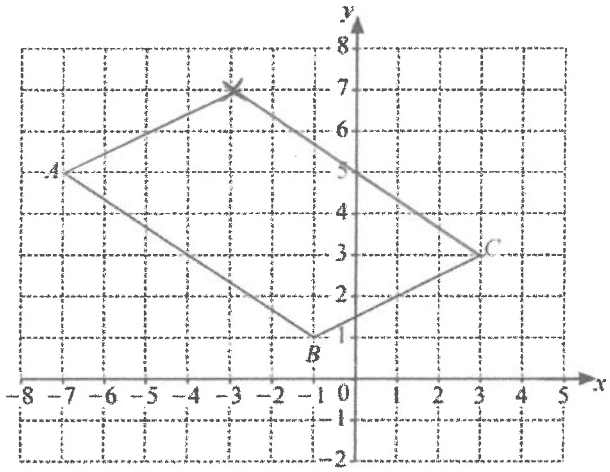
1

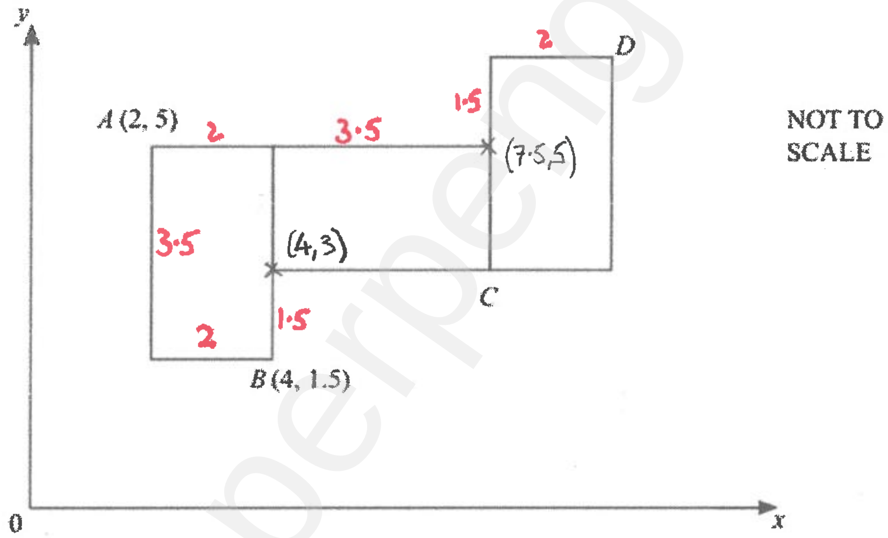


The diagram shows two sides of a parallelogram *ABCD*.

Find the coordinates of point *D*.

(-3 , 7) [2]

5



A pattern is formed by 3 congruent rectangles.
Each rectangle is a rotation of 90° around one vertex of the rectangle next to it.
The point *A* has coordinates (2, 5).
The point *B* has coordinates (4, 1.5).

Work out the coordinates of point *C* and point *D*.

C (7.5 , 3)
D (9.5 , 6.5)
[3]

9 The line $y = 2x - 5$ intersects the line $y = 3$ at the point P.

Find the coordinates of the point P.

$$3 = 2x - 5$$
$+5 \qquad +5$
$$8 = 2x$$
$\div 2 \qquad \div 2$
$$4 = x$$

$\rightarrow x = 4,\ y = 3$

(4 , 3) [2]

5 **(a)** Write down the gradient of the line $y = 5x + 7$.

5 (not $5x$) [1]

(b) Find the coordinates of the point where the line $y = 5x + 7$ crosses the y-axis.

y-intercept

(0 , 7) [1]

10 A is the point $(\overset{x_1}{-1}, \overset{y_1}{13})$ and B is the point $(\overset{x_2}{3}, \overset{y_2}{1})$.

Find the equation of the line AB, giving your answer in the form $y = mx + c$.

Gradient:
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 13}{3 - -1} = \frac{-12}{4} = -3$$

Equation: $y = -3x + c$

Sub. either point, e.g. $(3,1)$:
$$1 = -3(3) + c$$
$$1 = -9 + c$$
$+9 \qquad +9$
$$10 = c$$

$$y = -3x + 10$$

$y =$ $-3x + 10$ [3]

16 A is the point $(\overset{x_1}{5}, \overset{y_1}{7})$ and B is the point $(\overset{x_2}{9}, \overset{y_2}{-1})$.

(b) Find the equation of the line AB.

