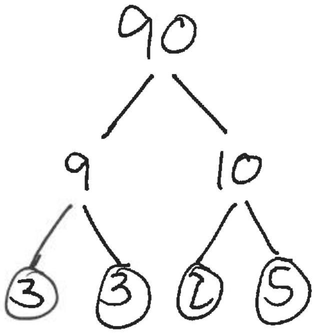
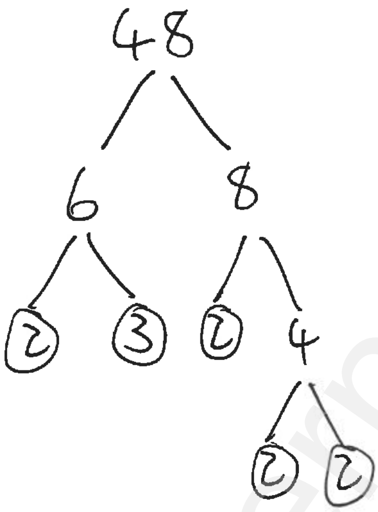
9 Write 90 as the product of its prime factors.



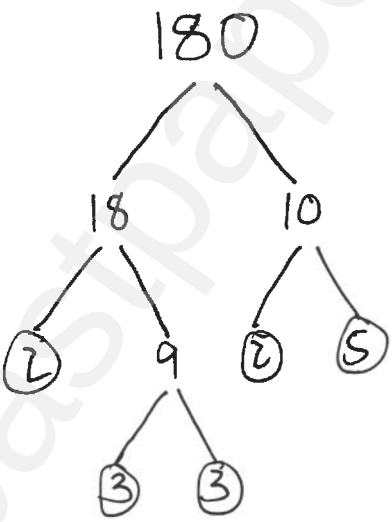
$2 \times 3^2 \times 5$ [2]

10 Write 48 as a product of its prime factors.



$2^4 \times 3$ [2]

6 Write 180 as a product of its prime factors.



$2^2 \times 3^2 \times 5$ [2]

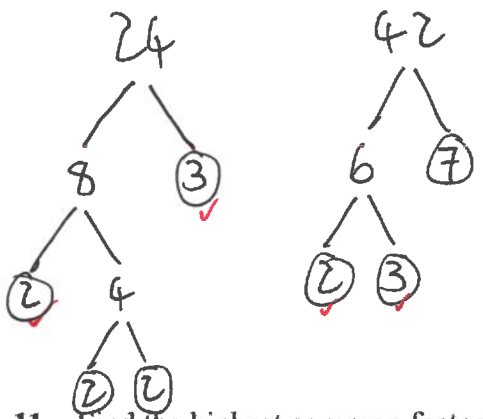
1 Write down a common multiple of 18 and 24.

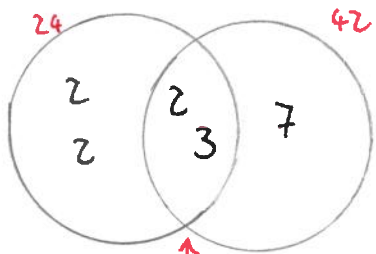
18 : 18, 36, 54, 72, ...

24 : 24, 48, 72, ...

72 (or 144, 216 etc.) .... [1]

(d) Find the highest common factor (HCF) of 24 and 42.





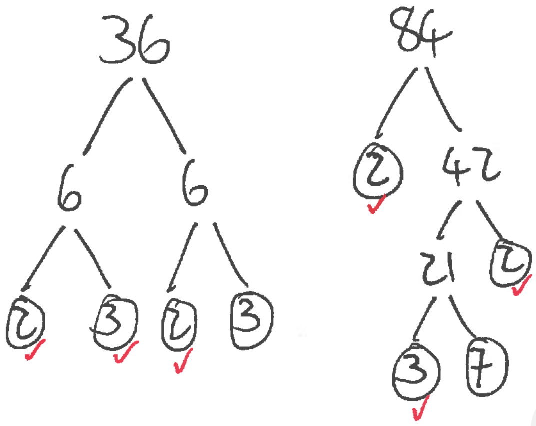
HCF = middle

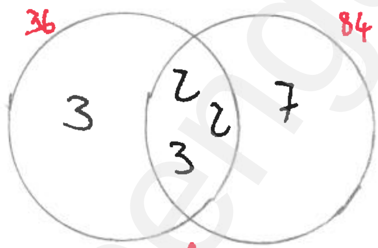
= 2×3

= 6

6 .... [1]

