{OT} = \overrightarrow{OS} + \overrightarrow{ST}$$
$$= -\underline{b} + \underline{a}$$
$$= \underline{a} - \underline{b}$$

$\underline{a} - \underline{b}$ [2]

(ii) $\overrightarrow{RV}$.

$$\overrightarrow{RV} = \overrightarrow{RT} + \overrightarrow{TV}$$
$$= -\underline{b} + \tfrac{1}{2}\underline{a}$$

$\overrightarrow{RV} = \tfrac{1}{2}\underline{a} - \underline{b}$ [1]

(b) Show that PT is parallel to RV.

$$\overrightarrow{PT} = \overrightarrow{PS} + \overrightarrow{ST}$$
$$= -2\underline{b} + \underline{a}$$
$$= \underline{a} - 2\underline{b}$$

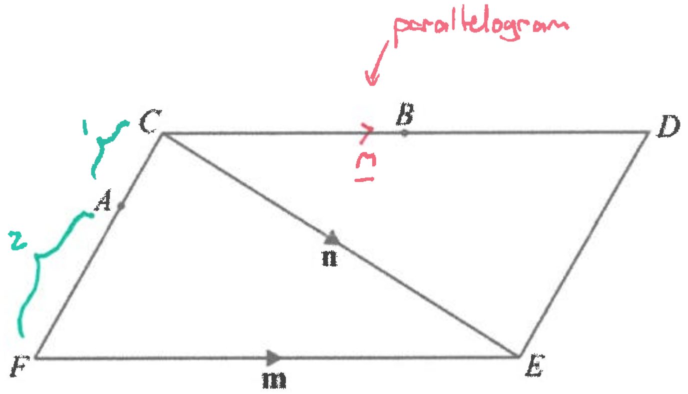
If parallel, $\overrightarrow{PT} = k\overrightarrow{RV}$:

$$\underline{a} - 2\underline{b} = k\left(\tfrac{1}{2}\underline{a} - \underline{b}\right)$$

$\rightarrow k = 2$

so $\overrightarrow{PT}$ is parallel to $\overrightarrow{RV}$ [2]

23 (a)



NOT TO SCALE

The diagram shows a parallelogram $CDEF$.
$\overrightarrow{FE} = \mathbf{m}$ and $\overrightarrow{CE} = \mathbf{n}$.
B is the midpoint of CD.
$FA = 2AC$

Find an expression, in terms of $\mathbf{m}$ and $\mathbf{n}$, for $\overrightarrow{AB}$.
Give your answer in its simplest form.

$\overrightarrow{AB} = \overrightarrow{AC} + \overrightarrow{CB}$

$\overrightarrow{AC} = \frac{1}{3}\overrightarrow{FC}$

$\overrightarrow{FC} = \underline{m} - \underline{n}$

$\overrightarrow{AC} = \frac{1}{3}(\underline{m} - \underline{n})$

$= \frac{1}{3}\underline{m} - \frac{1}{3}\underline{n}$

$\overrightarrow{CB} = \frac{1}{2}\overrightarrow{CD}$

$= \frac{1}{2}\underline{m}$

$\overrightarrow{AB} = \frac{1}{3}\underline{m} - \frac{1}{3}\underline{n} + \frac{1}{2}\underline{m}$

$= \frac{5}{6}\underline{m} - \frac{1}{3}\underline{n}$

$\overrightarrow{AB} =$ $\frac{5}{6}\underline{m} - \frac{1}{3}\underline{n}$ [3]

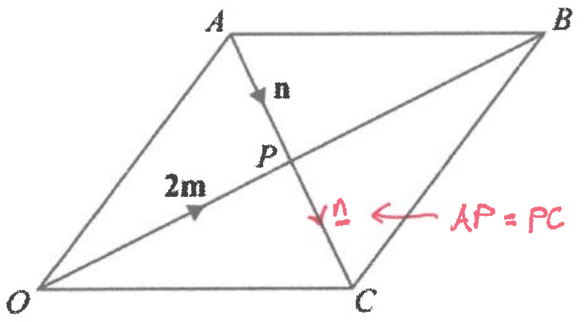
(b) $\overrightarrow{GH} = \frac{5}{6}(2\mathbf{p} + \mathbf{q})$ $\overrightarrow{JK} = \frac{5}{18}(2\mathbf{p} + \mathbf{q})$

Write down **two** facts about vectors $\overrightarrow{GH}$ and $\overrightarrow{JK}$.

① They are parallel

② $\overrightarrow{GH} = 3\overrightarrow{JK}$ [2]

21



NOT TO SCALE

$\leftarrow AP = PC$ (diagonals of a rhombus bisect each other)

$OABC$ is a rhombus and O is the origin.
The diagonals of the rhombus intersect at P.
$\overrightarrow{OP} = 2\mathbf{m}$ and $\overrightarrow{AP} = \mathbf{n}$.