Gradient:
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 7}{9 - 5} = \frac{-8}{4} = -2$$

Equation: $y = -2x + c$

Sub. $(5,7)$:
$$7 = -2(5) + c$$
$$7 = -10 + c$$
$+10 \qquad +10$
$$17 = c$$

$$y = -2x + 17$$

$y = -2x + 17$ [3]

9 **(a)** The equation of line L is $3x - 8y + 20 = 0$.

(i) Find the gradient of line L.

$3x - 8y + 20 = 0$

$+8y \qquad\qquad +8y$

$3x + 20 = 8y$

$\div 8 \quad \div 8 \quad \div 8$

$\frac{3}{8}x + \frac{5}{2} = y$

$y = \frac{3}{8}x + \frac{5}{2}$

$\frac{3}{8}$ [2]

(ii) Find the coordinates of the point where line L cuts the y-axis.

(0 , $\frac{5}{2}$) [1]

18 Find the equation of the straight line that passes through the points $(\overset{x_1}{2}, \overset{y_1}{-2})$ and $(\overset{x_2}{3}, \overset{y_2}{10})$.

Give your answer in the form $y = mx + c$.

Gradient:

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{10 - -2}{3 - 2}$

$= \frac{12}{1}$

$= 12$

Equation: $y = 12x + c$

Sub. $(3, 10)$:

$10 = 12(3) + c$

$10 = 36 + c$

$-36 \qquad -36$

$-26 = c$

$y = 12x - 26$

$y =$ $12x - 26$ [3]

x_1 y_1 x_2 y_2

24 **(a)** A is the point $(a, 12)$ and B is the point $(b, 27)$.

(i) Find the y-coordinate of the midpoint of AB.

$$\frac{12+27}{2} = 19.5$$

........19.5........ [1]

(ii) The line AB has gradient 3.

Find an expression for a in terms of b.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$3 = \frac{27-12}{b-a}$$

$$3 = \frac{15}{b-a}$$

$\times(b-a)$ $\quad$ $\times(b-a)$

$$3(b-a) = 15$$

$$3b - 3a = 15$$

$-3b$ $\quad$ $-3b$

$$-3a = 15 - 3b$$

$\div -3$ $\quad$ $\div 3$ $\quad$ $\div -3$

$$a = -5 + b$$

$a =$ ....$b - 5$........ [3]

14 Find the gradient of a line that is perpendicular to $8y+4x=5$.

$8y + 4x = 5$

$-4x \quad -4x$

$8y = -4x + 5$

$\div 8 \quad \div 8 \quad \div 8$

$y = -\frac{1}{2}x + \frac{5}{8}$

$m = -\frac{1}{2}$

m_{perp}: (negative reciprocal)

$= 2$

2 .... [2]

15 (a) A is the point $(3, 16)$ and B is the point $(8, 31)$.

$x_1 \; y_1 \qquad x_2 \; y_2$

Find the equation of the line that passes through A and B.
Give your answer in the form $y = mx + c$.

Gradient:

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{31 - 16}{8 - 3}$

$= \frac{15}{5}$

$= 3$

Equation: $y = 3x + c$

Sub. (3,16):

$16 = 3(3) + c$

$16 = 9 + c$

$-9 \quad -9$

$7 = c$

$y = 3x + 7$

$y =$ $3x + 7$ .... [3]

(b) The line CD has equation $y = 0.5x - 11$.

Find the gradient of a line that is perpendicular to the line CD.

$m = \frac{1}{2}$

$m_{perp} = -2$ (negative reciprocal)

-2 .... [1]

17 Find the gradient of the line that is perpendicular to the line $3y = 4x - 5$.

$3y = 4x - 5$

$\div 3 \quad \div 3 \quad \div 3$

$y = \frac{4}{3}x - \frac{5}{3}$

$m = \frac{4}{3}$

$m_{perp} = \frac{-3}{4}$

$\frac{-3}{4}$ .... [2]

12 A straight line, l, has equation $y = 5x + 12$.

(a) Write down the gradient of line l.

5 .... [1]

(b) Find the coordinates of the point where line l crosses the x-axis.