11 Find the highest common factor (HCF) of 36 and 84.



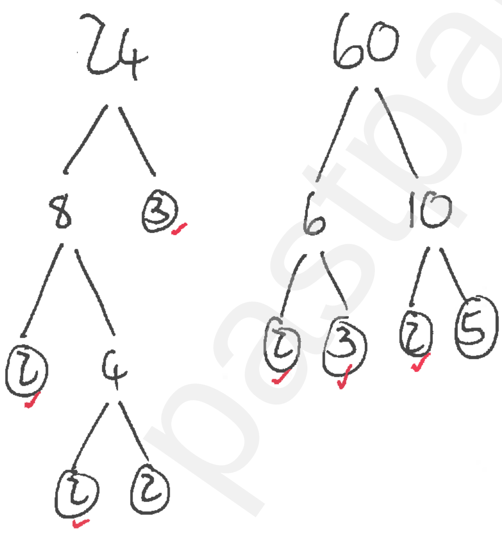


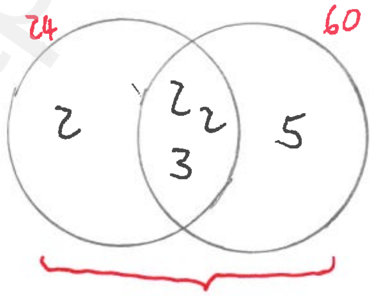
HCF = 2×2×3

= 12

12 .... [2]

6 Find the lowest common multiple (LCM) of 24 and 60.





LCM = whole thing

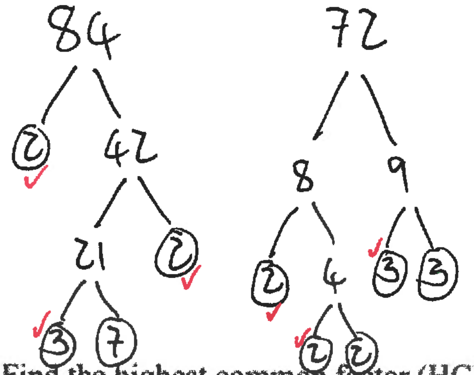
= 2×2×2×3×5

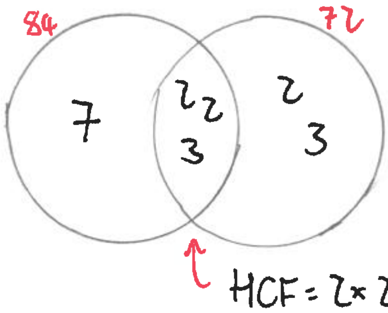
= 8 × 15

= 120

120 .... [2]

3 Find the highest common factor (HCF) of 84 and 72.

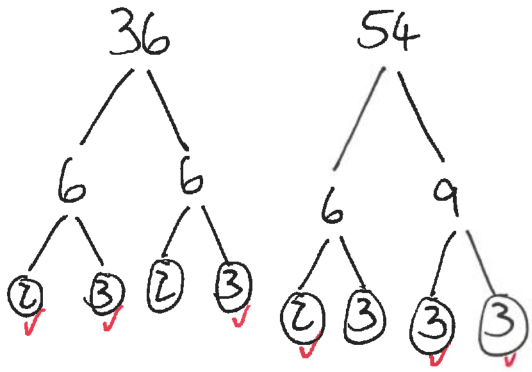


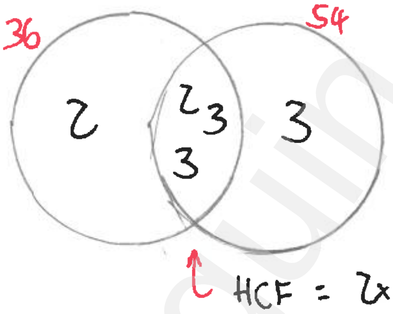


HCF = 2 × 2 × 3
= 12

12 .... [1]

