21 The line $y = x + 1$ ① intersects the curve $y = x^2 + x - 3$ ② at two points.

Find the coordinates of the two points.

① = ②:

$$x^2 + x - 3 = x + 1$$
$$ -x - 1 \quad\quad -x - 1$$
$$x^2 - 4 = 0$$
$$(x + 2)(x - 2) = 0$$
$$\underline{x = -2} \text{ or } \underline{x = 2}$$

$x = 2: \rightarrow$ ①: $y = 2 + 1$
$$\underline{y = 3}$$

$x = -2: \rightarrow$ ①: $y = -2 + 1$
$$\underline{y = -1}$$

(....2...... , ...3......)
(...-2...... , ...-1......) [4]

18 Solve the simultaneous equations.
You must show all your working.

$$y = x^2 - 9x + 21 \quad ①$$
$$y = 2x - 3 \quad ②$$

① = ②:

$$x^2 - 9x + 21 = 2x - 3$$
$$ -2x + 3 \quad\quad -2x + 3$$
$$x^2 - 11x + 24 = 0$$
$$(x - 8)(x - 3) = 0$$

$$\underline{x = 8} \text{ or } \underline{x = 3}$$

$x = 8: \rightarrow$ ②:
$$y = 2(8) - 3$$
$$= 16 - 3$$
$$\underline{y = 13}$$

$x = 3: \rightarrow$ ②:
$$y = 2(3) - 3$$
$$= 6 - 3$$
$$\underline{y = 3}$$

$x = $ ......8...... $y = $ ......13......
$x = $ ......3...... $y = $ ......3...... [5]

27 The line $y = x + 1$ ① intersects the graph of $y = x^2 - 3x - 11$ ② at the points A and B.

Find the coordinates of A and the coordinates of B.
You must show all your working.

① = ② :

$$x^2 - 3x - 11 = x + 1$$
$$-x \quad -1 \qquad -x \quad -1$$
$$x^2 - 4x - 12 = 0$$
$$(x - 6)(x + 2) = 0$$
$$\underline{x = 6} \text{ or } \underline{x = -2}$$

$\underline{x = 6:} \rightarrow$ ① :
$$y = 6 + 1$$
$$\underline{y = 7}$$

$\underline{x = -2:} \rightarrow$ ① :
$$y = -2 + 1$$
$$\underline{y = -1}$$

A (......6...... , ...7......)

B (......−2...... , ...−1......) [4]

(b) $y = 2 - x$ ①
          $y = x^2 + 2x + 2$ ②

① = ②:

$$x^2 + 2x + 2 = 2 - x$$
$$+x \quad -2 \quad -2 \quad +x$$

$$x^2 + 3x = 0$$

$$x(x + 3) = 0$$

$$\underline{x = 0} \quad \text{or} \quad \underline{x = -3}$$

$\underline{x = 0}$: → ①: $\underline{x = -3}$: → ①:

$$y = 2 - 0 \qquad\qquad y = 2 - -3$$
$$\underline{y = 2} \qquad\qquad\quad \underline{y = 5}$$

$x =$ ....0........ $y =$ ....2........

$x =$ ......-3...... $y =$ ......5...... [4]

22 Solve the simultaneous equations.
You must show all your working.

$$y = x^2 - 3x - 13 \quad \text{①}$$
$$y = x - 1 \quad \text{②}$$

① = ②:

$$x^2 - 3x - 13 = x - 1$$
$$ -x \quad +1 \quad\quad -x \quad +1$$
$$x^2 - 4x - 12 = 0$$
$$(x - 6)(x + 2) = 0$$
$$\underline{x = 6} \quad \text{or} \quad \underline{x = -2}$$

$\underline{x = 6}: \to ②:$

$$y = 6 - 1$$
$$\underline{y = 5}$$

$\underline{x = -2}: \to ②:$

$$y = -2 - 1$$
$$\underline{y = -3}$$

$x =$ ......6......, $y =$ ......5......