At x-axis, $y = 0$:

$0 = 5x + 12$

$-12 \quad -12$

$-12 = 5x$

$\div 5 \quad \div 5$

$-\frac{12}{5} = x$

($\frac{-12}{5}$, 0) [2]

(c) A line perpendicular to line l has gradient k.

Find the value of k.

$m = 5$

$m_{perp} = \frac{-1}{5}$

$k =$ $\frac{-1}{5}$ .... [1]

11 Line L has equation $y = 4 - 5x$.

Find the equation of a line that is perpendicular to line L and passes through the point (0, 6).

$y = -5x + 4$

$m = -5$

$m_{perp} = \frac{1}{5}$

Equation: $y = \frac{1}{5}x + c$

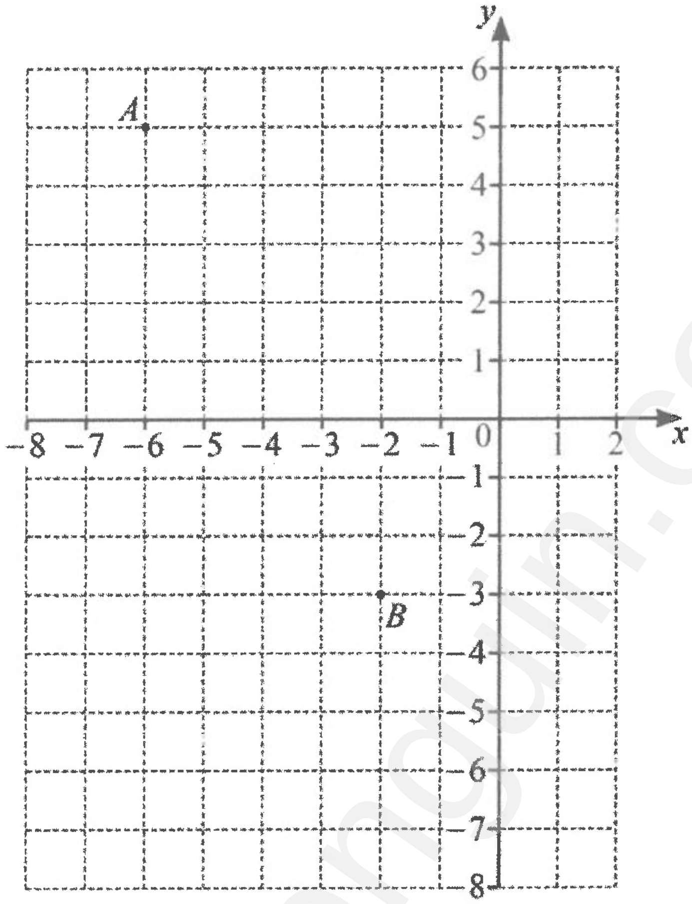
Sub. $(0,6)$:

$6 = \frac{1}{5}(0) + c$

$6 = c$

$y = \frac{1}{5}x + 6$

$y = \frac{1}{5}x + 6$ .... [3]



A is the point $(-6, 5)$ and B is the point $(-2, -3)$.

$x_1 \quad y_1 \qquad x_2 \quad y_2$

(a) Find the equation of the straight line, l, that passes through point A and point B.
Give your answer in the form $y = mx + c$.

Gradient:

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-3 - 5}{-2 - -6}$

$= \frac{-8}{4}$

$= -2$

Equation: $y = -2x + c$

Sub. $(-6,5)$:

$5 = -2(-6) + c$

$5 = 12 + c$

$-12 \quad -12$

$-7 = c$

$y =$ $-2x - 7$ [2]

(b) Find the equation of the line that is perpendicular to l and passes through the origin. $(0,0)$

$m = -2$

$m_{perp} = \frac{1}{2}$

Equation: $y = \frac{1}{2}x + c$

Sub. $(0,0)$: $0 = \frac{1}{2}(0) + c$

$c = 0$

$y = \frac{1}{2}x$ [2]

16 A is the point $(7, 2)$ and B is the point $(-5, 8)$.

$x_1 \; y_1 \qquad x_2 \; y_2$

(a) Calculate the length of AB.

