

14 Simplify.

$$\sqrt{125} + \sqrt{80}$$

$$\sqrt{25} \times \sqrt{5} + \sqrt{16} \times \sqrt{5}$$

$$= 5\sqrt{5} + 4\sqrt{5}$$

$$= 9\sqrt{5}$$

$$\underline{\hspace{10em} 9\sqrt{5} \hspace{10em}} \quad [2]$$

11 Simplify $\sqrt{27} + \sqrt{12} - \sqrt{108}$.

$$\sqrt{9} \times \sqrt{3} + \sqrt{4} \times \sqrt{3} - \sqrt{36} \times \sqrt{3}$$

$$= 3\sqrt{3} + 2\sqrt{3} - 6\sqrt{3}$$

$$= 5\sqrt{3} - 6\sqrt{3}$$

$$= -\sqrt{3}$$

$$\underline{\hspace{10em} -\sqrt{3} \hspace{10em}} \quad [2]$$

8 Expand and simplify $(2\sqrt{3}-5)(4+\sqrt{3})$.

$$\begin{aligned} & 8\sqrt{3} + 2\sqrt{9} - 20 - 5\sqrt{3} \\ &= 8\sqrt{3} + 2 \times 3 - 20 - 5\sqrt{3} \\ &= 3\sqrt{3} + 6 - 20 \\ &= 3\sqrt{3} - 14 \end{aligned}$$

$$\dots\dots\dots 3\sqrt{3} - 14 \quad [2]$$

11 Expand and simplify. $(4+2\sqrt{3})(5-\sqrt{3})$

$$\begin{aligned} & 20 - 4\sqrt{3} + 10\sqrt{3} - 2\sqrt{9} \\ &= 20 + 6\sqrt{3} - 2 \times 3 \\ &= 20 + 6\sqrt{3} - 6 \\ &= 14 + 6\sqrt{3} \end{aligned}$$

$$\dots\dots\dots 14 + 6\sqrt{3} \quad [2]$$

13 $(2\sqrt{3}-3\sqrt{2})^2 = p+q\sqrt{6}$

Find the value of p and the value of q .

$$\begin{aligned} & (2\sqrt{3}-3\sqrt{2})(2\sqrt{3}-3\sqrt{2}) \\ &= 4\sqrt{9} - 6\sqrt{6} - 6\sqrt{6} + 9\sqrt{4} \\ &= 4 \times 3 - 12\sqrt{6} + 9 \times 2 \\ &= 12 - 12\sqrt{6} + 18 \\ &= 30 - 12\sqrt{6} \end{aligned}$$

$$\begin{aligned} p &= \dots\dots\dots 30 \\ q &= \dots\dots\dots -12 \quad [3] \end{aligned}$$

15 (a) Expand and simplify.

$$(2 - \sqrt{5})(1 - 3\sqrt{5})$$

$$\begin{aligned} & 2 - 6\sqrt{5} - \sqrt{5} + 3\sqrt{25} \\ & = 2 - 7\sqrt{5} + 3 \times 5 \\ & = 2 - 7\sqrt{5} + 15 \\ & = 17 - 7\sqrt{5} \end{aligned}$$

$$\underline{\hspace{10em} 17 - 7\sqrt{5} \hspace{10em}} \quad [2]$$

(b) Rationalise the denominator.
Give your answer in its simplest form.

$$\begin{aligned} \frac{6}{\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}} &= \frac{6\sqrt{10}}{10} \quad \begin{matrix} \div 2 \\ \div 2 \end{matrix} \\ &= \frac{3\sqrt{10}}{5} \end{aligned}$$

$$\underline{\hspace{10em} \frac{3\sqrt{10}}{5} \hspace{10em}} \quad [2]$$

18 (a) $\frac{9}{\sqrt{3}}$

Rationalise the denominator.
Give your answer in its simplest form.

$$\begin{aligned} \frac{9}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} &= \frac{9\sqrt{3}}{3} \\ &= 3\sqrt{3} \end{aligned}$$

$$\underline{\hspace{10em} 3\sqrt{3} \hspace{10em}} \quad [2]$$

(b) $(5 - \sqrt{2})(1 + 3\sqrt{2}) = c + k\sqrt{2}$

Find the value of c and the value of k .