(a) Find, in terms of $\mathbf{m}$ and $\mathbf{n}$, in its simplest form

(i) $\overrightarrow{OA}$

$\overrightarrow{OA} = \overrightarrow{OP} + \overrightarrow{PA}$

$= 2\underline{m} - \underline{n}$

$\overrightarrow{OA} =$ .... $2\underline{m} - \underline{n}$ .... [1]

(ii) $\overrightarrow{OC}$.

$\overrightarrow{OC} = \overrightarrow{OP} + \overrightarrow{PC}$

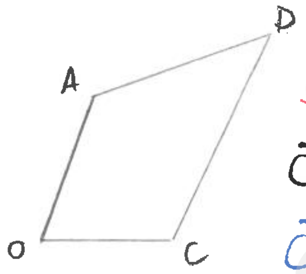
$= 2\underline{m} + \underline{n}$

$\overrightarrow{OC} =$ .... $2\underline{m} + \underline{n}$ .... [1]

(b) D is the point such that $\overrightarrow{AD} = 10\mathbf{m} - 3\mathbf{n}$.

Show that $OADC$ is a trapezium.

Need to show OADC has a pair of parallel lines. $\overrightarrow{AD}$ is not parallel to $\overrightarrow{OC}$ because no value of k works for $(10\underline{m} - 3\underline{n}) = k(2\underline{m} + \underline{n})$, so it must be $\overrightarrow{OA}$ and $\overrightarrow{CD}$:



$\overrightarrow{CD} = \overrightarrow{CO} + \overrightarrow{OA} + \overrightarrow{AD}$

$\overrightarrow{CO} = -\overrightarrow{OC}$

$= -2\underline{m} - \underline{n}$

$\overrightarrow{CD} = -2\underline{m} - \underline{n} + 2\underline{m} - \underline{n} + 10\underline{m} - 3\underline{n}$

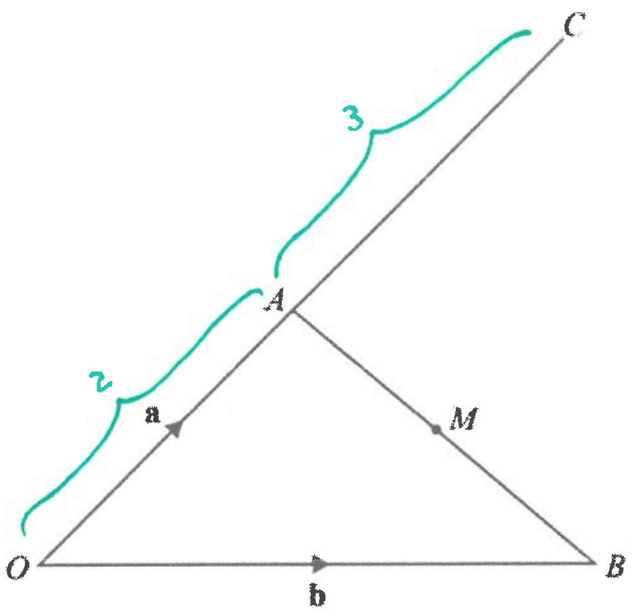
$= 10\underline{m} - 5\underline{n}$

Are they parallel?

$\overrightarrow{CD} = k\overrightarrow{OA}$

$10\underline{m} - 5\underline{n} = k(2\underline{m} - \underline{n})$ [3]

works when $k = 5$, therefore parallel, so OADC is a trapezium.



NOT TO SCALE

The diagram shows a triangle OAB and a straight line OAC.
$OA : OC = 2 : 5$ and M is the midpoint of AB.
$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

Find, in terms of **a** and **b**, in its simplest form

(a) $\overrightarrow{AB}$,

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= -\underline{a} + \underline{b}$

$= \underline{b} - \underline{a}$

$\overrightarrow{AB} =$ ...... $\underline{b} - \underline{a}$ ...... [1]

(b) $\overrightarrow{MC}$.

$\overrightarrow{MC} = \overrightarrow{MA} + \overrightarrow{AC}$

$\overrightarrow{MA} = \frac{1}{2}\overrightarrow{BA}$

$\overrightarrow{BA} = -\overrightarrow{AB}$

$= -(\underline{b} - \underline{a})$

$= \underline{a} - \underline{b}$

$\overrightarrow{MA} = \frac{1}{2}(\underline{a} - \underline{b})$

$= \frac{1}{2}\underline{a} - \frac{1}{2}\underline{b}$

$\overrightarrow{AC} = \frac{3}{2}\overrightarrow{OA}$

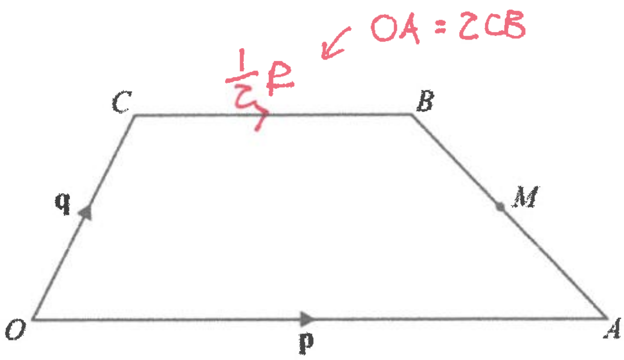
$= \frac{3}{2}\underline{a}$

$\overrightarrow{MC} = \frac{1}{2}\underline{a} - \frac{1}{2}\underline{b} + \frac{3}{2}\underline{a}$