$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

$$= \sqrt{(-5-7)^2 + (8-2)^2}$$

$$= \sqrt{(-12)^2 + 6^2}$$

$$= \sqrt{144+36}$$

$$= \sqrt{180}$$

$6\sqrt{5}$ [3]

$(\text{or } 13\cdot4)$

(b) Find the equation of the line that is perpendicular to AB and that passes through the point $(-1, 3)$.
Give your answer in the form $y = mx + c$.

Gradient:

$$m = \frac{y_2-y_1}{x_2-x_1}$$

$$= \frac{8-2}{-5-7}$$

$$= \frac{6}{-12}$$

$$m = \frac{-1}{2}$$

$$m_{perp} = \underline{2}$$

Equation:

$$y = 2x + c$$

Sub. $(-1,3)$:

$$3 = 2(-1) + c$$

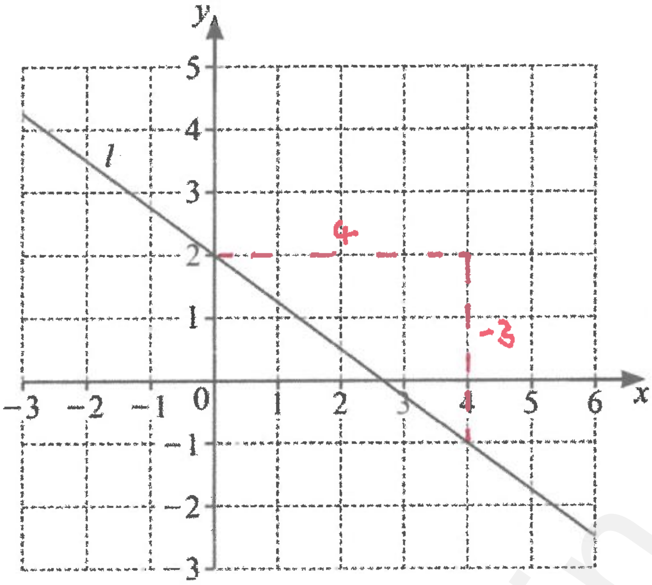
$$3 = -2 + c$$

$$+2 \qquad +2$$

$$\underline{5 = c}$$

$$y = 2x + 5$$

$y =$ $2x + 5$ [4]



(a) Find the gradient of line l.

From diagram: $m = \frac{\text{change in } y}{\text{change in } x}$

$= \frac{-3}{4}$

$\frac{-3}{4}$ [2]

(b) Find the equation of line l in the form $y = mx + c$.

$y = \frac{-3}{4}x + c$

y-intercept is $(0,2)$ (from diagram) : $y = \frac{-3}{4}x + 2$

$y = \frac{-3}{4}x + 2$ [2]

(c) Find the equation of the line that is perpendicular to line l and passes through the point $(12, -7)$.
Give your answer in the form $y = mx + c$.

$m_{perp} = \frac{4}{3}$

Equation: $y = \frac{4}{3}x + c$

Sub. $(12, -7)$: $-7 = \frac{4}{3}(12) + c$

$-7 = 16 + c$

$-16 \quad -16$

$-23 = c$

$y = \frac{4}{3}x - 23$ [3]

9 A is the point $(1, 3)$ and B is the point $(3, -7)$.
The line l passes through A and is perpendicular to AB.

$x_1 \; y_1 \qquad x_2 \; y_2$

Find the equation of line l.
Give your answer in the form $py + qx = r$ where p, q and r are integers.

Gradient$_{AB}$:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7-3}{3-1} = \frac{-10}{2} = -5$$

$$m_{perp} = \frac{1}{5}$$

Equation:

$$y = \frac{1}{5}x + c$$

Sub. $A(1,3)$:

$$3 = \frac{1}{5}(1) + c$$

$$3 = \frac{1}{5} + c$$

$$-\frac{1}{5} \qquad -\frac{1}{5}$$

$$\frac{14}{5} = c$$

$$y = \frac{1}{5}x + \frac{14}{5}$$

$$\times 5 \qquad \times 5 \qquad \times 5$$

$$5y = x + 14$$

$$-x \qquad -x$$

$$5y - x = 14$$

$5y - x = 14$ [4]

26 A is the point $(6, 1)$ and B is the point $(2, 7)$.

$x_1 \; y_1 \qquad x_2 \; y_2$

Find the equation of the perpendicular bisector of AB.
Give your answer in the form $y = mx + c$.

