19 Solve the equation $x^2+5x-7=0$.
You must show all your working and give your answers correct to 2 decimal places.

$$x=\frac{-5\pm\sqrt{5^2-4(1)(-7)}}{2(1)}$$

$$=\frac{-5\pm\sqrt{53}}{2}$$

$x=1.14$ or $x=-6.14$

$x=$ ....1.14.... or $x=$ ....-6.14.... [4]

(c) Solve $x^3+4x^2-17x=x^3-9$.
You must show all your working and give your answers correct to 2 decimal places.

$x^3+4x^2-17x=x^3-9$

$-x^3$ $\qquad$ $-x^3$

$4x^2-17x=-9$

$+9$ $\qquad$ $+9$

$4x^2-17x+9=0$

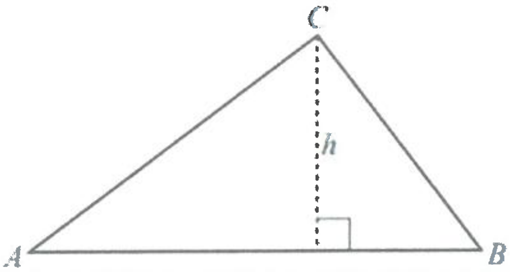
$$x=\frac{17\pm\sqrt{(-17)^2-4(4)(9)}}{2(4)}$$

$$=\frac{17\pm\sqrt{145}}{8}$$

$x=3.63$ or $x=0.62$

$x=$ ....3.63.... or $x=$ ....0.62.... [5]

(b)



NOT TO SCALE

$AB = (2x+3)$ cm and $h = (x+5)$ cm.
The area of triangle $ABC = 50\text{ cm}^2$.

Find the value of x, giving your answer correct to 2 decimal places.
You must show all your working.

Area $= \frac{1}{2} \times b \times h$:

$50 = \frac{1}{2} \times (2x+3) \times (x+5)$

$\times 2$ $\times 2$

$100 = (2x+3)(x+5)$

$100 = 2x^2 + 10x + 3x + 15$

$100 = 2x^2 + 13x + 15$

-100 -100

$2x^2 + 13x - 85 = 0$

$x = \dfrac{-13 \pm \sqrt{13^2 - 4(2)(-85)}}{2(2)}$

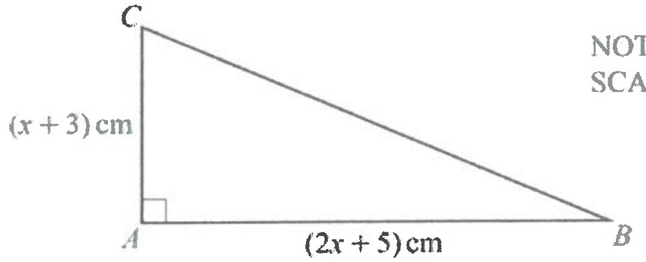
$= \dfrac{-13 \pm \sqrt{849}}{4}$

$x = 4.03$ or $x = -10.53$

can't be this because lengths can't be negative

$x =$ 4.03 [6]

3



The diagram shows a right-angled triangle ABC.

(a) **(i)** The area of the triangle is $60\,\text{cm}^2$.

Show that $2x^2 + 11x - 105 = 0$.

Area $= \frac{1}{2} \times b \times h$

$60 = \frac{1}{2} \times (2x+5)(x+3)$

$\times 2$ $\times 2$

$120 = (2x+5)(x+3)$

$120 = 2x^2 + 6x + 5x + 15$

$120 = 2x^2 + 11x + 15$

-120 -120

$2x^2 + 11x - 105 = 0$

[3]

(ii) Solve by factorisation.

$2x^2 + 11x - 105 = 0$

$ac = -210$

two numbers: $21, -10$

$\rightarrow 2x^2 - 10x + 21x - 105 = 0$

$2x(x-5) + 21(x-5) = 0$

$(2x+21)(x-5) = 0$

$2x + 21 = 0$ $x - 5 = 0$

$2x = -21$ $x = 5$

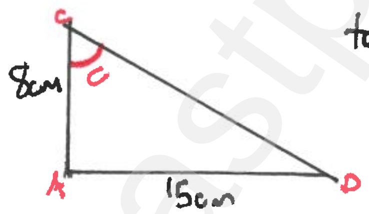
$x = -10.5$

$x =$ ....-10.5.... or $x =$ ....5.... [3]

(iii) Calculate angle ACB. (trigonometry required)

x can't be -10.5 because lengths can't be negative.

