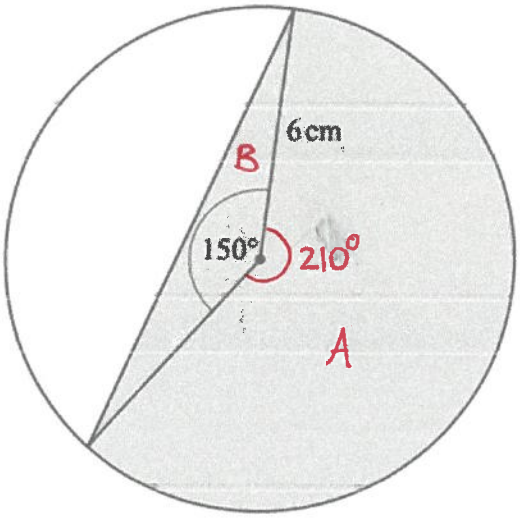
14



NOT TO SCALE

A sector of a circle with radius 6 cm has a sector angle of 150°.

Find the exact value of the area of the shaded region.
Give your answer in its simplest form.

Area of sector A:

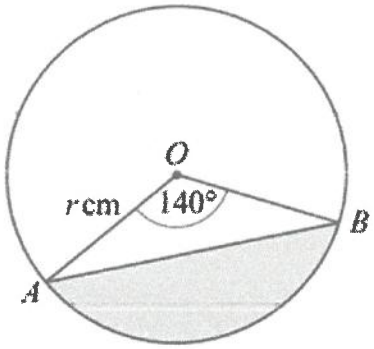
$$\text{Area} = \frac{210}{360} \times \pi r^2$$
$$= \frac{7}{12} \times \pi \times 6^2$$
$$= \frac{7}{12} \times 36\pi$$
$$= 21\pi$$

Area of triangle B:

$$\text{Area} = \frac{1}{2}ab\sin C$$
$$= \frac{1}{2} \times 7 \times 7 \times \sin 150$$
$$= \frac{1}{2} \times 49 \times \frac{1}{2}$$
$$= \frac{49}{4}$$

$\frac{49}{4} + 21\pi$ cm² [4]

(b)



NOT TO SCALE

A and B are points on a circle, centre O, radius r cm.
The area of the shaded segment is 65 cm^2.

Calculate the value of r.

Area of sector:

$$\text{Area} = \frac{140}{360} \times \pi r^2$$

$$= \frac{7}{18}\pi r^2$$

Area of triangle:

$$\text{Area} = \frac{1}{2}ab\sin C$$

$$= \frac{1}{2} \times r \times r \times \sin 140$$

$$= 0.32139 r^2$$

Shaded Area = sector - triangle:

$$\text{Area} = \frac{7}{18}\pi r^2 - 0.32139 r^2$$

$$65 = \left(\frac{7}{18}\pi - 0.32139\right) r^2$$

$$65 = 0.9003 r^2$$

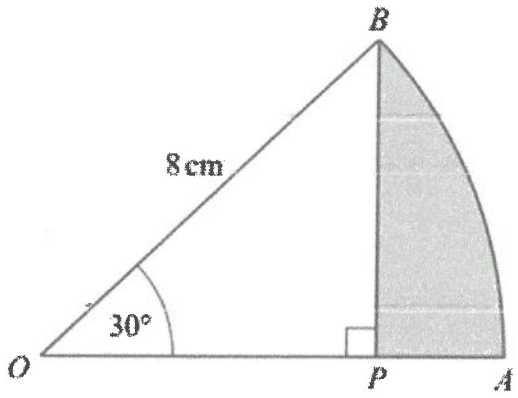
$$r^2 = \frac{65}{0.9003}$$

$$r = \sqrt{\frac{65}{0.9003}}$$

8.50 cm ..... [4]

(34)

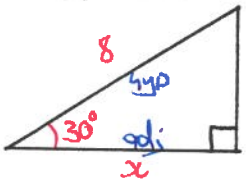
21



NOT TO SCALE

OAB is the sector of a circle, centre O.
$OB = 8$ cm and angle $AOB = 30°$.
BP is perpendicular to OA.

(a) Calculate AP.



$S^{o}{}_{H} C^{A}{}_{H} T^{o}{}_{A}$

$\cos 30 = \frac{x}{8}$ (×8 ×8)

$x = 8\cos 30$

$= 8 \times \frac{\sqrt{3}}{2}$

$= 4\sqrt{3}$

$AP = OA - x$:

$AP = 8 - 4\sqrt{3}$

$= 1.07$

$AP =$ 1.07 cm [3]
(3sf)

(b) Work out the area of the shaded region APB.