$= 2\underline{a} - \frac{1}{2}\underline{b}$

$\overrightarrow{MC} =$ ...... $2\underline{a} - \frac{1}{2}\underline{b}$ ...... [3]

(b)



NOT TO SCALE

OABC is a trapezium and *O* is the origin.
M is the midpoint of *AB*.
$\overrightarrow{OA} = \mathbf{p}$, $\overrightarrow{OC} = \mathbf{q}$ and $OA = 2CB$.

Find, in terms of **p** and **q**, the position vector of *M*.
Give your answer in its simplest form.

$$\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM}$$

$$\overrightarrow{AM} = \frac{1}{2}\overrightarrow{AB}$$

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OC} + \overrightarrow{CB}$$

$$= -p + q + \frac{1}{2}p$$

$$= q - \frac{1}{2}p$$

$$\overrightarrow{AM} = \frac{1}{2}\left(q - \frac{1}{2}p\right)$$

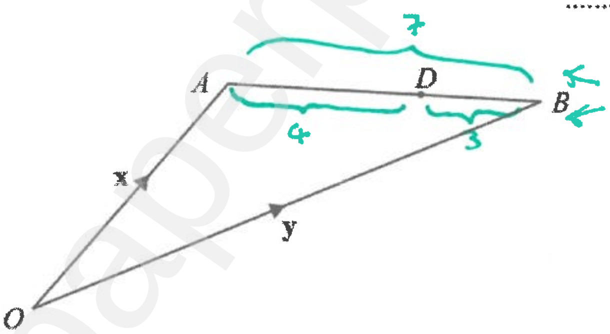
$$= \frac{1}{2}q - \frac{1}{4}p$$

$$\overrightarrow{OM} = p + \frac{1}{2}q - \frac{1}{4}p$$

$$= \frac{3}{4}p + \frac{1}{2}q$$

.... $\frac{3}{4}p + \frac{1}{2}q$ .... [3]

22



from final step.

NOT TO SCALE

$\overrightarrow{OA} = \mathbf{x}$, $\overrightarrow{OB} = \mathbf{y}$ and $\overrightarrow{OD} = \frac{3}{7}\mathbf{x} + \frac{4}{7}\mathbf{y}$.

Calculate the ratio $AD : DB$.

$$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD}$$

$$\overrightarrow{AD} = k\overrightarrow{AB}$$

$$\overrightarrow{AB} = -x + y$$

$$= y - x$$

$$\overrightarrow{AD} = k(y - x)$$

$$= ky - kx$$

$$\overrightarrow{OD} = x + ky - kx$$

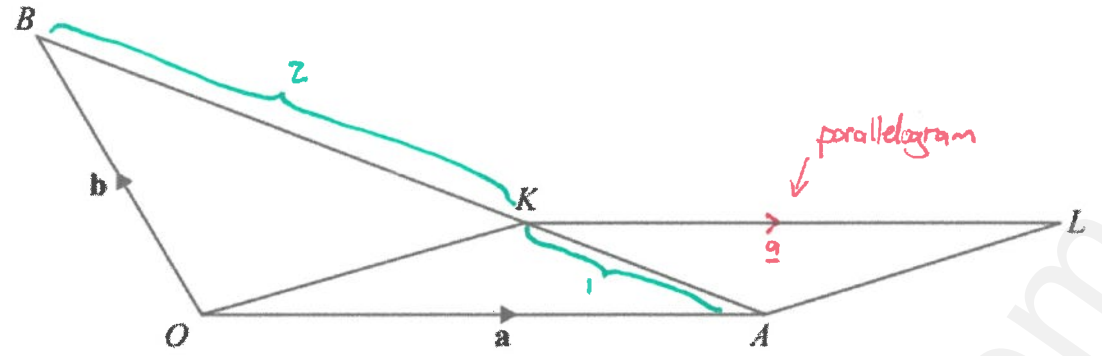
$$\frac{3}{7}x + \frac{4}{7}y = (1-k)x + ky$$

$$k = \frac{4}{7}$$

So $\overrightarrow{AD} = \frac{4}{7}\overrightarrow{AB}$ → see diagram

.... 4 : 3 .... [2]

26



NOT TO SCALE

The diagram shows a triangle OAB and a parallelogram $OALK$.
The position vector of A is **a** and the position vector of B is **b**.
K is a point on AB so that $AK : KB = 1 : 2$.