$$\begin{aligned} & 5 + 15\sqrt{2} - \sqrt{2} - 3\sqrt{4} \\ & = 5 + 14\sqrt{2} - 3 \times 2 \\ & = 5 + 14\sqrt{2} - 6 \\ & = -1 + 14\sqrt{2} \end{aligned}$$

$$c = \underline{\hspace{10em} -1 \hspace{10em}}$$

$$k = \underline{\hspace{10em} 14 \hspace{10em}}$$

[2]

15 (a) Simplify.

$$\begin{aligned} & \sqrt{27} + \sqrt{12} \\ & \sqrt{9} \times \sqrt{3} + \sqrt{4} \times \sqrt{3} \\ & = 3\sqrt{3} + 2\sqrt{3} \\ & = 5\sqrt{3} \end{aligned}$$

$$\dots\dots\dots \frac{5\sqrt{3}}{\dots\dots\dots} \quad [2]$$

(b) $\frac{40\sqrt{8}}{5\sqrt{2}} = k$, where k is an integer.

Find the value of k .

$$\begin{aligned} \frac{40}{5} \times \frac{\sqrt{8}}{\sqrt{2}} &= 8 \times \sqrt{\frac{8}{2}} \\ &= 8 \times \sqrt{4} \\ &= 8 \times 2 \\ &= 16 \end{aligned}$$

$$k = \dots\dots\dots \frac{16}{\dots\dots\dots} \quad [2]$$

13 Rationalise the denominator.

$$\frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\dots\dots\dots \frac{2\sqrt{3}}{3} \dots\dots\dots \quad [1]$$

20 (a) Simplify.

$$\sqrt{300} + \sqrt{48}$$

$$\sqrt{100} \times \sqrt{3} + \sqrt{16} \times \sqrt{3}$$

$$= 10\sqrt{3} + 4\sqrt{3}$$

$$= 14\sqrt{3}$$

$$\dots\dots\dots 14\sqrt{3} \dots\dots\dots [2]$$

(b) Rationalise the denominator and simplify.

$$\frac{9}{2+\sqrt{7}}$$

$$\frac{9}{2+\sqrt{7}} \times \frac{(2-\sqrt{7})}{(2-\sqrt{7})}$$

$$= \frac{9(2-\sqrt{7})}{4-2\sqrt{7}+2\sqrt{7}-\sqrt{49}}$$

$$= \frac{18-9\sqrt{7}}{4-7}$$

$$= \frac{18-9\sqrt{7}}{-3} \quad \begin{matrix} \div 3 \\ \div 3 \end{matrix}$$

$$= -6+3\sqrt{7}$$

$$\dots\dots\dots -6+3\sqrt{7} \dots\dots\dots [3]$$

(c) Rationalise the denominator.

$$\frac{1}{3-\sqrt{5}}$$

$$\frac{1}{3-\sqrt{5}} \times \frac{(3+\sqrt{5})}{(3+\sqrt{5})}$$

$$= \frac{3+\sqrt{5}}{9+3\sqrt{5}-3\sqrt{5}-\sqrt{25}}$$

$$= \frac{3+\sqrt{5}}{9-5}$$

$$= \frac{3+\sqrt{5}}{4}$$

$$\dots\dots\dots \frac{3+\sqrt{5}}{4} \dots\dots\dots [2]$$

19 (a) Simplify.

$$\sqrt{32} + \sqrt{98}$$

$$\begin{aligned} & \sqrt{16} \times \sqrt{2} + \sqrt{49} \times \sqrt{2} \\ & = 4\sqrt{2} + 7\sqrt{2} \\ & = 11\sqrt{2} \end{aligned}$$

$$\dots\dots\dots 11\sqrt{2} \dots\dots\dots [2]$$

(b) Rationalise the denominator.

$$\begin{aligned} & \frac{1}{\sqrt{2}+1} \times \frac{(\sqrt{2}-1)}{(\sqrt{2}-1)} = \frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} \\ & = \frac{\sqrt{2}-1}{\sqrt{4}-\sqrt{2}+\sqrt{2}-1} \\ & = \frac{\sqrt{2}-1}{2-1} \\ & = \frac{\sqrt{2}-1}{1} \end{aligned}$$

$$\dots\dots\dots \sqrt{2}-1 \dots\dots\dots [2]$$

13 Rationalise the denominator and simplify.

$$\begin{aligned} & \frac{2}{\sqrt{5}+1} \times \frac{(\sqrt{5}-1)}{(\sqrt{5}-1)} = \frac{2(\sqrt{5}-1)}{\sqrt{25}-\sqrt{5}+\sqrt{5}-1} \\ & = \frac{2\sqrt{5}-2}{5-1} \\ & = \frac{2\sqrt{5}-2}{4} \end{aligned}$$

$$\begin{aligned} & = \frac{\sqrt{5}-1}{2} \\ & \dots\dots\dots \frac{\sqrt{5}-1}{2} \dots\dots\dots [3] \end{aligned}$$