Gradient$_{AB}$:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-1}{2-6} = \frac{6}{-4} = \frac{-3}{2}$$

$$m_{perp} = \frac{2}{3}$$

Mid-point of AB:

$$\left(\frac{6+2}{2}, \frac{1+7}{2}\right) = (4, 4)$$

Equation:

$$y = \frac{2}{3}x + c$$

Sub. $(4,4)$:

$$4 = \frac{2}{3}(4) + c$$

$$4 = \frac{8}{3} + c$$

$$-\frac{8}{3} \qquad -\frac{8}{3}$$

$$\frac{4}{3} = c$$

$y = \frac{2}{3}x + \frac{4}{3}$ [5]

13 A is the point $(\overset{x_1}{1}, \overset{y_1}{7})$ and B is the point $(\overset{x_2}{4}, \overset{y_2}{1})$.

Find the equation of the perpendicular bisector of AB in the form $y = mx + c$.

Gradient:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{1-7}{4-1}$$

$$= \frac{-6}{3}$$

$$= -2$$

$$m_{perp} = \frac{1}{2}$$

Mid-point of AB:

$$\left(\frac{1+4}{2}, \frac{7+1}{2}\right)$$

$$= \left(\frac{5}{2}, 4\right)$$

Equation:

$$y = \frac{1}{2}x + C$$

Sub. $\left(\frac{5}{2}, 4\right)$:

$$4 = \frac{1}{2}\left(\frac{5}{2}\right) + C$$

$$4 = \frac{5}{4} + C$$

$$-\frac{5}{4} \qquad -\frac{5}{4}$$

$$\frac{11}{4} = C$$

$$y = \frac{1}{2}x + \frac{11}{4}$$

$y =$ $\frac{1}{2}x + \frac{11}{4}$ [5]

(b) The coordinates of P are $(\overset{x_1}{-3}, \overset{y_1}{8})$ and the coordinates of Q are $(\overset{x_2}{9}, \overset{y_2}{-2})$.

(ii) Find the equation of the line parallel to PQ that passes through the point $(6, -1)$.

Gradient$_{PQ}$: $m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-2-8}{9--3}$

$= \frac{-10}{12}$

$= -\frac{5}{6}$

Equation: $y = \frac{-5}{6}x + c$

Sub. $(6,-1)$:

$-1 = \frac{-5}{6}(6) + c$

$-1 = -5 + c$

$+5 \quad +5$

$4 = c$

$y = \frac{-5}{6}x + 4$ [3]

(iii) Find the equation of the perpendicular bisector of PQ.

$m = \frac{-5}{6}$

$m_{perp} = \frac{6}{5}$

Mid-point of PQ:

$\left(\frac{-3+9}{2}, \frac{8+-2}{2}\right)$

$= (3, 3)$

Equation:

$y = \frac{6}{5}x + c$

Sub. $(3,3)$:

$3 = \frac{6}{5}(3) + c$

$3 = \frac{18}{5} + c$

$-\frac{18}{5} \quad -\frac{18}{5}$

$-\frac{3}{5} = c$

$y = \frac{6}{5}x - \frac{3}{5}$ [4]

8 AB is a line with midpoint M.
A is the point (2, 3) and M is the point (12, 7).

(a) Find the coordinates of B. ← (x, y)

x-coord: $\frac{x+2}{2} = 12$ ×2 ×2

$x + 2 = 24$ → $x = 22$

y-coord: $\frac{y+3}{2} = 7$ ×2 ×2

$y + 3 = 14$ → $y = 11$

(22 , 11) [2]

(b) Show that the equation of the perpendicular bisector of AB is $2y + 5x = 74$.

Gradient$_{AB}$:

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{11-3}{22-2}$

$= \frac{8}{20}$

$= \frac{2}{5}$

$m_{perp} = \frac{-5}{2}$

Equation: $y = \frac{-5}{2}x + c$

Sub. (12, 7):

$7 = \frac{-5}{2}(12) + c$

$7 = -30 + c$
+30 +30

$37 = c$

$y = \frac{-5}{2}x + 37$
×2 ×2 ×2

$2y = -5x + 74$
+5x +5x

$2y + 5x = 74$

[4]

10 A is the point $(-5, 7)$ and C is the point $(1, -2)$.

(a) B is the mid-point of AC.