Find the position vector of L, in terms of **a** and **b**.
Give your answer in its simplest form.

$\vec{OL} = \vec{OA} + \vec{AL}$

$\vec{AL} = \vec{AK} + \vec{KL}$

$\vec{AK} = \frac{1}{3}\vec{AB}$

$\vec{AB} = -\underline{a} + \underline{b}$

$= \underline{b} - \underline{a}$

$\vec{AK} = \frac{1}{3}(\underline{b} - \underline{a})$

$= \frac{1}{3}\underline{b} - \frac{1}{3}\underline{a}$

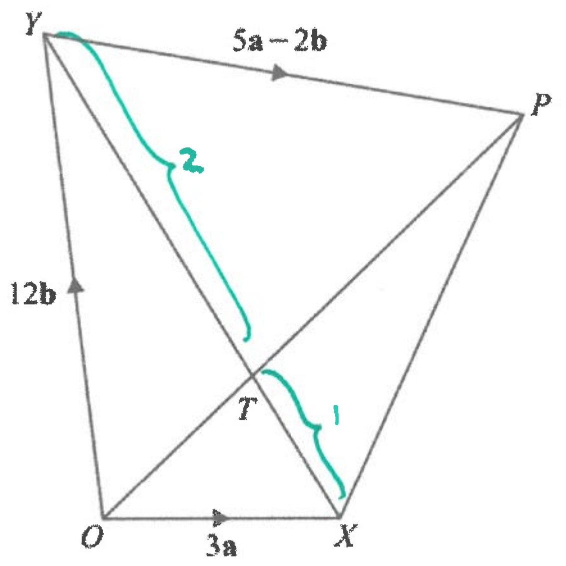
$\vec{AL} = \frac{1}{3}\underline{b} - \frac{1}{3}\underline{a} + \underline{a}$

$= \frac{2}{3}\underline{a} + \frac{1}{3}\underline{b}$

$\vec{OL} = \underline{a} + \frac{2}{3}\underline{a} + \frac{1}{3}\underline{b}$

$= \frac{5}{3}\underline{a} + \frac{1}{3}\underline{b}$

.... $\frac{5}{3}\underline{a} + \frac{1}{3}\underline{b}$ .... [4]



The diagram shows a quadrilateral $OXPY$ with diagonals meeting at T.
$\overrightarrow{OX} = 3\mathbf{a}$ and $\overrightarrow{OY} = 12\mathbf{b}$.

(a) Find $\overrightarrow{XY}$ in terms of **a** and **b**.

$\vec{XY} = \vec{XO} + \vec{OY}$

$= -3\underline{a} + 12\underline{b}$

$-3\underline{a} + 12\underline{b}$ [1]

(b) $XT : TY = 1 : 2$ and $\overrightarrow{YP} = 5\mathbf{a} - 2\mathbf{b}$.

Find the ratio $OT : TP$.

$\vec{OT} = \vec{OX} + \vec{XT}$

$\vec{XT} = \frac{1}{3}\vec{XY}$

$= \frac{1}{3}(12\underline{b} - 3\underline{a})$

$= 4\underline{b} - \underline{a}$

$\vec{OT} = 3\underline{a} + 4\underline{b} - \underline{a}$

$= 2\underline{a} + 4\underline{b}$

$\vec{TP} = \vec{TY} + \vec{YP}$

$\vec{TY} = \frac{2}{3}\vec{XY}$

$= \frac{2}{3}(12\underline{b} - 3\underline{a})$

$= 8\underline{b} - 2\underline{a}$

$\vec{TP} = 8\underline{b} - 2\underline{a} + 5\underline{a} - 2\underline{b}$

$= 3\underline{a} + 6\underline{b}$

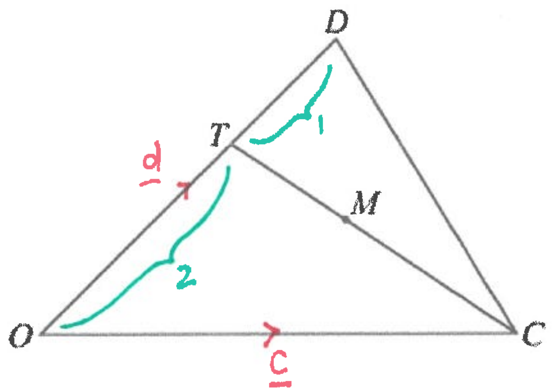
$\vec{OT} : \vec{TP}$

$2\underline{a} + 4\underline{b} : 3\underline{a} + 6\underline{b}$

compare $\underline{a}$'s or $\underline{b}$'s: $2\underline{a} : 3\underline{a} = 2 : 3$

2 : 3 [4]

(d)



NOT TO SCALE

In the diagram, O is the origin, $OT = 2TD$ and M is the midpoint of TC.
$\overrightarrow{OC} = \mathbf{c}$ and $\overrightarrow{OD} = \mathbf{d}$.