13 Rationalise the denominator and simplify.

$$\begin{aligned} & \frac{2}{3-\sqrt{5}} \times \frac{(3+\sqrt{5})}{(3+\sqrt{5})} = \frac{2(3+\sqrt{5})}{9-3\sqrt{5}+3\sqrt{5}-\sqrt{25}} \\ & = \frac{6+2\sqrt{5}}{9-5} \\ & = \frac{6+2\sqrt{5}}{4} \end{aligned}$$

$$\begin{aligned} & = \frac{3+\sqrt{5}}{2} \\ & \dots\dots\dots \frac{3+\sqrt{5}}{2} \dots\dots\dots [3] \end{aligned}$$

7 (a) Simplify $\sqrt{98}$.

$$\begin{aligned} \sqrt{49} \times \sqrt{2} \\ = 7\sqrt{2} \end{aligned}$$

$$\underline{\hspace{1cm} 7\sqrt{2} \hspace{1cm}} \quad [1]$$

(b) Rationalise the denominator.

$$\begin{aligned} \frac{3}{\sqrt{5}-2} & \times \frac{\sqrt{5}+2}{\sqrt{5}+2} = \frac{3(\sqrt{5}+2)}{\sqrt{25}+2\sqrt{5}-2\sqrt{5}-4} \\ & = \frac{3\sqrt{5}+6}{5-4} \\ & = \frac{3\sqrt{5}+6}{1} \end{aligned}$$

$$\underline{\hspace{1cm} 3\sqrt{5} + 6 \hspace{1cm}} \quad [2]$$

13 (a) Simplify fully.

$$\begin{aligned} \sqrt{75} - \sqrt{48} + \sqrt{12} \\ \sqrt{25} \times \sqrt{3} - \sqrt{16} \times \sqrt{3} + \sqrt{4} \times \sqrt{3} \\ = 5\sqrt{3} - 4\sqrt{3} + 2\sqrt{3} \\ = 7\sqrt{3} - 4\sqrt{3} \end{aligned}$$

$$\underline{\hspace{1cm} = 3\sqrt{3} \hspace{1cm}} \quad [2]$$

(b) Rationalise the denominator, giving your answer in its simplest form.

$$\begin{aligned} \frac{1}{\sqrt{3}+5} & \times \frac{\sqrt{3}-5}{\sqrt{3}-5} = \frac{\sqrt{3}-5}{\sqrt{9}-5\sqrt{3}+5\sqrt{3}-25} \\ & = \frac{\sqrt{3}-5}{3-25} \\ & = \frac{\sqrt{3}-5}{-22} \times -1 \\ & = \frac{-\sqrt{3}+5}{22} \end{aligned}$$

$$\underline{\hspace{1cm} \frac{5-\sqrt{3}}{22} \hspace{1cm}} \quad [2]$$

6 (a) Simplify.

$$\sqrt{75} - \sqrt{27}$$

$$\begin{aligned} & \sqrt{25} \times \sqrt{3} - \sqrt{9} \times \sqrt{3} \\ & = 5\sqrt{3} - 3\sqrt{3} \\ & = 2\sqrt{3} \end{aligned}$$

$$\dots\dots\dots 2\sqrt{3} \dots\dots\dots [2]$$

(b) Rationalise the denominator and simplify your answer.

$$\frac{10}{5 - \sqrt{5}}$$

$$\begin{aligned} \frac{10}{5 - \sqrt{5}} \times \frac{(5 + \sqrt{5})}{(5 + \sqrt{5})} &= \frac{10(5 + \sqrt{5})}{25 + 5\sqrt{5} - 5\sqrt{5} - \sqrt{25}} = \frac{50 + 10\sqrt{5}}{25 - 5} \\ &= \frac{50 + 10\sqrt{5}}{20} \quad \begin{array}{l} \div 10 \\ \div 10 \end{array} = \frac{5 + \sqrt{5}}{2} \end{aligned}$$
$$\dots\dots\dots \frac{5 + \sqrt{5}}{2} \dots\dots\dots [3]$$