8 Find the highest common factor (HCF) of 36 and 54.

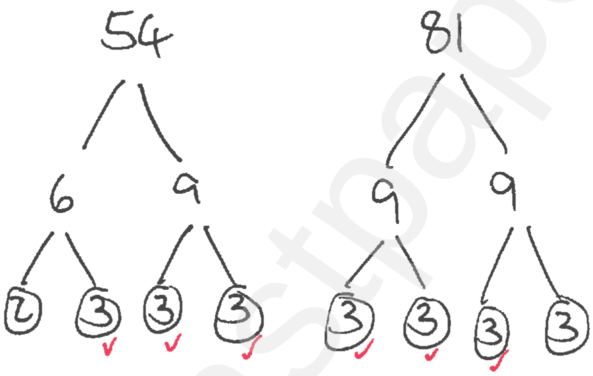




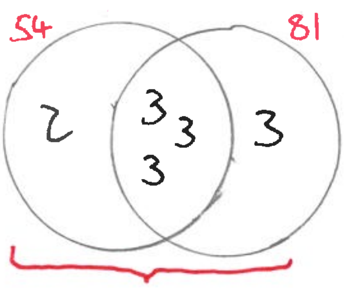
HCF = 2 × 3 × 3
= 18

18 .... [2]

(b) Find the lowest common multiple (LCM) of 54 and 81.

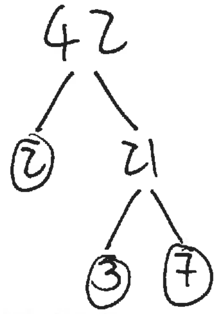


162 .... [2]



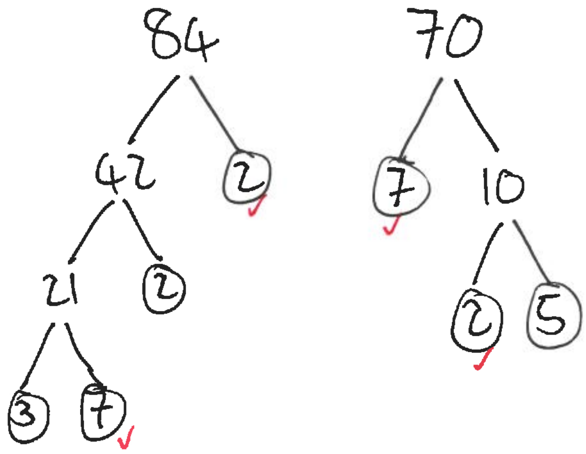
LCM = 2 × 3 × 3 × 3 × 3

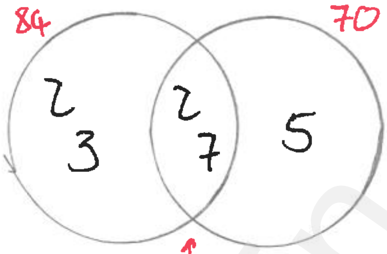
8 **(a)** Write 42 as a product of its prime factors.



2×3×7 .... [2]

(b) Find the highest common factor (HCF) of 84 and 70.

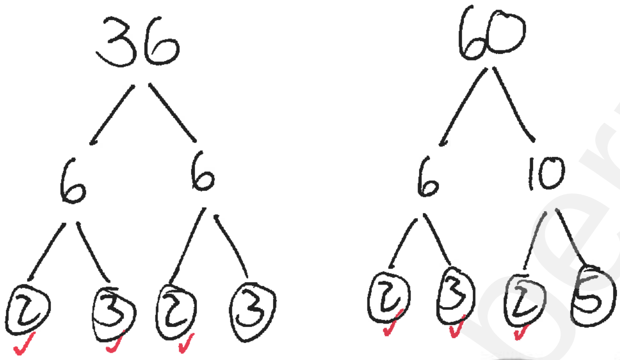




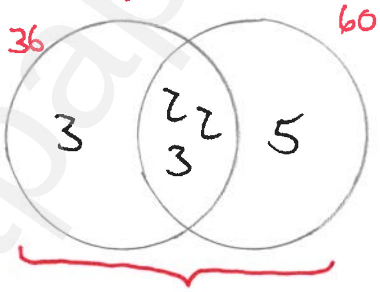
HCF = 2×7
= 14

14 .... [2]