Area of Sector:

$\text{Area} = \frac{30}{360} \times \pi r^2$

$= \frac{1}{12} \times \pi \times 8^2$

$= \frac{1}{12} \times \pi \times 64$

$= \frac{16}{3}\pi$

Area of Triangle:

$\text{Area} = \frac{1}{2}ab\sin C$

$= \frac{1}{2} \times 4\sqrt{3} \times 8 \times \sin 30$

$= \frac{1}{2} \times 32\sqrt{3} \times \frac{1}{2}$

$= 8\sqrt{3}$

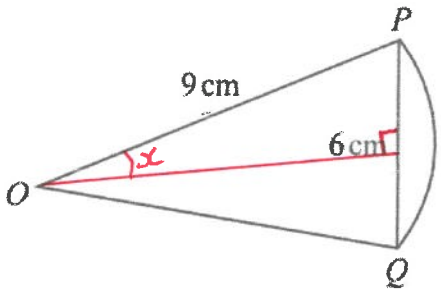
Shaded Area = Sector − Triangle:

$\text{Area} = \frac{16}{3}\pi - 8\sqrt{3}$

$= 2.90 \text{ cm}^2$ (3sf)

2.90 cm² [3]

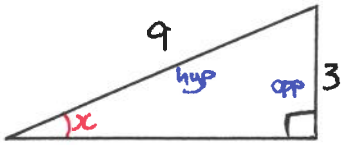
14



NOT TO SCALE

The diagram shows a sector of a circle with centre O and radius 9 cm.
The length of the chord PQ is 6 cm.

Calculate the length of the arc PQ.



$S^{\circ}{}_{H} C^{A}{}_{H} T^{\circ}{}_{A}$

$\sin x = \frac{3}{9}$

$\sin x = \frac{1}{3}$

$x = \sin^{-1}\left(\frac{1}{3}\right)$

$= 19.471^{\circ}$

Angle of sector $= 2x$:

$2 \times 19.471.. = 38.942^{\circ}$

Arc length:

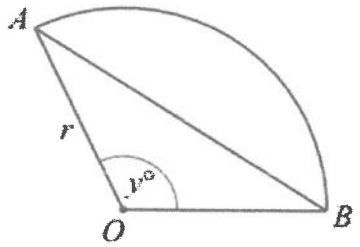
$\text{length} = \frac{38.942}{360} \times 2\pi r$

$= \frac{38.942}{360} \times 2 \times \pi \times 9$

$= 6.12 \text{ cm}$

6.12 ..... cm [3]

(b)



NOT TO SCALE

The diagram shows a sector, OAB, of a circle with centre O and radius r.
The area of triangle OAB is half the area of the sector.
Angle $AOB = y°$ and is obtuse.

(i) Show that $360 \sin y = \pi y$.

Area of Triangle:

$$\text{Area} = \frac{1}{2}ab\sin C$$
$$= \frac{1}{2} \times r \times r \times \sin y$$
$$= \frac{1}{2}r^2 \sin y$$

Area of Sector:

$$\text{Area} = \frac{y}{360} \times \pi r^2$$

Area of sector = 2 × triangle:

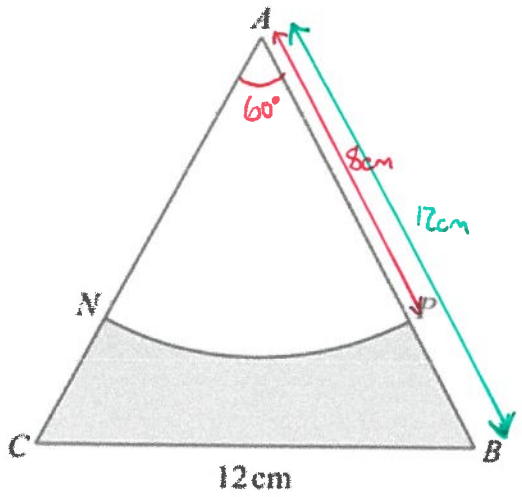
$$\frac{y}{360} \times \pi r^2 = 2 \times \frac{1}{2}r^2 \sin y$$

$$\frac{y}{360} \times \pi r^2 = r^2 \sin y \quad (\times 360)$$

$$y\pi r^2 = 360 r^2 \sin y \quad (\div r^2)$$

$$\pi y = 360 \sin y \quad [2]$$

21



NOT TO SCALE

The diagram shows an equilateral triangle ABC with all sides of length 12 cm.
ANP is a sector of a circle, centre A.
N lies on AC such that $AN : NC = 2 : 1$.
P lies on AB such that $AP : PB = 2 : 1$.

Calculate the area of the shaded region.