Sub. $x = 5$:



$\tan C = \frac{15}{8}$

$C = \tan^{-1}\left(\frac{15}{8}\right)$

$C =$ ....$61.9°$.... [3]

9 **(a)**

NOT TO SCALE

The total of the areas of rectangles A and B is $20\,\text{cm}^2$.

(i) Show that $3x^2 + 6x - 22 = 0$.

A: Area $= x(3x+4)$
$= 3x^2 + 4x$

B: Area $= 2(x-1)$
$= 2x - 2$

A+B=20:

$3x^2 + 4x + 2x - 2 = 20$

$3x^2 + 6x - 2 = 20$ -20 -20

$3x^2 + 6x - 22 = 0$

[2]

(ii) Solve the equation $3x^2 + 6x - 22 = 0$, giving your answers correct to 4 significant figures.
You must show all your working.

$$x = \frac{-6 \pm \sqrt{6^2 - 4(3)(-22)}}{2(3)}$$

$$= \frac{-6 \pm \sqrt{300}}{6}$$

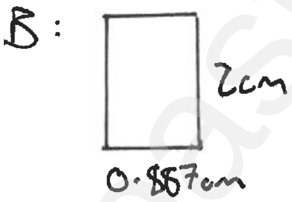
$x = 1.887$ or $x = -3.887$

$x =$ ......1.887...... or $x =$ ......−3.887...... [4]

(iii) Find the perimeter of rectangle B.

x can't be -3.887, so sub. $x = 1.887$:

B:



P= 2+2+0.887+0.887
= 5.774cm

......5.77...... cm [1]

(3sf)

9 (a)

The area of the rectangle is 29 cm^2 greater than the area of the square.
The difference between the perimeters of the two shapes is k cm.

Find the value of k.
You must show all your working.

Rectangle:

Area $= (x-1)(2x+1)$

$= 2x^2 + x - 2x - 1$

$= 2x^2 - x - 1$

Square:

Area $= x^2$

Rectangle = Square + 29:

$2x^2 - x - 1 = x^2 + 29$

$-x^2 \qquad -x^2$

$x^2 - x - 1 = 29$

$-29 \qquad -29$

$x^2 - x - 30 = 0$

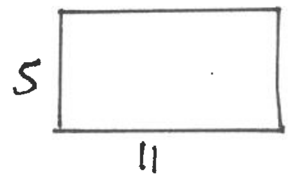
$(x+5)(x-6) = 0$

$x+5=0$ or $x-6=0$

$x=-5$ ✗ $\qquad x=6$ ✓

lengths can't be negative

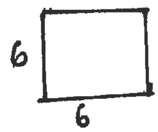
Rectangle:



Perimeter: $11+11+5+5$

$= 32$cm

Square:

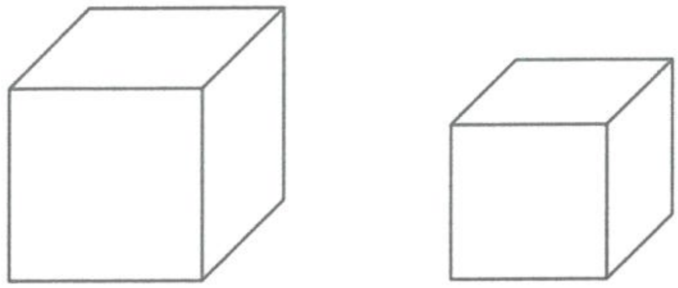


Perimeter $= 4\times6 = 24$cm

Difference: $32 - 24 = 8$

$k =$ 8 [6]

(b)



$(y+1)$ cm $\qquad$ y cm

NOT TO SCALE

The volume of the larger cube is 5 cm^3 greater than the volume of the smaller cube.

(i) Show that $3y^2 + 3y - 4 = 0$.