7 Find the lowest common multiple (LCM) of 36 and 60.

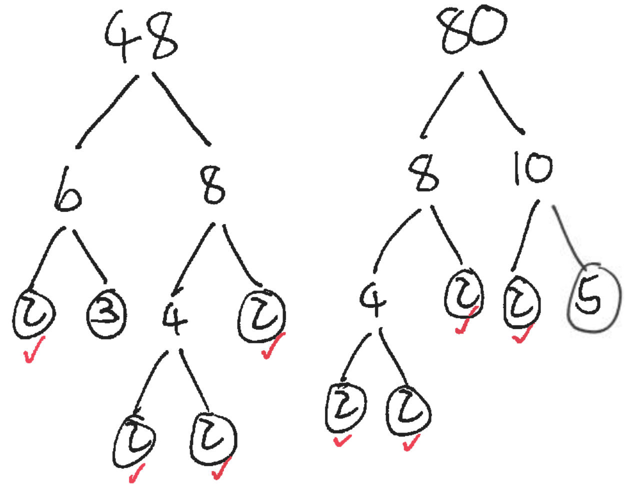


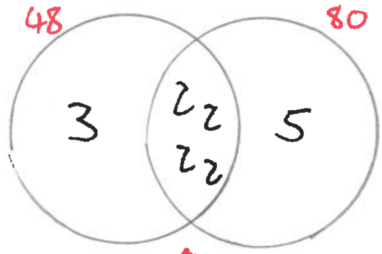
180 .... [2]



LCM = 2×2×3×3×5
= 4 × 9 ×5
= 20 ×9
= 180

8 Find the highest common factor (HCF) of 48 and 80.

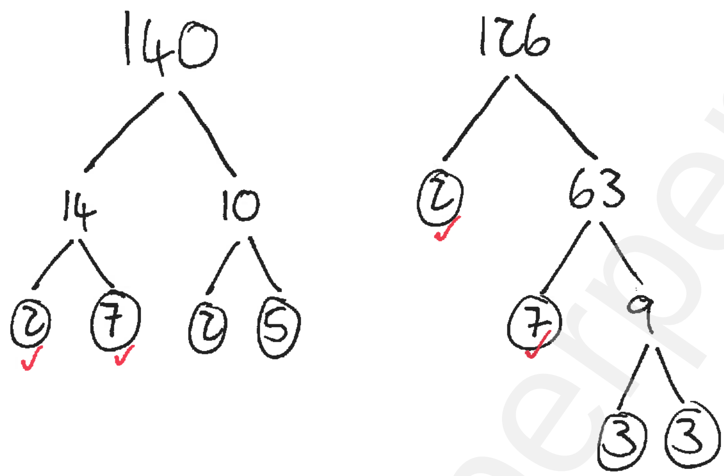


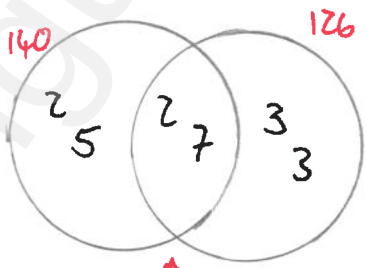


HCF = 2×2×2×2
= 4 × 4
= 16

16 .... [2]

9 Find the highest common factor (HCF) of 140 and 126.

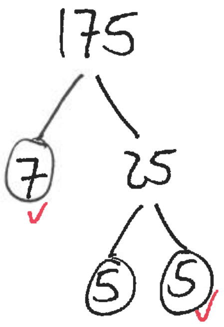




HCF = 2×7
= 14

14 .... [2]

4 (a) Express 175 as the product of its prime factors.

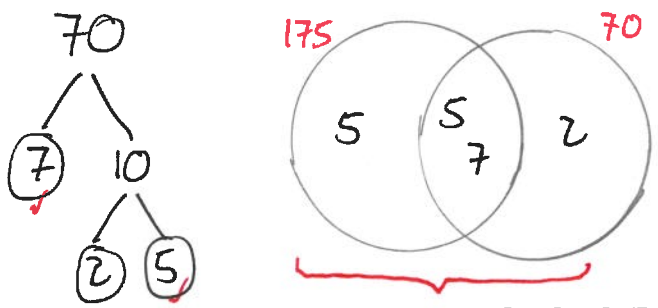


$5^2 \times 7$ .... [2]

(b) Kurt has two timers.
One is set to ring every 175 minutes.
The other is set to ring every 70 minutes.