Find the position vector of M.
Give your answer in terms of $\mathbf{c}$ and $\mathbf{d}$ in its simplest form.

$\overrightarrow{OM} = \overrightarrow{OT} + \overrightarrow{TM}$

$\overrightarrow{OT} = \frac{2}{3}\overrightarrow{OD}$

$= \frac{2}{3}\underline{d}$

$\overrightarrow{TM} = \frac{1}{2}\overrightarrow{TC}$

$\overrightarrow{TC} = \overrightarrow{TO} + \overrightarrow{OC}$

$= \frac{-2}{3}\underline{d} + \underline{c}$

$\overrightarrow{TM} = \frac{1}{2}\left(\frac{-2}{3}\underline{d} + \underline{c}\right)$

$= \frac{-2}{6}\underline{d} + \frac{1}{2}\underline{c}$

$= \frac{1}{2}\underline{c} - \frac{1}{3}\underline{d}$

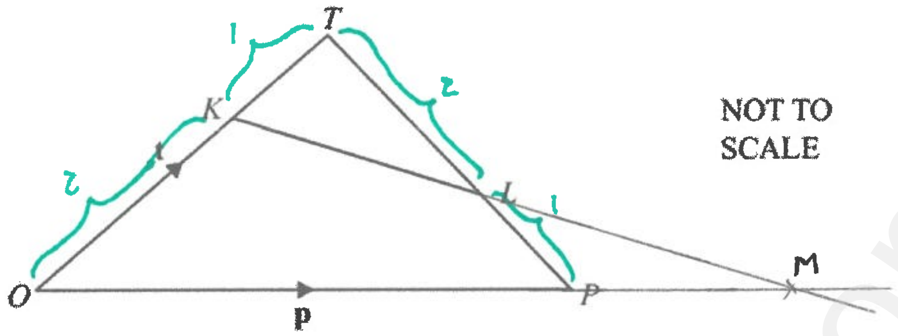
$\overrightarrow{OM} = \overrightarrow{OT} + \overrightarrow{TM}$

$= \frac{2}{3}\underline{d} + \frac{1}{2}\underline{c} - \frac{1}{3}\underline{d}$

$= \frac{1}{2}\underline{c} + \frac{1}{3}\underline{d}$

$\frac{1}{2}\underline{c} + \frac{1}{3}\underline{d}$ .... [3]

22 The diagram shows triangle OPT.



In the diagram $\overrightarrow{OT} = \mathbf{t}$ and $\overrightarrow{OP} = \mathbf{p}$.
$OK : KT = 2 : 1$ and $TL : LP = 2 : 1$.

(a) Find, in terms of $\mathbf{t}$ and $\mathbf{p}$, in its simplest form

(i) $\overrightarrow{PL}$

$\overrightarrow{PL} = \frac{1}{3}\overrightarrow{PT}$

$\overrightarrow{PT} = \overrightarrow{PO} + \overrightarrow{OT}$
$= -\mathbf{p} + \mathbf{t}$
$= \mathbf{t} - \mathbf{p}$

$\overrightarrow{PL} = \frac{1}{3}(\mathbf{t} - \mathbf{p})$
$= \frac{1}{3}\mathbf{t} - \frac{1}{3}\mathbf{p}$

$\frac{1}{3}\mathbf{t} - \frac{1}{3}\mathbf{p}$ .... [2]

(ii) $\overrightarrow{KL}$.

$\overrightarrow{KL} = \overrightarrow{KT} + \overrightarrow{TL}$
$\overrightarrow{KT} = \frac{1}{3}\overrightarrow{OT}$
$= \frac{1}{3}\mathbf{t}$
$\overrightarrow{TL} = \frac{2}{3}\overrightarrow{TP}$

$\overrightarrow{TP} = -\mathbf{t} + \mathbf{p}$
$= \mathbf{p} - \mathbf{t}$
$\overrightarrow{TL} = \frac{2}{3}(\mathbf{p} - \mathbf{t})$
$= \frac{2}{3}\mathbf{p} - \frac{2}{3}\mathbf{t}$

$\overrightarrow{KL} = \frac{1}{3}\mathbf{t} + \frac{2}{3}\mathbf{p} - \frac{2}{3}\mathbf{t}$
$= \frac{2}{3}\mathbf{p} - \frac{1}{3}\mathbf{t}$

$\frac{2}{3}\mathbf{p} - \frac{1}{3}\mathbf{t}$ .... [2]

(b) KL is extended to the point M.

$\overrightarrow{KM} = -\frac{2}{3}\mathbf{t} + \frac{4}{3}\mathbf{p}$.