13 Write in the form $a + b\sqrt{3}$ where a and b are integers.

(a) $(5 + 2\sqrt{3})^2$

$$\begin{aligned} & (5 + 2\sqrt{3})(5 + 2\sqrt{3}) \\ & = 25 + 10\sqrt{3} + 10\sqrt{3} + 4\sqrt{9} \\ & = 25 + 20\sqrt{3} + 4 \times 3 \\ & = 25 + 20\sqrt{3} + 12 \\ & = 37 + 20\sqrt{3} \end{aligned}$$

$$\dots\dots\dots 37 + 20\sqrt{3} \dots\dots\dots [2]$$

(b) $\frac{5}{2 + \sqrt{3}}$

$$\begin{aligned} \frac{5}{2 + \sqrt{3}} \times \frac{(2 - \sqrt{3})}{(2 - \sqrt{3})} &= \frac{5(2 - \sqrt{3})}{4 - 2\sqrt{3} + 2\sqrt{3} - \sqrt{9}} \\ &= \frac{10 - 5\sqrt{3}}{4 - 3} \\ &= \frac{10 - 5\sqrt{3}}{1} \end{aligned}$$
$$\dots\dots\dots 10 - 5\sqrt{3} \dots\dots\dots [2]$$

13 Rationalise the denominator.

$$\frac{9}{\sqrt{7}-2}$$

$$\frac{9}{\sqrt{7}-2} \times \frac{(\sqrt{7}+2)}{(\sqrt{7}+2)} = \frac{9(\sqrt{7}+2)}{\sqrt{49} + 2\sqrt{7} - 2\sqrt{7} - 4}$$

$$= \frac{9\sqrt{7} + 18}{7-4}$$

$$= \frac{9\sqrt{7} + 18}{3} \stackrel{\div 3}{=} 3\sqrt{7} + 6 \quad [2]$$

15 Rationalise the denominator.

$$\frac{\sqrt{5}}{\sqrt{5}-1}$$

$$\frac{\sqrt{5}}{\sqrt{5}-1} \times \frac{(\sqrt{5}+1)}{(\sqrt{5}+1)} = \frac{\sqrt{5}(\sqrt{5}+1)}{\sqrt{25} + \sqrt{5} - \sqrt{5} - 1}$$

$$= \frac{\sqrt{25} + \sqrt{5}}{5-1}$$

$$= \frac{5 + \sqrt{5}}{4} \quad [2]$$

11 (a) Simplify.

$$\frac{\sqrt{50} - \sqrt{8}}{\sqrt{25} \times \sqrt{2} - \sqrt{4} \times \sqrt{2}}$$

$$\frac{5\sqrt{2} - 2\sqrt{2}}{5\sqrt{2} - 2\sqrt{2}} = 3\sqrt{2} \quad [2]$$

(b) By rationalising the denominator, simplify

$$\frac{12}{\sqrt{7}-\sqrt{3}}$$

$$\frac{12}{\sqrt{7}-\sqrt{3}} \times \frac{(\sqrt{7}+\sqrt{3})}{(\sqrt{7}+\sqrt{3})} = \frac{12(\sqrt{7}+\sqrt{3})}{\sqrt{49} + \sqrt{21} - \sqrt{21} - \sqrt{9}}$$

$$= \frac{12\sqrt{7} + 12\sqrt{3}}{7-3}$$

$$= \frac{12\sqrt{7} + 12\sqrt{3}}{4} \stackrel{\div 4}{=} 3\sqrt{7} + 3\sqrt{3} \quad [3]$$

14 Rationalise the denominator.

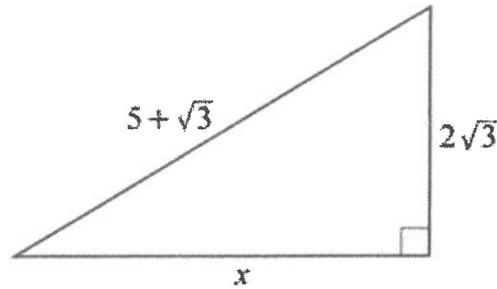
$$\frac{5}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \frac{5(\sqrt{3}+\sqrt{2})}{\sqrt{9}+\sqrt{6}-\sqrt{6}-\sqrt{4}}$$
$$= \frac{5\sqrt{3}+5\sqrt{2}}{3-2} = \frac{5\sqrt{3}+5\sqrt{2}}{1}$$
$$\underline{\underline{5\sqrt{3}+5\sqrt{2}}} \quad [2]$$