Both timers ring together at 09 15.

Find the time when the timers next ring together. → LCM



LCM = $2 \times 5 \times 5 \times 7$
= 10×35
= 350

→ ring together after 350 mins
= 5 hours and 50 mins

09:15 $\xrightarrow{+50\text{mins}}$ 10:05 $\xrightarrow{+5\text{hrs}}$ 15:05

15:05 .... [3]

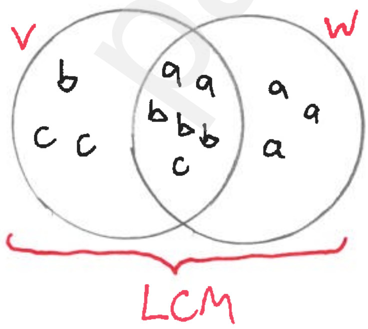
7 a, b and c are prime numbers.

$V = a^2b^4c^3$
$W = a^5b^3c$

Find the lowest common multiple (LCM) of V and W in terms of a, b and c.

$V = a \times a \times b \times b \times b \times b \times c \times c \times c$

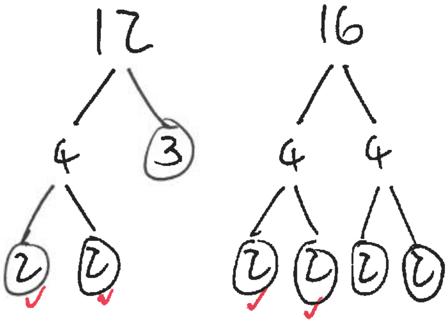
$W = a \times a \times a \times a \times a \times b \times b \times b \times c$

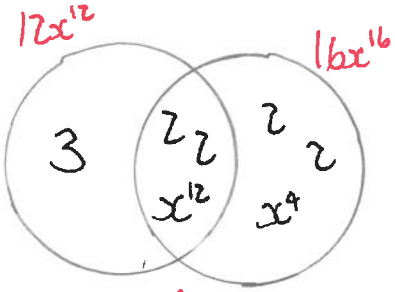


LCM = $a \times a \times a \times a \times a \times b \times b \times b \times b \times c \times c \times c$
= $a^5 b^4 c^3$

$a^5b^4c^3$ .... [2]

11 Find the highest common factor (HCF) of $12x^{12}$ and $16x^{16}$.



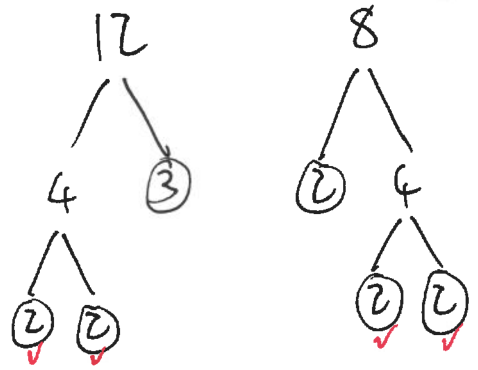


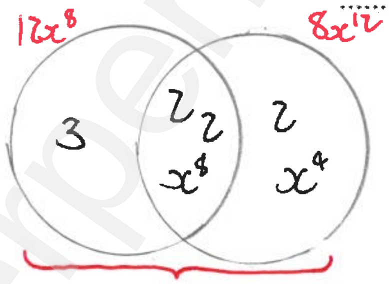
$4x^{12}$ .... [2]

HCF $= 2 \times 2 \times x^{12}$

$= 4x^{12}$

16 Find the lowest common multiple (LCM) of $12x^8$ and $8x^{12}$.



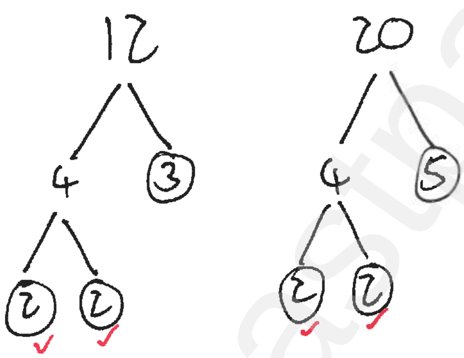


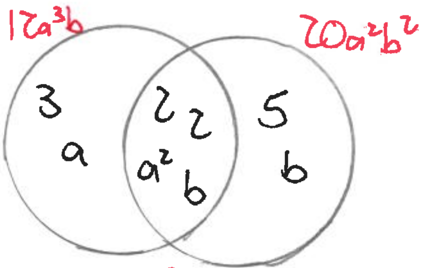
$24x^{12}$ .... [2]