$x =$ ......-2......, $y =$ ......-3...... [5]

24 The line $y = 7x + 3$ intersects the curve $y = x^2 + 5x - 12$ at the points A and B.

Find the coordinates of A and B.

①＝②: $x^2 + 5x - 12 = 7x + 3$
$\qquad\qquad\quad -7x \quad -3 \qquad\; -7x \; -3$

$x^2 - 2x - 15 = 0$

$(x - 5)(x + 3) = 0$

$\underline{x = 5}$ or $\underline{x = -3}$

$\underline{x = 5:} \rightarrow ①:$

$y = 7(5) + 3$
$\quad = 35 + 3$
$\underline{y = 38}$

$\underline{x = -3:} \rightarrow ①:$

$y = 7(-3) + 3$
$\quad = -21 + 3$
$\underline{y = -18}$

A (....5.... , ...38....)

B (....-3.... , ...-18....)

[5]

(b) The graphs of $y + 5x = 8$ and $y = 2x^2 + 6x - 13$ intersect at the points P and Q.

Find the coordinates of P and the coordinates of Q.
Show all your working.

$$y + 5x = 8$$
$$\quad -5x \quad -5x$$
$$y = 8 - 5x \quad ①$$

$$① = ②:$$

$$2x^2 + 6x - 13 = 8 - 5x$$
$$\quad +5x \quad -8 \quad -8 + 5x$$

$$2x^2 + 11x - 21 = 0$$

$ac = -42$

two numbers: $14, -3$

$$2x^2 + 14x - 3x - 21 = 0$$
$$2x(x + 7) - 3(x + 7) = 0$$
$$(2x - 3)(x + 7) = 0$$

$2x - 3 = 0$ or $x + 7 = 0$
$2x = 3 \qquad \underline{x = -7}$
$$\underline{x = \frac{3}{2}}$$

$x = \frac{3}{2}: \rightarrow ①:$
$$y = 8 - 5\left(\frac{3}{2}\right)$$
$$= 8 - \frac{15}{2}$$
$$\underline{y = \frac{1}{2}}$$

$x = -7: \rightarrow ①:$
$$y = 8 - 5(-7)$$
$$= 8 + 35$$
$$\underline{y = 43}$$

$P(\ \underline{\frac{3}{2}}\ ,\ \underline{\frac{1}{2}}\ )$

$Q(\ \underline{-7}\ ,\ \underline{43}\ )$ [6]

24 The straight line $y = 2x + 1$ ① intersects the curve $y = x^2 + 3x - 4$ ② at the points A and B.

Find the coordinates of A and B.
Give your answers correct to 2 decimal places.

① = ②:

$$x^2 + 3x - 4 = 2x + 1$$
$$ -2x \quad -1 \quad\quad -2x \quad -1$$

$$x^2 + x - 5 = 0$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-5)}}{2(1)}$$

$$x = \frac{-1 \pm \sqrt{21}}{2}$$

$$\underline{x = 1.791...} \quad \text{or} \quad \underline{x = -2.791...}$$
$$ \text{STO} \quad\quad\quad\quad \text{STO}$$

$$\underline{x = 1.791..} \rightarrow ①:$$
$$y = 2(1.791..) + 1$$
$$\underline{y = 4.58}$$

$$A(\underline{\quad 1.79 \quad}, \underline{\quad 4.58 \quad})$$
$$B(\underline{\quad -2.79 \quad}, \underline{\quad -4.58 \quad})$$
$$[6]$$

$$\underline{x = -2.791..} \rightarrow ①:$$
$$y = 2(-2.791..) + 1$$
$$\underline{y = -4.58}$$

22 The line $y = 4x + 12$ intersects the curve $y = 2x^2 - x - 3$ at point P and point Q.

Find the coordinates of P and Q.
You must show all your working and give your answers correct to 2 decimal places.