17 Simplify by rationalising the denominator.

$$\frac{3}{2\sqrt{2}-1}$$

$$\frac{3}{2\sqrt{2}-1} \times \frac{(2\sqrt{2}+1)}{(2\sqrt{2}+1)} = \frac{3(2\sqrt{2}+1)}{4\sqrt{4}+2\sqrt{2}-2\sqrt{2}-1}$$
$$= \frac{6\sqrt{2}+3}{4 \times 2 - 1}$$
$$= \frac{6\sqrt{2}+3}{8-1}$$
$$= \underline{\underline{\frac{6\sqrt{2}+3}{7}}}$$

11 In this question all lengths are in centimetres.



NOT TO SCALE

Find the value of x^2 .

Give your answer in the form $a + b\sqrt{3}$ where a and b are integers.

Pythagoras:

$$x^2 + (2\sqrt{3})^2 = (5 + \sqrt{3})^2$$

$$x^2 + 4 \times \sqrt{9} = (5 + \sqrt{3})(5 + \sqrt{3})$$

$$x^2 + 4 \times 3 = 25 + 5\sqrt{3} + 5\sqrt{3} + \sqrt{9}$$

$$x^2 + 12 = 25 + 10\sqrt{3} + 3$$

$$x^2 + 12 = 28 + 10\sqrt{3}$$

$$x^2 = 16 + 10\sqrt{3}$$

$$x^2 = \underline{16 + 10\sqrt{3}} \quad [4]$$

4 You are given that $\sqrt{7} = 2.65$ and $\sqrt{70} = 8.37$, each correct to 2 decimal places.

Use this information to find the value of

(a) $\sqrt{700}$, $\sqrt{700} = \sqrt{7} \times \sqrt{100}$

$$= \sqrt{7} \times 10$$

$$= 2.65 \times 10 = 26.5$$

$$\underline{26.5} \quad [1]$$

(b) $\sqrt{280}$.

$$\sqrt{280} = \sqrt{70} \times \sqrt{4}$$

$$= \sqrt{70} \times 2$$

$$= 8.37 \times 2$$

$$\begin{array}{r} 8.37 \\ \times \quad 2 \\ \hline 16.74 \end{array}$$

$$\underline{16.74} \quad [1]$$

17 (a) Expand the brackets and simplify.

$$\begin{aligned}
 & (\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) \\
 & \sqrt{a^2} - \cancel{\sqrt{ab}} + \cancel{\sqrt{ab}} - \sqrt{b^2} \\
 & = a - b
 \end{aligned}$$

..... $a - b$ [2]

(b) Rationalise the denominator.

$$\begin{aligned}
 \frac{1}{\sqrt{7} + \sqrt{6}} & \times \frac{(\sqrt{7} - \sqrt{6})}{(\sqrt{7} - \sqrt{6})} = \frac{\sqrt{7} - \sqrt{6}}{\sqrt{49} - \sqrt{42} + \sqrt{42} - \sqrt{36}} \\
 & = \frac{\sqrt{7} - \sqrt{6}}{7 - 6} \\
 & = \frac{\sqrt{7} - \sqrt{6}}{1}
 \end{aligned}$$

..... $\sqrt{7} - \sqrt{6}$ [1]

(c) Work out the value of

$$\frac{1}{\sqrt{9} + \sqrt{8}} + \frac{1}{\sqrt{8} + \sqrt{7}} + \frac{1}{\sqrt{7} + \sqrt{6}} + \frac{1}{\sqrt{6} + \sqrt{5}} + \frac{1}{\sqrt{5} + \sqrt{4}}$$

using part (b):

$$\frac{1}{\sqrt{9} + \sqrt{8}} = \sqrt{9} - \sqrt{8} \text{ and same for the others.}$$

so expression becomes:

$$\begin{aligned}
 & \sqrt{9} - \sqrt{8} + \sqrt{8} - \sqrt{7} + \sqrt{7} - \sqrt{6} + \sqrt{6} - \sqrt{5} + \sqrt{5} - \sqrt{4} \\
 & = \sqrt{9} - \sqrt{4} \\
 & = 3 - 2 \\
 & = \underline{1}
 \end{aligned}$$

..... 1 [2]