Large cube:

$$\text{Volume} = (y+1)(y+1)(y+1)$$
$$= (y+1)(y^2+2y+1)$$
$$= y^3+2y^2+y+y^2+2y+1$$
$$= y^3+3y^2+3y+1$$

Small cube:

$$\text{Volume} = y^3$$

large cube = small cube + 5

$$y^3+3y^2+3y+1 = y^3+5$$
$$-y^3 \qquad\qquad -y^3$$
$$3y^2+3y+1 = 5$$
$$-5 \qquad -5$$
$$3y^2+3y-4 = 0$$

[4]

(ii) Find the volume of the smaller cube.
Show all your working and give your answer correct to 2 decimal places.

$$y = \frac{-3 \pm \sqrt{3^2 - 4(3)(-4)}}{2(3)}$$

$$= \frac{-3 \pm \sqrt{57}}{6}$$

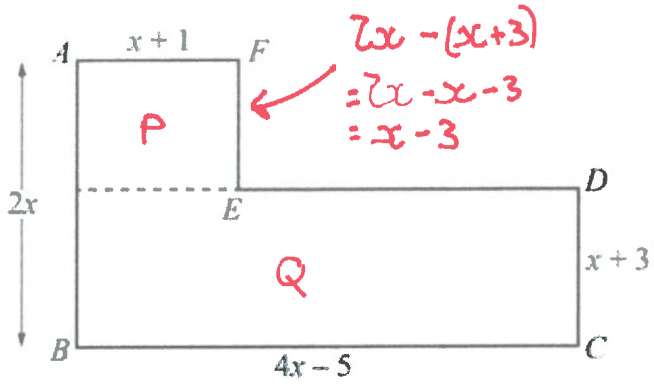
$y = 0.758$ or $y = -1.758$

STO $\qquad$ × length can't be negative.

$$V = (0.758\ldots)^3$$
$$= \underline{0.436 \text{ cm}^3}$$

0.436 cm^3 [4]

5 All the lengths in this question are in centimetres.



NOT TO SCALE

The diagram shows a shape *ABCDEF* made from two rectangles.
The total area of the shape is 342 cm^2.

(a) Show that $x^2 + x - 72 = 0$.

P: Area $= (x+1)(x-3)$
$= x^2 - 3x + x - 3$
$= x^2 - 2x - 3$

Q: Area $= (x+3)(4x-5)$
$= 4x^2 - 5x + 12x - 15$
$= 4x^2 + 7x - 15$

P + Q = 342:
$x^2 - 2x - 3 + 4x^2 + 7x - 15 = 342$
$5x^2 + 5x - 18 = 342$
$-342 \qquad -342$
$5x^2 + 5x - 360 = 0 \qquad \div 5$
$x^2 + x - 72 = 0$

[5]

(b) Solve by factorisation.

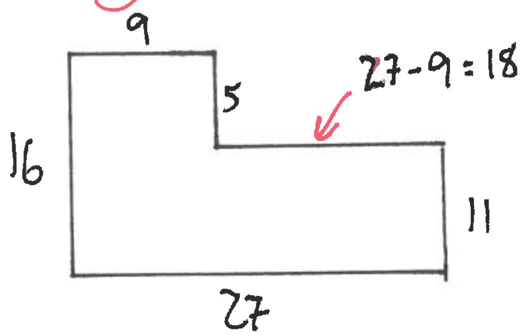
$$x^2 + x - 72 = 0$$

$(x + 9)(x - 8) = 0$
$x + 9 = 0$ or $x - 8 = 0$
$x = -9 \qquad x = 8$

$x =$ ....-9.... or $x =$ ....8.... [3]

(c) Work out the perimeter of the shape *ABCDEF*. (continued from previous page)

Lengths can't be negative, so use $x = 8$:



Perimeter $= 27 + 16 + 9 + 5 + 18 + 11$

$= 86$ cm

86 cm [2]

18 One day, Anya runs 12 km at a speed of x km/h.
The next day she walks 10 km at a speed of $(x-4)$ km/h.

(a) Write down an expression, in terms of x, for the time she spends running.

$\text{time} = \frac{\text{distance}}{\text{speed}}$

$\text{time} = \frac{12}{x}$

$\frac{12}{x}$ h [1]

(b) Write down an expression, in terms of x, for the time she spends walking.