LCM $= 2 \times 2 \times 2 \times 3 \times x^4 \times x^8$

$= 4 \times 6 \times x^{12} = 24x^{12}$

15 Find the highest common factor (HCF) of $12a^3b$ and $20a^2b^2$.



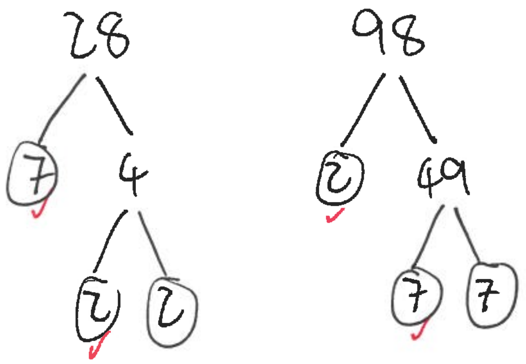


$4a^2b$ .... [2]

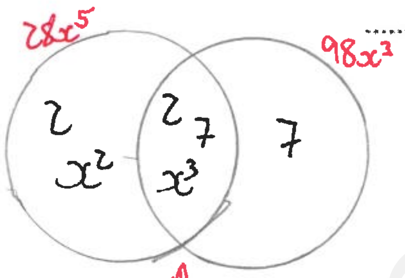
HCF $= 2 \times 2 \times a^2 \times b$

$= 4a^2b$

18 Find the highest common factor (HCF) of $28x^5$ and $98x^3$.



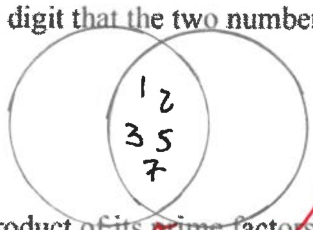
$14x^3$ .... [2]



HCF $= 2 \times 7 \times x^3 = 14x^3$

8 **(a)** 1, 2, 3, 5 and 7 are all common factors of two numbers.

Write down the digit that the two numbers must end in.

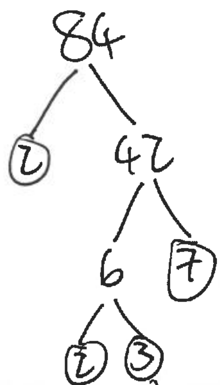


(anything with 2 and 5 as factors is divisible by 10)

0 .... [1]

(b) Write 84 as a product of its prime factors.

$1 \times 2 \times 3 \times 5 \times 7$
$= 6 \times 35$
$= 210$ ← must end in zero



$2^2 \times 3 \times 7$ .... [2]

7 $234 = 2 \times 3^2 \times 13$ $\quad 1872 = 2^4 \times 3^2 \times 13$ $\quad 234 \times 1872 = 438\,048$

Use this information to write 438 048 as a product of its prime factors.

$234 \times 1872 = 438\,048$

$2 \times 3^2 \times 13 \times 2^4 \times 3^2 \times 13 = 438\,048$

$2^5 \times 3^4 \times 13^2 = 438\,048$

$2^5 \times 3^4 \times 13^2$ .... [1]

(b) $k = 2 \times 3^2 \times p^3$, where p is a prime number greater than 3.

Write $6k^2$ as a product of prime factors in terms of p.

$$6k^2 = 6(2 \times 3^2 \times p^3)^2$$

$$= \underbrace{2 \times 3} \times (2^2 \times 3^4 \times p^6)$$

$$= 2^3 \times 3^5 \times p^6$$

$2^3 \times 3^5 \times p^6$ [2]

12 $x = 3^2 \times 5^2 \times 7 \times 199^{57}$ when written as a product of its prime factors.

Write $x \div 315$ as a product of its prime factors.