Show that M lies on OP extended.

$\overrightarrow{OM}$ via K:

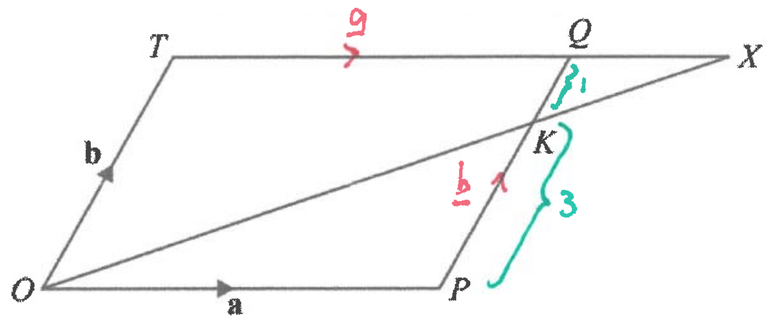
$\overrightarrow{OM} = \overrightarrow{OK} + \overrightarrow{KM}$
$\overrightarrow{OK} = \frac{2}{3}\mathbf{t}$
$\overrightarrow{KM} = -\frac{2}{3}\mathbf{t} + \frac{4}{3}\mathbf{p}$
$\overrightarrow{OM} = \frac{2}{3}\mathbf{t} - \frac{2}{3}\mathbf{t} + \frac{4}{3}\mathbf{p}$
$\overrightarrow{OM} = \frac{4}{3}\mathbf{p}$ ①

$\overrightarrow{OP}$ extended:

$\overrightarrow{OM} = k\overrightarrow{OP}$ ②
① = ②: $= k\mathbf{p}$
$\frac{4}{3}\mathbf{p} = k\mathbf{p}$
$\frac{4}{3} = k$

so $\overrightarrow{OM}$ is $\frac{4}{3} \times \overrightarrow{OP}$

[2]



NOT TO SCALE

The diagram shows a parallelogram *OPQT*.
The position vector of *P* is **a** and the position vector of *T* is **b**.

K is on *PQ* so that *PK* : *KQ* = 3 : 1.
The lines *OK* and *TQ* are extended to meet at *X*.

Find the position vector of *X* in terms of **a** and **b**.
Give your answer in its simplest form.

Via P:

$\vec{OX} = k\vec{OK}$

$\vec{OK} = \vec{OP} + \vec{PK}$

$\vec{PK} = \frac{3}{4}\vec{PQ}$

$= \frac{3}{4}\underline{b}$

$\vec{OK} = \underline{a} + \frac{3}{4}\underline{b}$

$\vec{OX} = k(\underline{a} + \frac{3}{4}\underline{b})$ ①

Via T:

$\vec{OX} = \vec{OT} + \vec{TX}$

$\vec{TX} = m\vec{TQ}$ can't re-use k, so use different letter

$= m\underline{a}$

$\vec{OX} = \underline{b} + m\underline{a}$ ②

Set both $\vec{OX}$ equal to each other:

$m\underline{a} + \underline{b} = k\underline{a} + \frac{3}{4}k\underline{b}$

compare $\underline{b}$:

$\underline{b} = \frac{3}{4}k\underline{b}$

$\div 3/4 \quad \div 3/4$

$\frac{4}{3}\underline{b} = k\underline{b}$

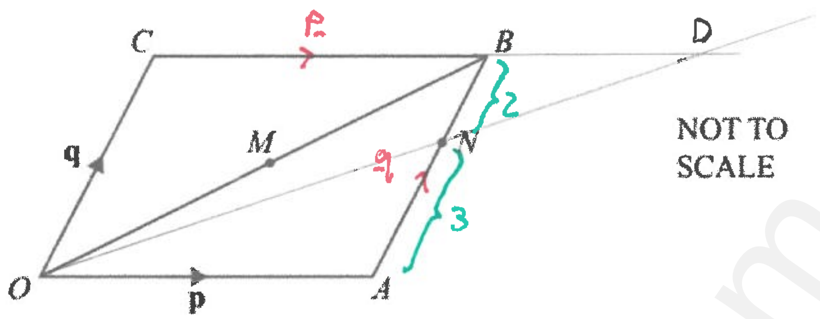
$k = \frac{4}{3}$

Sub. into ①:

$\vec{OX} = \frac{4}{3}(\underline{a} + \frac{3}{4}\underline{b})$

$\vec{OX} = \frac{4}{3}\underline{a} + \underline{b}$

(b)



OABC is a parallelogram and *O* is the origin.
M is the midpoint of *OB*.
N is the point on *AB* such that *AN* : *NB* = 3 : 2.
$\overrightarrow{OA} = \mathbf{p}$ and $\overrightarrow{OC} = \mathbf{q}$.

(i) Find, in terms of **p** and **q**, in its simplest form.