$\text{time} = \frac{10}{x-4}$

$\frac{10}{x-4}$ h [1]

(c) The time Anya spends walking is 1 hour more than the time she spends running.

Write an equation in terms of x and show that it simplifies to $x^2 - 2x - 48 = 0$.

walking = running + 1:

$\frac{10}{x-4} = \frac{12}{x} + 1 \times \frac{x}{x}$

$\frac{10}{x-4} = \frac{12}{x} + \frac{x}{x}$

$\frac{10}{x-4} = \frac{12+x}{x}$

$10x = (x-4)(12+x)$

$10x = 12x + x^2 - 48 - 4x$

$10x = x^2 + 8x - 48$

$-10x \qquad -10x$

$x^2 - 2x - 48 = 0$

[4]

(d) Use factorisation to solve the equation $x^2 - 2x - 48 = 0$.

$(x+6)(x-8) = 0$

$x + 6 = 0$ or $x - 8 = 0$

$x = -6 \qquad x = 8$

$x =$ -6 or $x =$ 8 [3]

(e) Find the time Anya spends running.

Speed can't be negative, so $x = 8$:

$\text{time} = \frac{12}{x} = \frac{12}{8}$

$= 1.5$ hours

1.5 h [1]

8 Darpan runs a distance of 12 km and then cycles a distance of 26 km.
His running speed is x km/h and his cycling speed is 10 km/h faster than his running speed.
He takes a total time of 2 hours 48 minutes.

(a) An expression for the time, in hours, Darpan takes to run the 12 km is $\frac{12}{x}$.

Write an equation, in terms of x, for the total time he takes in hours.

Cycling speed: $x + 10$

Cycling time: $\frac{26}{x+10}$

48 mins in hours: $48 \div 60 = 0.8$

So 2hrs 48m = 2.8 hours

Total time: $\frac{12}{x} + \frac{26}{x+10} = 2.8$

.... [3]

(b) Show that this equation simplifies to $7x^2 - 25x - 300 = 0$.

$$\frac{12}{x} \times \frac{(x+10)}{(x+10)} + \frac{26}{x+10} \times \frac{x}{x} = 2.8$$

$$\frac{12(x+10) + 26x}{x(x+10)} = 2.8$$

$$\frac{12x + 120 + 26x}{x(x+10)} = 2.8$$

$$\frac{38x + 120}{x(x+10)} = 2.8 \quad \times x(x+10) \quad \times x(x+10)$$

$$38x + 120 = 2.8x(x+10)$$

$$38x + 120 = 2.8x^2 + 28x$$

$$2.8x^2 - 10x - 120 = 0$$

$\times 2.5 \quad \times 2.5 \quad \times 2.5 \quad \times 2.5$

$$7x^2 - 25x - 300 = 0$$

[4]

(c) Use the quadratic formula to solve $7x^2 - 25x - 300 = 0$.
You must show all your working.

$$x = \frac{25 \pm \sqrt{(-25)^2 - 4(7)(-300)}}{2(7)}$$

$$x = \frac{25 \pm \sqrt{9025}}{14}$$

$$= \frac{25 \pm 95}{14}$$

$x = \frac{60}{7}$ or $x = -5$

$x =$ ......$\frac{60}{7}$...... or $x =$ ......-5...... [4]

(d) Calculate the number of minutes Darpan takes to run the 12 km.

$x = \frac{60}{7}$ because speed can't be negative.

$$\text{time} = \frac{12}{x}$$

$$= \frac{12 \times 7}{\frac{60}{7} \times 7} = \frac{84}{60}$$

$= 1.4$ hours

$1.4 \times 60 = 84$ mins

......84...... min [2]

11 Gaya spends \$48 to buy books that cost \$$x$ each.

(a) Write down an expression, in terms of x, for the number of books Gaya buys.

$\text{number} = \frac{\text{money}}{\text{cost}}$ $\quad$ $\text{number} = \frac{48}{x}$

$\frac{48}{x}$ [1]

(b) Myra spends \$60 to buy books that cost \$$(x+2)$ each.
Gaya buys 4 more books than Myra.

Show that $x^2 + 5x - 24 = 0$.