Area of Sector:

$$\text{Area} = \frac{60}{360} \times \pi r^2$$

$$= \frac{1}{6} \times \pi \times 8^2$$

$$= \frac{32}{3}\pi$$

Area of Triangle:

$$\text{Area} = \frac{1}{2}ab\sin C$$

$$= \frac{1}{2} \times 12 \times 12 \times \sin 60$$

$$= \frac{1}{2} \times 144 \times \frac{\sqrt{3}}{2}$$

$$= 36\sqrt{3}$$

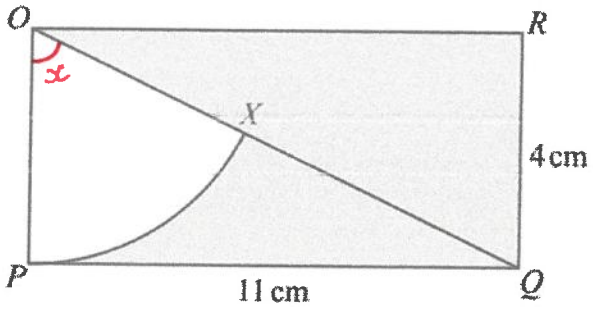
Shaded Area = Triangle - Sector:

$$\text{Area} = 36\sqrt{3} - \frac{32}{3}\pi$$

$$= 28.8\text{cm}^2$$

28.8 ..... cm^2 [4]

17

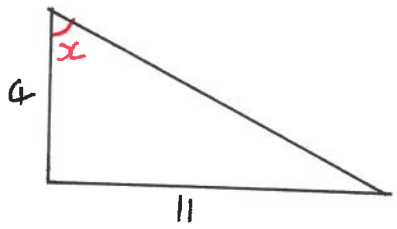


NOT TO SCALE

The diagram shows a rectangle $OPQR$ with length 11 cm and width 4 cm.
OQ is a diagonal and OPX is a sector of a circle, centre O.

Calculate the percentage of the rectangle that is shaded.

Angle x: $S^{o}_{H}C^{A}_{H}T^{o}_{A}$



$\tan x = \frac{11}{4}$

$x = \tan^{-1}\left(\frac{11}{4}\right)$

$= 70.017°$

Area of sector:

$\text{Area} = \frac{70.017}{360} \times \pi \times r^2$

$= \frac{70.017}{360} \times \pi \times 4^2$

$= 9.7762$

Shaded Area = Rectangle - Sector:

$\text{Area} = 11 \times 4 - 9.7762$

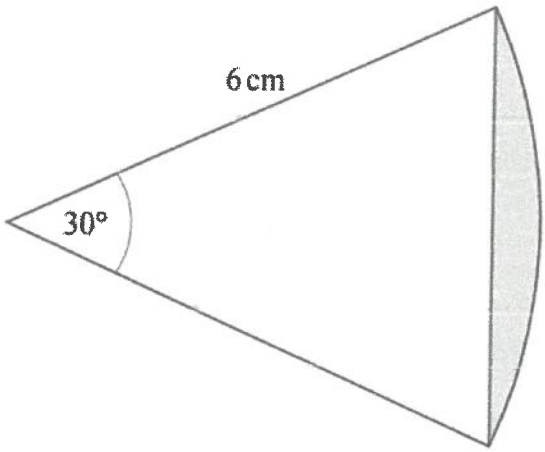
$= 44 - 9.7762$

$= 34.2238$

Percentage Shaded:

$\frac{34.2238}{44} \times 100 = 77.8\%$

13



NOT TO SCALE

The area of the shaded segment is $(a\pi + b)$ cm^2.

Find the value of a and the value of b.

Area of Sector:

$$\text{Area} = \frac{30}{360} \times \pi r^2$$

$$= \frac{1}{12} \times \pi \times 6^2$$

$$= \frac{1}{12} \times 36\pi$$

$$= 3\pi$$

Area of Triangle:

$$\text{Area} = \frac{1}{2}ab\sin C$$

$$= \frac{1}{2} \times 6 \times 6 \times \sin 30$$

$$= \frac{1}{2} \times 36 \times \frac{1}{2}$$

$$= 9$$

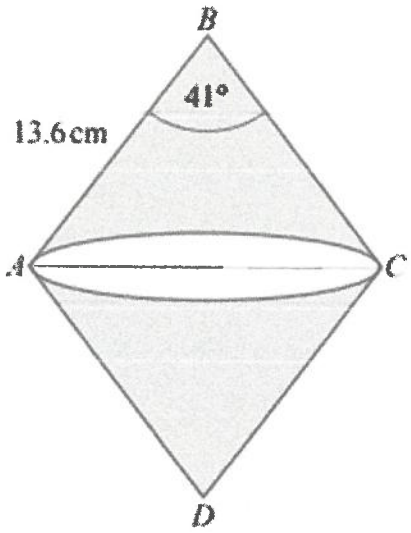
Shaded segment:

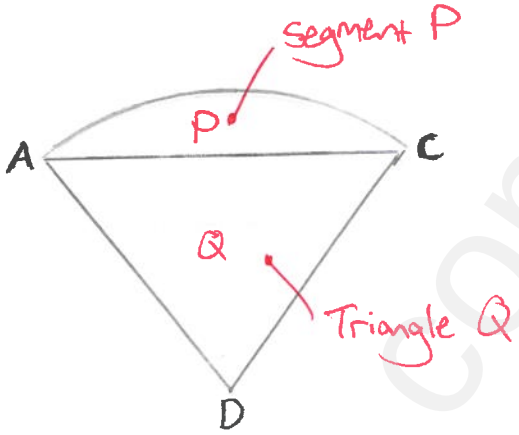
$$\text{Area} = 3\pi - 9$$

$a =$ 3

$b =$ −9 [4]

22





ABCD is a rhombus with side length 13.6cm.
Angle *ABC* = 41°.
BAC is a sector of a circle with centre *B*.
DAC is a sector of a circle with centre *D*.

Calculate the shaded area.

Shaded Area = 2x Triangle Q - 2x Segment P.

Triangle:

$$\text{Area} = \frac{1}{2}ab\sin C$$
$$= \frac{1}{2} \times 13.6 \times 13.6 \times \sin 41$$
$$= 60.672$$

Sector:

$$\text{Area} = \frac{41}{360} \times \pi r^2$$
$$= \frac{41}{360} \times \pi \times 13.6^2$$
$$= 66.177$$

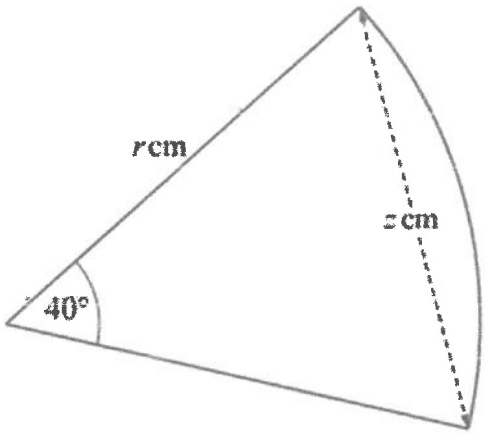
Segment = Sector - Triangle:

$$\text{Area} = 66.177 - 60.672$$
$$= 5.505$$

Shaded Area = 2x Triangle - 2x Segment:

$$\text{Area} = 2 \times 60.672 - 2 \times 5.505$$
$$= 121.344 - 11.01$$
$$= 110.334$$
$$= 110 \text{ cm}^2 \quad (3\text{sf})$$

(c)



NOT TO SCALE

This sector of a circle has radius r and perimeter 20 cm.

Find the value of z.

Arc length:

$$l = \frac{40}{360} \times 2\pi r$$

$$= \frac{1}{9} \times 2\pi r$$

$$= \frac{2}{9}\pi r$$

Perimeter = 20cm:

$$\text{Perimeter} = r + r + \frac{2}{9}\pi r$$

$$= 2r + \frac{2}{9}\pi r$$

$$2r + \frac{2}{9}\pi r = 20$$

$$r\left(2 + \frac{2}{9}\pi\right) = 20$$

$$r = \frac{20}{2 + \frac{2}{9}\pi}$$

$$= 7.4125\ldots \text{ STO}$$

Cosine Rule:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$z^2 = r^2 + r^2 - 2 \times r \times r \times \cos 40$$

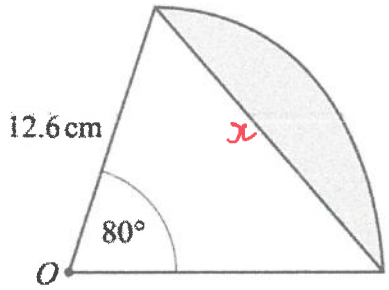
$$z^2 = (7.4125..)^2 + (7.4125..)^2 - 2 \times 7.4125..^2 \cos 40$$

$$z = \sqrt{2(7.4125..)^2 - 2(7.4125..)^2\cos 40}$$

$$= 5.07 \text{ cm}$$

$z =$ 5.07 cm [6]

19



NOT TO SCALE

The diagram shows a sector of a circle, centre O, radius 12.6 cm.

Calculate the perimeter of the shaded segment.

Arc Length:

$$l = \frac{80}{360} \times 2\pi r$$

$$= \frac{2}{9} \times 2 \times \pi \times 12.6$$

$$= \frac{28}{5}\pi$$

Side x (Cosine Rule):

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$x^2 = 12.6^2 + 12.6^2 - 2 \times 12.6 \times 12.6 \times \cos 80$$

$$x = \sqrt{12.6^2 + 12.6^2 - 2 \times 12.6^2 \cos 80}$$

$$= 16.198$$

Perimeter = Arc + x:

$$\frac{28}{5}\pi + 16.198 = 33.8\text{cm}$$