Find the coordinates of B.

$\left(\frac{-5+1}{2}, \frac{7+-2}{2}\right) = \left(\frac{-4}{2}, \frac{5}{2}\right)$

$= (-2, 5/2)$

(-2 , $\frac{5}{2}$) [2]

(b) The line CD is perpendicular to the line AC.

Find the equation of line CD.

CD means the line passes through point C, so this is the point we'll substitute.

Gradient: $m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-2-7}{1--5}$

$= \frac{-9}{6}$

$= \frac{-3}{2}$

$m_{perp} = \frac{2}{3}$

Equation:
$y = \frac{2}{3}x + c$

Sub. (1, −2):

$-2 = \frac{2}{3}(1) + c$

$-2 = \frac{2}{3} + c$
$-\frac{2}{3}$ $-\frac{2}{3}$

$-\frac{8}{3} = c$

$y = \frac{2}{3}x - \frac{8}{3}$ [4]

15 C is the point $(5, -1)$ and D is the point $(13, 15)$.

$x_1 \quad y_1 \qquad x_2 \quad y_2$

(a) Find the midpoint of CD.

$$\left(\frac{5+13}{2}, \frac{-1+15}{2}\right)$$

$$= \left(\frac{18}{2}, \frac{14}{2}\right)$$

$$= (9, 7)$$

(........9........ , ...7........) [2]

(b) Find the gradient of CD.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{15 - -1}{13 - 5}$$

$$= \frac{16}{8}$$

$$= 2$$

........2........ [2]

(c) Find the equation of the perpendicular bisector of CD.
Give your answer in the form $y = mx + c$.

$$m_{perp} = \frac{-1}{2}$$

Equation: $y = \frac{-1}{2}x + c$

Sub. $(9,7)$: $7 = \frac{-1}{2}(9) + c$

$$7 = \frac{-9}{2} + c$$

$+\frac{9}{2} \qquad +\frac{9}{2}$

$$\frac{23}{2} = c$$

$y =$ ....$\frac{-1}{2}x + \frac{23}{2}$........ [3]

14 The line L is perpendicular to the line $2y = 5 - x$ and passes through the point $(2, 3)$.

Find the equation of line L.
Give your answer in the form $y = mx + c$.

$$2y = -x + 5$$

$\div 2 \qquad \div 2 \qquad \div 2$

$$y = -\frac{1}{2}x + \frac{5}{2}$$

$$m = -\frac{1}{2}$$

$$m_{perp} = 2$$

Equation: $y = 2x + c$

Sub. $(2,3)$:

$$3 = 2(2) + c$$

$$3 = 4 + c$$

$-4 \qquad -4$

$$-1 = c$$

$$y = 2x - 1$$

$y =$ ....$2x - 1$........ [4]

10 **(a)** A rhombus $ABCD$ has a diagonal AC where A is the point $(-3, 10)$ and C is the point $(4, -4)$.

$x_1 \quad y_1 \qquad x_2 \quad y_2$

(i) Calculate the length AC.

$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$
$$= \sqrt{(4 - -3)^2 + (-4-10)^2}$$
$$= \sqrt{7^2 + (-14)^2}$$
$$= \sqrt{49 + 196}$$
$$= \sqrt{245}$$

$7\sqrt{5}$ (or 15.7 to 3sf) [3]

(ii) Show that the equation of the line AC is $y = -2x + 4$.

Gradient: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{-4 - 10}{4 - -3}$

$= \dfrac{-14}{7}$

$= -2$

Equation: $y = -2x + c$

sub. $(-3, 10)$:

$10 = -2(-3) + c$

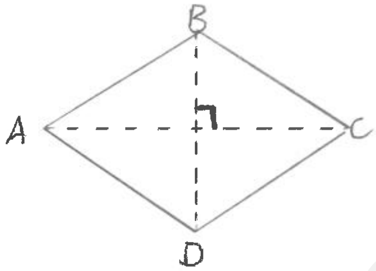
$10 = 6 + c$

$-6 \quad -6$

$4 = c$

$\rightarrow y = -2x + 4$ [2]

(iii) Find the equation of the line BD.