$①$ $②$

$① = ②$:

$$2x^2 - x - 3 = 4x + 12$$
$$ -4x - 12 \quad -4x - 12$$

$$2x^2 - 5x - 15 = 0$$

$$x = \frac{5 \pm \sqrt{(-5)^2 - 4(2)(-15)}}{2(2)}$$

$$= \frac{5 \pm \sqrt{25 + 120}}{4}$$

$$= \frac{5 \pm \sqrt{145}}{4}$$

$$x = 4.260\ldots \quad \text{or} \quad x = -1.760\ldots$$
$$\text{STO} \qquad\qquad\qquad \text{STO}$$

$x = 4.260\ldots \rightarrow ①$:

$$y = 4(4.260\ldots) + 12$$
$$y = 29.04$$

$x = -1.760\ldots \rightarrow ①$:

$$y = 4(-1.760\ldots) + 12$$
$$y = 4.96$$

(4.26 , 29.04)

(-1.76 , 4.96)

[6]

19 Find the values of x when $6x + y = 10$ and $y = x^2 - 3x + 10$.

②

$6x + y = 10$

$-6x \qquad -6x$

$y = 10 - 6x$ ①

① = ②:

$x^2 - 3x + 10 = 10 - 6x$

$+6x \quad -10 \quad -10 \quad +6x$

$x^2 + 3x = 0$

$x(x + 3) = 0$

$\underline{x = 0}$ or $\underline{x = -3}$

not needed

$x = 0: \rightarrow$ ①:

$y = 10 - 6(0)$

$= 10 - 0$

$\underline{y = 10}$

$x = -3: \rightarrow$ ①:

$y = 10 - 6(-3)$

$= 10 + 18$

$\underline{y = 28}$

$x = \underline{\quad 0 \quad}$ or $x = \underline{\quad -3 \quad}$ [3]

19 Solve the simultaneous equations.
You must show all your working.

$x - y = 7$

$x^2 + y = 149$ ②

$x - y = 7$

$+y \quad +y$

$x = y + 7$ ①

Sub. ① into ②:

$(y + 7)^2 + y = 149$

$y^2 + 14y + 49 + y = 149$

$y^2 + 15y + 49 = 149$

$\qquad -149 \qquad -149$

$y^2 + 15y - 100 = 0$

$(y + 20)(y - 5) = 0$

$\underline{y = -20}$ or $\underline{y = 5}$

$y = 5: \rightarrow$ ①:

$x = 5 + 7$

$\underline{x = 12}$

$y = -20: \rightarrow$ ①:

$x = -20 + 7$

$\underline{x = -13}$

$x = \underline{\ 12\ }$ $y = \underline{\ 5\ }$

$x = \underline{\ -13\ }$ $y = \underline{\ -20\ }$ [5]

20 Solve the simultaneous equations.
    You must show all your working.

$$3x + y = 11$$
$$x^2 - 2y = 18 \,②$$

$$3x + y = 11$$
$$-3x \qquad -3x$$
$$y = 11 - 3x \,①$$

Sub. ① into ②:

$$x^2 - 2(11 - 3x) = 18$$
$$x^2 - 22 + 6x = 18$$
$$x^2 + 6x - 22 = 18$$
$$\qquad\qquad -18 \quad -18$$
$$x^2 + 6x - 40 = 0$$
$$(x + 10)(x - 4) = 0$$
$$\underline{x = -10} \text{ or } \underline{x = 4}$$

$x = 4: \to ①:$
$$y = 11 - 3(4)$$
$$= 11 - 12$$
$$\underline{y = -1}$$

$x = -10: \to ①:$
$$y = 11 - 3(-10)$$
$$= 11 + 30$$
$$\underline{y = 41}$$

$x = $ ....4.......... $y = $ ....-1..........