Myra: $\frac{60}{x+2}$

Gaya = Myra + 4:

$$\frac{48}{x} = \frac{60}{x+2} + \frac{4}{1} \times \frac{(x+2)}{(x+2)}$$

$$\frac{48}{x} = \frac{60 + 4(x+2)}{x+2}$$

$$\frac{48}{x} = \frac{60 + 4x + 8}{x+2}$$

$$\frac{48}{x} = \frac{4x + 68}{x+2}$$

$$48(x+2) = x(4x + 68)$$

$$48x + 96 = 4x^2 + 68x$$

$-48x$ $\quad$ $-48x$

$$96 = 4x^2 + 20x$$

-96 $\quad$ -96

$$4x^2 + 20x - 96 = 0 \quad \div 4$$

$$x^2 + 5x - 24 = 0$$

[4]

(c) Solve by factorisation.

$$x^2 + 5x - 24 = 0$$

$$(x+8)(x-3) = 0$$

$$x + 8 = 0 \text{ or } x - 3 = 0$$

$$x = -8 \qquad x = 3$$

$x =$ -8 or $x =$ 3 [3]

(d) Find the number of books Myra buys.

x can't be negative, so $x = 3$:

$$\text{Myra}: \frac{60}{3+2} = \frac{60}{5}$$

$$= \underline{12}$$

12 [1]

(c) Antonio travels 100 km at an average speed of x km/h.
He then travels a further 150 km at an average speed of $(x + 10)$ km/h.
The time taken for the whole journey is 4 hours 20 minutes.

$20 \text{ mins} = \frac{20}{60} = \frac{1}{3} \text{ hour}$

(i) Show that $13x^2 - 620x - 3000 = 0$.

First 100km: time $= \frac{100}{x}$

Next 150km: time $= \frac{150}{x+10}$

Total Time: $\frac{100}{x} + \frac{150}{x+10} = 4\frac{1}{3}$

$$\frac{100}{x} \times \frac{(x+10)}{(x+10)} + \frac{150}{x+10} \times \frac{x}{x} = \frac{13}{3}$$

$$\frac{100(x+10) + 150x}{x(x+10)} = \frac{13}{3}$$

$$\frac{100x + 1000 + 150x}{x(x+10)} = \frac{13}{3}$$

$$\frac{250x + 1000}{x(x+10)} = \frac{13}{3}$$

$$3(250x + 1000) = 13x(x+10)$$

$$750x + 3000 = 13x^2 + 130x$$

$-750x \qquad -750x$

$$3000 = 13x^2 - 620x$$

$-3000 \qquad -3000$

$$13x^2 - 620x - 3000 = 0$$

[4]

(ii) Solve $13x^2 - 620x - 3000 = 0$ to find the speed Antonio travels for the first 100 km of the journey.
You must show all your working and give your answer correct to 1 decimal place.

$$x = \frac{620 \pm \sqrt{(-620)^2 - 4(13)(-3000)}}{2(13)}$$

$$= \frac{620 \pm \sqrt{384\,400 + 156\,000}}{26}$$

$$= \frac{620 \pm \sqrt{540\,400}}{26}$$

$x = 52.1$ or $x = -4.4$

× can't be negative

52.1 km/h [3]

5 (a) In a shop the cost of a fiction book is $\$x$ and the cost of a reference book is $\$(x+2)$.
The cost of 11 fiction books is the same as the cost of 10 reference books.

Find the value of x.

$$11x = 10(x+2)$$
$$11x = 10x + 20$$
$$-10x \quad -10x$$
$$x = 20$$

$x =$ ......20...... [2]

(b) In another shop, the cost of a fiction book is $\$y$ and the cost of a reference book is $\$(y+2)$.
Maria spends \$95 on fiction books and \$147 on reference books.
She buys a total of 12 books.

(i) Show that $6y^2 - 109y - 95 = 0$.

Number of fiction books: $\frac{95}{y}$

Number of reference books: $\frac{147}{y+2}$

Total books = 12:

$$\frac{95}{y} + \frac{147}{y+2} = 12$$

$$\frac{95}{y} \times \frac{(y+2)}{(y+2)} + \frac{147}{y+2} \times \frac{y}{y} = 12$$

$$\frac{95(y+2) + 147y}{y(y+2)} = 12$$

$$\frac{95y + 190 + 147y}{y(y+2)} = 12$$

$$\frac{242y + 190}{y(y+2)} = 12 \quad \times y(y+2) \quad \times y(y+2)$$

$$242y + 190 = 12y(y+2)$$

$$242y + 190 = 12y^2 + 24y$$

$$12y^2 - 218y - 190 = 0 \quad \div 2$$

$$6y^2 - 109y - 95 = 0$$ [4]

(ii) Factorise $6y^2 - 109y - 95$.