(a) $\overrightarrow{OB}$

$\vec{OB} = \vec{OA} + \vec{AB}$
$= p + q$

$\overrightarrow{OB} =$ ......$p + q$...... [1]

(b) $\overrightarrow{CM}$

$\vec{CM} = \vec{CO} + \vec{OM}$
$\vec{OM} = \frac{1}{2}\vec{OB}$
$= \frac{1}{2}p + \frac{1}{2}q$

$\vec{CM} = -q + \frac{1}{2}p + \frac{1}{2}q$

$\overrightarrow{CM} =$ ......$\frac{1}{2}p - \frac{1}{2}q$...... [2]

(c) $\overrightarrow{MN}$

$\vec{MN} = \vec{MB} + \vec{BN}$
$\vec{MB} = \frac{1}{2}\vec{OB}$
$= \frac{1}{2}p + \frac{1}{2}q$
$\vec{BN} = \frac{2}{5}\vec{BA}$
$= -\frac{2}{5}q$

$\vec{MN} = \frac{1}{2}p + \frac{1}{2}q - \frac{2}{5}q$
$= \frac{1}{2}p + \frac{1}{10}q$

$\overrightarrow{MN} =$ ......$\frac{1}{2}p + \frac{1}{10}q$...... [2]

(ii) *CB* and *ON* are extended to meet at *D*.

Find the position vector of *D* in terms of **p** and **q**.
Give your answer in its simplest form.

$\vec{OD}$ via N:

$\vec{OD} = k\vec{ON}$
$\vec{ON} = \vec{OA} + \vec{AN}$
$= p + \frac{3}{5}q$
$\vec{OD} = kp + \frac{3}{5}kq$ (1)

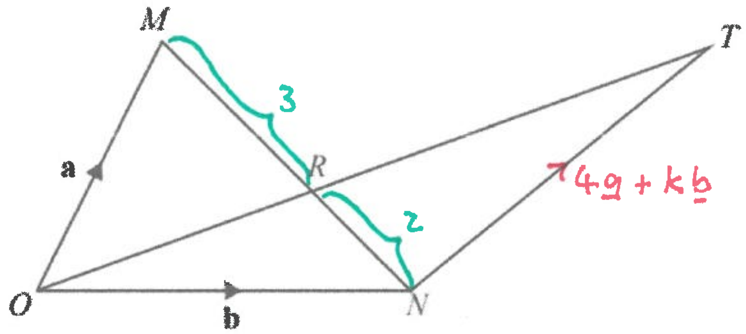
$\vec{OD}$ via C:

$\vec{OD} = \vec{OC} + \vec{CD}$
$\vec{CD} = mp$
$\vec{OD} = q + mp$ (2)

(1) = (2):
$kp + \frac{3}{5}kq = mp + q$
compare q: $\frac{3}{5}kq = q$
$\div\frac{3}{5}$ $\div\frac{3}{5}$
$kq = \frac{5}{3}q$
$k = \frac{5}{3}$

Sub. into (1):
$\vec{OD} = \frac{5}{3}p + q$

(b)



NOT TO SCALE

OMN is a triangle.
$\overrightarrow{OM} = \mathbf{a}$ and $\overrightarrow{ON} = \mathbf{b}$.
R is a point on *MN* such that *MR* : *RN* = 3 : 2.
ORT is a straight line.

(i) Show that $\overrightarrow{OR} = \frac{2}{5}\mathbf{a} + \frac{3}{5}\mathbf{b}$.

$\vec{OR} = \vec{OM} + \vec{MR}$

$\vec{MR} = \frac{3}{5}\vec{MN}$

$\vec{MN} = -\underline{a} + \underline{b}$

$= \underline{b} - \underline{a}$

$\vec{MR} = \frac{3}{5}(\underline{b} - \underline{a})$

$= \frac{3}{5}\underline{b} - \frac{3}{5}\underline{a}$

$\vec{OR} = \underline{a} + \frac{3}{5}\underline{b} - \frac{3}{5}\underline{a}$

$= \frac{2}{5}\underline{a} + \frac{3}{5}\underline{b}$

[3]

(ii) **(a)** $\overrightarrow{NT} = 4\mathbf{a} + k\mathbf{b}$ and $\overrightarrow{OT} = c\overrightarrow{OR}$.

Find the value of *k* and the value of *c*.