$x = $ ....-10.......... $y = $ ....41.......... [5]

(d) Solve the simultaneous equations.
You must show all your working.

$$x + 2y = 12$$
$$5x + y^2 = 39 \quad ②$$

$$x + 2y = 12$$
$$-2y \qquad -2y$$
$$x = 12 - 2y \quad ①$$

sub. ① into ② :

$$5(12 - 2y) + y^2 = 39$$
$$60 - 10y + y^2 = 39$$
$$y^2 - 10y + 60 = 39$$
$$\qquad\quad -39 \qquad -39$$
$$y^2 - 10y + 21 = 0$$
$$(y - 7)(y - 3) = 0$$

$$\underline{y = 7} \quad \text{or} \quad \underline{y = 3}$$

$$\underline{y = 3} : \rightarrow ① :$$
$$x = 12 - 2(3)$$
$$= 12 - 6$$
$$\underline{x = 6}$$

$$\underline{y = 7} : \rightarrow ① :$$
$$x = 12 - 2(7)$$
$$= 12 - 14$$
$$\underline{x = -2}$$

$x =$ ......6...... $y =$ .....3.........

$x =$ ......-2...... $y =$ .....7......... [5]

(b) Solve the simultaneous equations.
You must show all your working.

$$x^2 + 4y = 37 \quad ①$$
$$5x + y = -8$$

$5x + y = -8$
$-5x \qquad -5x$

$\qquad y = -8 - 5x \quad ②$

Sub. ② into ①:

$x^2 + 4(-8 - 5x) = 37$

$x^2 - 32 - 20x = 37$

$x^2 - 20x - 32 = 37$
$\qquad -37 \qquad -37$

$x^2 - 20x - 69 = 0$

$(x - 23)(x + 3) = 0$

$\underline{x = 23}$ or $\underline{x = -3}$

$\underline{x = 23} \rightarrow ②:$

$y = -8 - 5(23)$

$\quad = -8 - 115$

$\underline{y = -123}$

$\underline{x = -3} \rightarrow ②:$

$y = -8 - 5(-3)$

$\quad = -8 + 15$

$\underline{y = 7}$

$x = \underline{\quad 23 \quad}$, $y = \underline{\quad -123 \quad}$

$x = \underline{\quad -3 \quad}$, $y = \underline{\quad 7 \quad}$ [5]

22 Find the coordinates of the point where the line $4x + y = 9$ intersects the curve $y + x^2 = 5$. ②
You must show all your working.

$4x + y = 9$
$-4x \qquad -4x$

$\qquad y = 9 - 4x \quad ①$

Sub. ① into ②:

$9 - 4x + x^2 = 5$

$x^2 - 4x + 9 = 5$
$\qquad -5 \qquad -5$

$x^2 - 4x + 4 = 0$

$(x - 2)(x - 2) = 0$

$\underline{x = 2}$

Sub. into ①:

$y = 9 - 4(2)$

$\quad = 9 - 8$

$\underline{y = 1}$

$(\underline{\quad 2 \quad}, \underline{\quad 1 \quad})$ [5]

(ii) The graphs of $y = (x-1)^2$ ① and $y = \frac{1}{2}x + 1$ ② intersect at A and B.

Find the length of AB.

① = ② :

$(x-1)^2 = \frac{1}{2}x + 1$

$(x-1)(x-1) = \frac{1}{2}x + 1$

$x^2 - 2x + 1 = \frac{1}{2}x + 1$
$\qquad\qquad -1 \qquad\qquad -1$

$x^2 - 2x = \frac{1}{2}x$
$\times 2 \quad \times 2 \qquad \times 2$

$2x^2 - 4x = x$
$\qquad -x \qquad -x$

$2x^2 - 5x = 0$

$x(2x - 5) = 0$

$\underline{x = 0}$ or $2x - 5 = 0$
$\qquad\qquad\qquad 2x = 5$
$\qquad\qquad\qquad \underline{x = \frac{5}{2}}$

$\underline{x = 0}$: → ② :

$y = \frac{1}{2}(0) + 1$

$\underline{y = 1}$ → $\underline{(0, 1)}$ A

$\underline{x = \frac{5}{2}}$: → ② :

$y = \frac{1}{2}\left(\frac{5}{2}\right) + 1$

$\quad = \frac{5}{4} + 1$

$\underline{y = \frac{9}{4}}$ → $\underline{\left(\frac{5}{2}, \frac{9}{4}\right)}$ B