Gradient$_{BD}$: $m_{perp} = \dfrac{1}{2}$

Mid-point of AC:

$\left(\dfrac{-3+4}{2}, \dfrac{10+-4}{2}\right)$

$= \left(\dfrac{1}{2}, 3\right)$

Equation:

$y = \dfrac{1}{2}x + c$

sub. $\left(\dfrac{1}{2}, 3\right)$:

$3 = \dfrac{1}{2}\left(\dfrac{1}{2}\right) + c$

$3 = \dfrac{1}{4} + c$

$-\dfrac{1}{4} \quad -\dfrac{1}{4}$

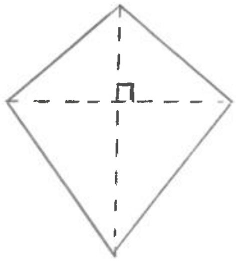
$\dfrac{11}{4} = c$

$y = \dfrac{1}{2}x + \dfrac{11}{4}$ [4]

16 A kite is drawn on a coordinate grid.
The diagonals of the kite intersect at the point $(-2, -5)$.

One diagonal has equation $y = 4x + 3$.

Find the equation of the other diagonal of the kite.
Give your answer in the form $y = mx + c$.



$M_{perp} = \frac{-1}{4}$

Equation:

$y = \frac{-1}{4}x + C$

Sub. $(-2, -5)$:

$-5 = \frac{-1}{4}(-2) + C$

$-5 = \frac{1}{2} + C$

$-\frac{1}{2} \quad -\frac{1}{2}$

$\frac{-11}{2} = C$

$y = \frac{-1}{4}x - \frac{11}{2}$ [3]

24 A line from the point $(2, 3)$ is perpendicular to the line $y = \frac{1}{3}x + 1$.
The two lines meet at the point P.

Find the coordinates of P.

$M_{perp} = -3$

Equation:

$y = -3x + C$

Sub. $(2, 3)$:

$3 = -3(2) + C$

$3 = -6 + C$

$+6 \quad +6$

$9 = C$

$y = -3x + 9$

Find where lines intersect:

$\frac{1}{3}x + 1 = -3x + 9$

$\times 3 \quad \times 3 \quad \times 3 \quad \times 3$

$x + 3 = -9x + 27$

$+9x \quad +9x$

$10x + 3 = 27$

$-3 \quad -3$

$10x = 24$

$\div 10 \quad \div 10$

$x = \frac{12}{5}$

Sub into either eq^n:

$y = \frac{1}{3}\left(\frac{12}{5}\right) + 1$

$= \frac{12}{15} + 1$

$= \frac{9}{5}$

$\left(\frac{12}{5} , \frac{9}{5} \right)$ [5]

16 The point A has coordinates (2, 3) and the point B has coordinates (6, 5).
The point C lies on the line AB.
The point D has coordinates (2, 5.5).
CD is perpendicular to AB.

Find the coordinates of C.

C lies on both lines, so find point of intersection of the two lines.

Gradient$_{AB}$: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{5-3}{6-2}$

$= \dfrac{2}{4}$

$= \dfrac{1}{2}$

Equation$_{AB}$: $y = \frac{1}{2}x + c$

Sub. (2,3): $3 = \frac{1}{2}(2) + c$

$3 = 1 + c$

$-1 \quad -1$

$2 = c$

line AB: $y = \frac{1}{2}x + 2$

Gradient$_{CD}$: $m_{perp} = -2$

Equation$_{CD}$: $y = -2x + c$

sub. (2, 5.5):

$5.5 = -2(2) + c$

$5.5 = -4 + c$

$+4 \quad +4$

$9.5 = c$

line CD: $y = -2x + 9.5$

line$_{AB}$ = line$_{CD}$

$\frac{1}{2}x + 2 = -2x + 9.5$

$\times 2 \quad \times 2 \quad \times 2 \quad \times 2$

$x + 4 = -4x + 19$

$+4x \quad +4x$

$5x + 4 = 19$

$-4 \quad -4$

$5x = 15$

$\div 5 \quad \div 5$

$x = 3$

Sub. into either equation:

$y = \frac{1}{2}x + 2$

$= \frac{1}{2}(3) + 2$

$= \frac{3}{2} + 2$

$= \frac{7}{2}$

$\rightarrow \left(3, \frac{7}{2}\right)$