315 → 5, 63; 63 → 7, 9; 9 → 3, 3

$\rightarrow 315 = 3^2 \times 5 \times 7$

$$\frac{x}{315} = \frac{\cancel{3^2} \times 5^{\cancel{2}} \times \cancel{7} \times 199^{57}}{\cancel{3^2} \times \cancel{5} \times \cancel{7}}$$

$$= 5 \times 199^{57}$$

5×199^{57} [2]

14 $N = 2^4 \times 3 \times 7^5$

$PN = K$, where P is an integer and K is a square number.

Find the smallest value of P.

For K to be a square number, all the indices on its prime factors must be <u>even</u>.

$N = 2^4 \times 3^1 \times 7^5$ ok, not ok, not ok

$K = \underbrace{3 \times 7}_{P} \times \underbrace{2^4 \times 3^1 \times 7^5}_{N}$

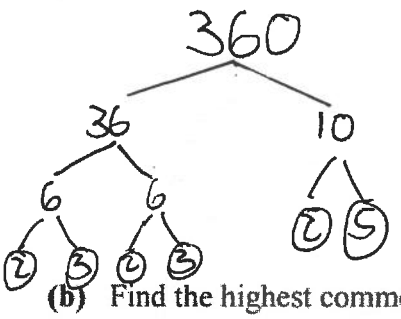
(because this makes: $2^4 \times 3^2 \times 7^6$)

$\rightarrow P = 3 \times 7$
$= 21$

$P =$ 21 [2]

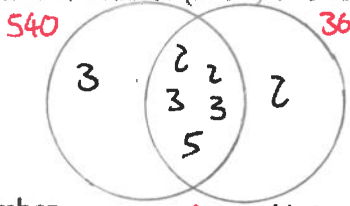
8 Written as the product of its prime factors, $540 = 2^2 \times 3^3 \times 5$.

(a) Write 360 as a product of its prime factors.



$2^3 \times 3^2 \times 5$ [2]

(b) Find the highest common factor (HCF) of 540 and 360.



HCF $= 2 \times 2 \times 3 \times 3 \times 5$

$= 10 \times 18$

$= 180$

180 [1]

(c) $540n$ is a cube number.

Find the smallest possible value of n.

For a number to be cube, all the indices on the prime factors must be a multiple of 3:

not ok ok not ok

$2^{2} \times 3^{3} \times 5^{1}$

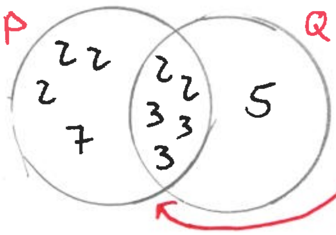
$540n = \underbrace{2^2 \times 3^3 \times 5^1}_{540} \times \underbrace{2 \times 5^2}_{50}$

$n = 50$

50 [1]

(c) $P = 2^5 \times 3^3 \times 7$ $\quad Q = 540$ $= 2^2 \times 3^3 \times 5$

(i) Find the highest common factor (HCF) of P and Q.



HCF $= 2 \times 2 \times 3 \times 3 \times 3$
$= 4 \times 27$

............108............ [2]

(ii) Find the lowest common multiple (LCM) of P and Q.

LCM $= 108 \times 2 \times 2 \times 2 \times 7 \times 5$
$= 108 \times 4 \times 10 \times 7$
$= 30\,240$

............30 240............ [2]

(iii) $P \times R$ is a cube number, where R is an integer.

Find the smallest possible value of R.

$P = 2^{5} \times 3^3 \times 7^{1}$

$P \times R = \underbrace{2^5 \times 3^3 \times 7^1}_{P} \times \underbrace{2^1 \times 7^2}_{R}$

$R = 2 \times 7^2$
$= 2 \times 49$
$= 98$

............98............ [2]

12 f is a common factor of 14 and 28.
m is a common multiple of 10 and 25.
p is a prime number.

Work out the largest possible value of $\dfrac{f}{mp}$.

$\dfrac{f}{mp}$

f ← need this to be as big as possible

mp ← needs to be as small as possible

f: HCF of 14 and 28 is $\underline{14}$

m: LCM of 10 and 25 is $\underline{50}$

p: smallest prime number is $\underline{2}$

$\dfrac{14}{50 \times 2}$

$= \dfrac{14}{100}$

$= \underline{0.14}$

............0.14............ [4]