Distance between A and B :

$d = \sqrt{\left(\frac{5}{2} - 0\right)^2 + \left(\frac{9}{4} - 1\right)^2}$

$\quad = \sqrt{\frac{25}{4} + \frac{25}{16}}$

$\quad = \frac{5\sqrt{5}}{4}$ or 2.80

$\underline{\dfrac{5\sqrt{5}}{4}}$ (or 2.80) [7]

---

20 The curve $y = x^2 - 2x + 1$ is drawn on a grid.
A line is drawn on the same grid.
The points of intersection of the line and the curve are used to solve the equation $x^2 - 7x + 5 = 0$.

Find the equation of the line in the form $y = mx + c$.

Start with $y = x^2 - 7x + 5$, but $= 0$:

$x^2 - 7x + 5 = 0$

Now add and subtract terms to make it the same as $x^2 - 2x + 1$:

$x^2 - 7x + 5 = 0$
$\qquad\qquad -4 \qquad -4$

$x^2 - 7x + 1 = -4$
$\qquad +5x \qquad\qquad +5x$

$x^2 - 2x + 1 = \underline{5x - 4}$

this is the line we need to draw

$\underline{y = 5x - 4}$ [1]

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(c) Solve the simultaneous equations.
You must show all your working and give your answers correct to 2 decimal places.

$$2x + y = 7$$
$$y = 5x^2 + 2x - 13 \quad ②$$

$$2x + y = 7$$
$$-2x \qquad \qquad -2x$$

$$y = 7 - 2x \quad ①$$

$$① = ②:$$

$$5x^2 + 2x - 13 = 7 - 2x$$
$$+2x \quad -7 \qquad -7 \ +2x$$

$$5x^2 + 4x - 20 = 0$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(5)(-20)}}{2(5)}$$

$$= \frac{-4 \pm \sqrt{416}}{10}$$

$$x = 1.639... \quad or \quad x = -2.439...$$
$$\text{STO} \qquad\qquad\qquad \text{STO}$$

$$x = 1.639.. \rightarrow ①:$$

$$y = 7 - 2(1.639..)$$
$$y = 3.72$$

$$x = -2.439.. \rightarrow ①:$$

$$y = 7 - 2(-2.439..)$$
$$y = 11.88$$

$x =$ .....1.64..... , $y =$ .....3.72.....

$x =$ .....-2.44..... , $y =$ .....11.88..... [6]

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16 Solve the simultaneous equations.
You must show all your working.

$$x = 7 - 3y \quad ①$$
$$x^2 - y^2 = 39 \quad ②$$

Sub. ① into ②:

$$(7 - 3y)^2 - y^2 = 39$$

$$(7 - 3y)(7 - 3y) - y^2 = 39$$

$$49 - 42y + 9y^2 - y^2 = 39$$

$$8y^2 - 42y + 49 = 39$$
$$\qquad\qquad -39 \qquad -39$$

$$8y^2 - 42y + 10 = 0$$
$$\div 2 \quad \div 2 \quad \div 2 \quad \div 2$$

$$4y^2 - 21y + 5 = 0$$

$ac = 20$

two numbers: $-20, -1$

$$4y^2 - 20y - 1y + 5 = 0$$

$$4y(y - 5) - 1(y - 5) = 0$$

$$(4y - 1)(y - 5) = 0$$

$$4y - 1 = 0 \qquad \text{or} \quad y - 5 = 0$$
$$4y = 1 \qquad\qquad\qquad \underline{y = 5}$$
$$\underline{y = \tfrac{1}{4}}$$

$y = \frac{1}{4} : \rightarrow ①$:

$$x = 7 - 3(\tfrac{1}{4})$$
$$= 7 - \tfrac{3}{4}$$
$$\underline{x = \tfrac{25}{4}}$$