$\vec{OT}$ via R:

$\vec{OT} = c\vec{OR}$

$= c(\frac{2}{5}\underline{a} + \frac{3}{5}\underline{b})$

$= \frac{2}{5}c\underline{a} + \frac{3}{5}c\underline{b}$ ①

$\vec{OT}$ via N:

$\vec{OT} = \vec{ON} + \vec{NT}$

$= \underline{b} + 4\underline{a} + k\underline{b}$

$= 4\underline{a} + (1+k)\underline{b}$ ②

① = ②:

$\frac{2}{5}c\underline{a} + \frac{3}{5}c\underline{b} = 4\underline{a} + (1+k)\underline{b}$

Compare $\underline{a}$:

$\frac{2}{5}c\underline{a} = 4\underline{a}$

$\div\frac{2}{5} \quad \div\frac{2}{5}$

$c\underline{a} = 10\underline{a}$

$\underline{c = 10}$

Sub. into ①:

$\vec{OT} = 4\underline{a} + 6\underline{b}$

Compare $6\underline{b}$ with $(1+k)$ in ②:

$6\underline{b} = (1+k)\underline{b}$

$\rightarrow \underline{k = 5}$

(b) Find $\overrightarrow{MT}$.

$\vec{MT} = \vec{MN} + \vec{NT}$

$= \underline{b} - \underline{a} + 4\underline{a} + 5\underline{b}$

$= 3\underline{a} + 6\underline{b}$

$\overrightarrow{MT} =$ ......$3\underline{a} + 6\underline{b}$...... [1]

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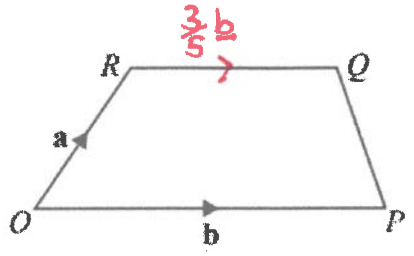


NOT TO SCALE

The diagram shows a trapezium $OPQR$.
O is the origin, $\overrightarrow{OR} = \mathbf{a}$ and $\overrightarrow{OP} = \mathbf{b}$.

$|\overrightarrow{RQ}| = \frac{3}{5}|\overrightarrow{OP}|$

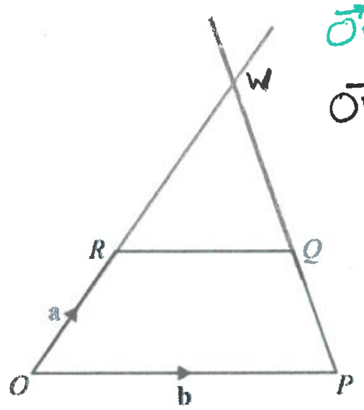
(a) Find $\overrightarrow{PQ}$ in terms of $\mathbf{a}$ and $\mathbf{b}$ in its simplest form.

$$\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OR} + \overrightarrow{RQ}$$
$$= -\underline{b} + \underline{a} + \frac{3}{5}\underline{b}$$
$$= \underline{a} - \frac{2}{5}\underline{b}$$

$\overrightarrow{PQ} =$ $\underline{a} - \frac{2}{5}\underline{b}$ [2]

(b) When PQ and OR are extended, they intersect at W.

Find the position vector of W.



$\overrightarrow{OW}$ via R:

$$\overrightarrow{OW} = k\overrightarrow{OR}$$
$$= k\underline{a} \quad ①$$

$\overrightarrow{OW}$ via P:

$$\overrightarrow{OW} = \overrightarrow{OP} + \overrightarrow{PW}$$
$$\overrightarrow{PW} = m\overrightarrow{PQ}$$
$$= m\left(\underline{a} - \frac{2}{5}\underline{b}\right)$$
$$= m\underline{a} - \frac{2}{5}m\underline{b}$$
$$\overrightarrow{OW} = \underline{b} + m\underline{a} - \frac{2}{5}m\underline{b}$$
$$= m\underline{a} + \left(1 - \frac{2}{5}m\right)\underline{b} \quad ②$$

① = ② :

$$k\underline{a} = m\underline{a} + \left(1 - \frac{2}{5}m\right)\underline{b}$$

Compare $\underline{b}$:

$$0\underline{b} = \left(1 - \frac{2}{5}m\right)\underline{b}$$
$$0 = 1 - \frac{2}{5}m$$
$$\frac{2}{5}m = 1$$
$$m = \frac{5}{2}$$

Sub. into ② :

$$\overrightarrow{OW} = \frac{5}{2}\underline{a} + 0\underline{b}$$
$$= \frac{5}{2}\underline{a}$$

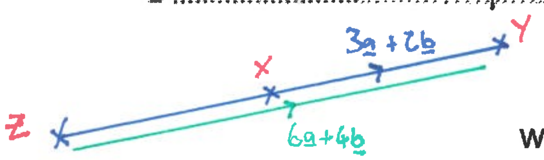
21 $\overrightarrow{XY} = 3\mathbf{a} + 2\mathbf{b}$ and $\overrightarrow{ZY} = 6\mathbf{a} + 4\mathbf{b}$.

points not vectors.

Write down two statements about the relationship between the points X, Y and Z.

1 They all lie on the same straight line.

2 X is the mid-point of Z and Y. [2]



$$\leftarrow \frac{6\underline{a} + 4\underline{b}}{2} = 3\underline{a} + 2\underline{b}$$