$ac = -570$

two numbers: $-114, 5$

$$\rightarrow 6y^2 - 114y + 5y - 95$$

$$6y(y-19) + 5(y-19)$$

$$(6y+5)(y-19)$$

......$(6y+5)(y-19)$...... [2]

(iii) Find the value of y.

$$6y+5=0 \quad \text{or} \quad y-19=0$$

$$6y = -5 \qquad y = 19$$

$$y = \frac{-5}{6}$$

× can't be negative

$y =$ ......19...... [1]

8 (a) Kaito runs along a 12 km path at an average speed of x km/h.

time = $\frac{\text{distance}}{\text{speed}}$

(i) Write down an expression, in terms of x, for the number of hours he takes.

time = $\frac{12}{x}$

$\frac{12}{x}$ hours [1]

(ii) Yuki takes 1.5 hours longer to walk along the same path as Kaito.
She walks at an average speed of $(x-4)$ km/h.

Write down an equation, in terms of x, and show that it simplifies to $x^2-4x-32=0$.

Yuki time: $\frac{12}{x-4}$

Yuki = Kaito + 1.5:

$$\frac{12}{x-4} = \frac{12}{x} + \frac{3}{2}$$

$$\frac{12}{x-4} = \frac{12}{x} \times \frac{2}{2} + \frac{3}{2} \times \frac{x}{x}$$

$$\frac{12}{x-4} = \frac{24}{2x} + \frac{3x}{2x}$$

$$\frac{12}{x-4} = \frac{24+3x}{2x}$$

$$24x = (x-4)(24+3x)$$

$$24x = 24x + 3x^2 - 96 - 12x$$

$$3x^2 - 12x - 96 = 0 \quad \div 3$$

$$x^2 - 4x - 32 = 0$$

[4]

(iii) Solve by factorisation.

$$x^2-4x-32=0$$

$$(x-8)(x+4) = 0$$

$x-8=0$ or $x+4=0$

$x=8$ $\quad$ $x=-4$

$x =$ 8 or $x =$ −4 [3]

(iv) Find the number of hours it takes Yuki to walk along the 12 km path.

x can't be negative, so $x=8$:

Yuki: $\frac{12}{8-4} = \frac{12}{4}$

$= 3$ hrs

3 hours [2]

(f) Alan invests \$200 at a rate of r% per year compound interest.
After 2 years the value of his investment is \$206.46 .

(i) Show that $r^2 + 200r - 323 = 0$.

$$200 \times \left(1 + \frac{r}{100}\right)^2 = 206.46$$

$$200 \times \left(1 + \frac{r}{100}\right)\left(1 + \frac{r}{100}\right) = 206.46$$

$$200 \times \left(1 + \frac{r}{100} + \frac{r}{100} + \frac{r^2}{10\,000}\right) = 206.46$$

$$200 \times \left(1 + \frac{r}{50} + \frac{r^2}{10\,000}\right) = 206.46$$

$$200 + 4r + \frac{r^2}{50} = 206.46$$

$-206.46 \qquad\qquad -206.46$

$$\frac{r^2}{50} + 4r - 6.46 = 0$$

$\times 50 \quad \times 50 \quad \times 50 \quad \times 50$

$$r^2 + 200r - 323 = 0$$

[3]

(ii) Solve the equation $r^2 + 200r - 323 = 0$ to find the rate of interest.
Show all your working and give your answer correct to 2 decimal places.

$$r = \frac{-200 \pm \sqrt{200^2 - 4(1)(-323)}}{2(1)}$$

$$= \frac{-200 \pm \sqrt{40000 + 1292}}{2}$$

$$= \frac{-200 \pm \sqrt{41292}}{2}$$

$r = 1.60$ or $r = -201.60$ X

Can't be negative

$r =$ ......1.60%...... [3]