$y = 5 : \rightarrow ①$:

$$x = 7 - 3(5)$$
$$= 7 - 15$$
$$\underline{x = -8}$$

$$\underline{x = \frac{25}{4}, \quad y = \frac{1}{4}}$$

$$\underline{x = -8, \quad y = 5}$$

[6]

(c) Solve the simultaneous equations.
You must show all your working.

$$y = 4 - x \quad ①$$
$$x^2 + 2y^2 = 67 \quad ②$$

Sub. ① into ②:

$$x^2 + 2(4-x)^2 = 67$$
$$x^2 + 2(4-x)(4-x) = 67$$
$$x^2 + 2(16 - 8x + x^2) = 67$$
$$x^2 + 32 - 16x + 2x^2 = 67$$
$$3x^2 - 16x + 32 = 67$$
$$\qquad\qquad -67 \quad -67$$
$$3x^2 - 16x - 35 = 0$$

$$ac = -105$$
two numbers: 5, -21

$$\rightarrow 3x^2 - 21x + 5x - 35 = 0$$
$$3x(x-7) + 5(x-7) = 0$$
$$(3x + 5)(x - 7) = 0$$

$$(3x + 5)(x - 7) = 0$$
$$3x + 5 = 0 \text{ or } x - 7 = 0$$
$$3x = -5 \qquad \underline{x = 7}$$
$$x = \frac{-5}{3}$$

$\underline{x = 7} : \rightarrow ①:$
$$y = 4 - 7$$
$$\underline{y = -3}$$

$x = \frac{-5}{3} : \rightarrow ①:$
$$y = 4 - \frac{-5}{3}$$
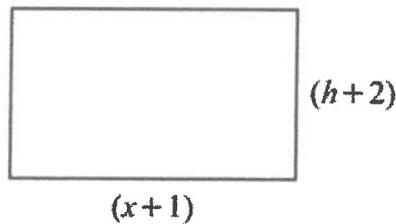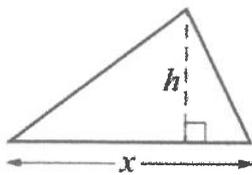$$\underline{y = \frac{17}{3}}$$

$x = \underline{\quad 7 \quad}, y = \underline{\quad -3 \quad}$
$x = \underline{\quad \frac{-5}{3} \quad}, y = \underline{\quad \frac{17}{3} \quad}$ [6]

21 In this question, all measurements are in centimetres.

NOT TO SCALE

The height of the triangle is h and the height of the rectangle is $(h + 2)$.
The length of the base of the triangle is x and the length of the rectangle is $(x + 1)$.
The area of the triangle is $11\,\text{cm}^2$ and the area of the rectangle is $39\,\text{cm}^2$.

(a) Write down an expression, in terms of x, for the height of the rectangle.

Area of triangle $= 11$:

$$\frac{1}{2} \times x \times h = 11 \quad \Rightarrow \quad h = \frac{22}{x} \quad \text{sub. into } (x+2):$$

$$xh = 22 \qquad \frac{22}{x} + 2$$

$$\underline{\frac{22}{x} + 2} \quad \text{..} [1]$$

(b) Show that $2x^2 - 15x + 22 = 0$.

Area of rectangle $= 39$:

$$(x + 1)\left(\frac{22}{x} + 2\right) = 39$$

$$22 + 2x + \frac{22}{x} + 2 = 39$$

$$2x + 24 + \frac{22}{x} = 39$$

$$2x^2 + 24x + 22 = 39x$$

$$-39x \qquad\qquad -39x$$

$$2x^2 - 15x + 22 = 0 \qquad\qquad\qquad [3]$$

(c) By factorising and solving $2x^2 - 15x + 22 = 0$, find the two possible heights of the triangle.

$ac = 44$

two numbers: $-4, -11$

$\rightarrow 2x^2 - 4x - 11x + 22 = 0$

$2x(x - 2) - 11(x - 2) = 0$

$(2x - 11)(x - 2) = 0$

$2x - 11 = 0 \quad$ or $\quad x - 2 = 0$
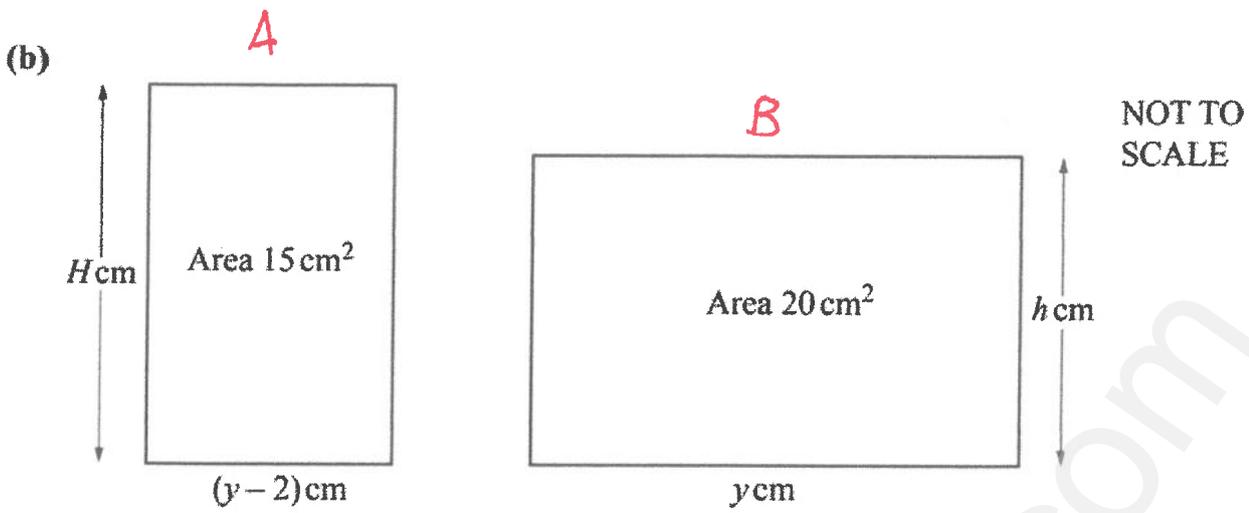
$2x = 11 \qquad\qquad \underline{x = 2}$

$\underline{x = 5.5}$

$h = \frac{22}{x}: \quad h = \frac{22}{5.5} \quad$ or $\quad h = \frac{22}{2}$

$\underline{h = 4} \qquad\qquad \underline{h = 11}$

$h = \text{.......}4\text{.......}$ or $h = \text{.......}11\text{.......}$ [5]

(b)

A

Area 15 cm²

H cm

$(y-2)$ cm

B

Area 20 cm²

h cm

y cm

NOT TO SCALE

The diagram shows two rectangles where $H - h = 1$.

By forming a quadratic equation and factorising, find the value of y.

A: Area = b×h

$15 = H(y-2)$

÷(y-2)   ÷(y-2)

$H = \dfrac{15}{y-2}$ ①

B: Area = b×h

$20 = hy$

÷y   ÷y

$h = \dfrac{20}{y}$ ②

$H - h = 1$:

$$\dfrac{15}{y-2} - \dfrac{20}{y} = 1$$

$$\dfrac{15}{y-2} \times \dfrac{y}{y} - \dfrac{20}{y} \times \dfrac{(y-2)}{(y-2)} = 1$$

$$\dfrac{15y - 20(y-2)}{y(y-2)} = 1$$

$$\dfrac{15y - 20y + 40}{y(y-2)} = 1$$

$$\dfrac{-5y + 40}{y(y-2)} = 1 \quad \times y(y-2)$$

$\times y(y-2)$

$-5y + 40 = y(y-2)$

$-5y + 40 = y^2 - 2y$

+5y − 40   +5y −40

$y^2 + 3y - 40 = 0$

$(y+8)(y-5) = 0$

$y = -8 \qquad y = 5$ ✓

✗ can't be negative

$y = \underline{\quad 5 \quad}$ [7]

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