(d) Factorise the following completely.

    (i) $x^2 - 3x - 28$

$$x^2 - 7x + 4x - 28$$
$$x(x - 7) + 4(x - 7)$$
$$(x + 4)(x - 7)$$

                $(x + 4)(x - 7)$ [2]

10 Factorise

    (a) $x^2 - x - 6$,

$$x^2 - 3x + 2x - 6$$
$$x(x - 3) + 2(x - 3)$$
$$(x + 2)(x - 3)$$

                $(x + 2)(x - 3)$ [2]

    (b) $h^2 - 144$

        ↖ difference of two squares

                $(h - 12)(h + 12)$ [1]

    (b) $9x^2 - 1$

                $(3x + 1)(3x - 1)$ [1]

12 Factorise fully.

    (a) $(3y)^2 - 16$

$$9y^2 - 16$$

                $(3y + 4)(3y - 4)$ [1]

7   **(a)** Factorise fully.

        (i) $27y^2 - 3$

Single bracket first:

$$3(9y^2 - 1)$$
$$3(3y + 1)(3y - 1)$$

                $3(3y + 1)(3y - 1)$ [3]

13 Factorise.

(a) $49 - 16u^2$

$$\underline{(7 + 4u)(7 - 4u)} \quad [1]$$

18 (a) Factorise.

$$18a^2 - 98$$

Single bracket first:

$$2(9a^2 - 49)$$
$$2(3a + 7)(3a - 7)$$

$$\underline{2(3a + 7)(3a - 7)} \quad [2]$$

9 Factorise completely.

(b) $20x^2 - 45y^2$

$$5(4x^2 - 9y^2)$$
$$5(2x + 3y)(2x - 3y)$$

$$\underline{5(2x + 3y)(2x - 3y)} \quad [3]$$

11 Factorise completely.

$$48x^2 - 75y^2$$

$$3(16x^2 - 25y^2)$$
$$3(4x + 5y)(4x - 5y)$$

$$\underline{3(4x + 5y)(4x - 5y)} \quad [3]$$

(b) Factorise completely.

(i) $2x^2 - 288y^2$

$$2(x^2 - 144y^2)$$
$$2(x + 12y)(x - 12y)$$

$$\underline{2(x + 12y)(x - 12y)} \quad [3]$$

(b) $2x^3 - 18xy^2$

$$2x(x^2 - 9y^2)$$
$$2x(x + 3y)(x - 3y)$$

$$\underline{2x(x+3y)(x-3y)} \quad [3]$$

(b) $5x^2 - 20y^2$

$$5(x^2 - 4y^2)$$
$$5(x + 2y)(x - 2y)$$

$$\underline{5(x+2y)(x-2y)} \quad [3]$$

(c) Factorise completely. $x^3 - 16xy^2$

$$x(x^2 - 16y^2)$$
$$x(x + 4y)(x - 4y)$$

$$\underline{x(x + 4y)(x - 4y)} \quad [3]$$

6 (a) Factorise $a^2 - b^2$.

$$\underline{(a+b)(a-b)} \quad [1]$$

(b) Work out $5.37^2 - 4.63^2$.

Factorise like part (a):

$$(5.37 + 4.63)(5.37 - 4.63)$$
$$= \quad (10)(0.74)$$

$$\underline{7.4} \quad [2]$$

$$= \quad 10 \times 0.74$$
$$= \quad 7.4$$

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10 Factorise.

$$2x + 6 - 3xy - 9y$$

$$2(x + 3) - 3y(x + 3)$$

$$(2 - 3y)(x + 3)$$

...$(2 - 3y)(x + 3)$... [2]

9 Factorise completely.

$$2x - 6y - ax + 3ay$$

$$2(x - 3y) - a(x - 3y)$$

$$(2 - a)(x - 3y)$$

...$(2 - a)(x - 3y)$... [2]

8 Factorise fully.

no obvious common factors, so use 1 or -1

$$2cx^2 - 2dx - cx + d$$

$$2x(cx - d) - 1(cx - d)$$

$$(2x - 1)(cx - d)$$

...$(2x - 1)(cx - d)$... [2]

(b) Factorise.

$$3x(a + 4y) - ay - 4y^2$$

$$3x(a + 4y) - y(a + 4y)$$

$$(3x - y)(a + 4y)$$

...$(3x - y)(a + 4y)$... [1]

16 Factorise.

$$3x + 6 - 2xy - 4y$$

$$3(x + 2) - 2y(x + 2)$$

$$(3 - 2y)(x + 2)$$

$$\underline{(3 - 2y)(x + 2)}$$ [2]

10 Factorise.

(c) $6ax + 9ay - 8bx - 12by$

$$3a(2x + 3y) - 4b(2x + 3y)$$

$$(3a - 4b)(2x + 3y)$$

$$\underline{(3a - 4b)(2x + 3y)}$$ [2]

20 Factorise completely.

(a) $1 + x - y - xy$

use 1 as the common factor:

$$1(1 + x) - y(1 + x)$$

$$(1 - y)(1 + x)$$

$$\underline{(1 - y)(1 + x)}$$ [2]

12 Factorise.

$$1 + a - c - ac$$

$$1(1 + a) - c(1 + a)$$

$$(1 - c)(1 + a)$$

$$\underline{(1 - c)(1 + a)}$$ [2]

21 Factorise completely.

$$1 - q - a + aq$$

$$1(1 - q) - a(1 - q)$$

$$(1 - a)(1 - q)$$

$$(1 - a)(1 - q)$$ [2]

10 (a) Write down **all** the factors of 18.

$$\begin{array}{cc} & 18 \\ \hline 1 & 18 \\ 2 & 9 \\ 3 & 6 \end{array}$$

$$1, 2, 3, 6, 9, 18$$ [2]

(b) Factorise.

$$3y - xy + 15 - 5x$$

$$y(3 - x) + 5(3 - x)$$

$$(y + 5)(3 - x)$$

$$(y + 5)(3 - x)$$ [2]

(c) $$3y - xy + 15 - 5x = 18$$

where x and y are positive integers.

Using your answers to **part (a)** and **part (b)**, find one possible value of x and the corresponding value of y.

from part (b): $(y + 5)(3 - x) = 18$

must be factors of 18

either: $(y + 5)(3 - x) = 18$
9×2
so: $y + 5 = 9$ and $3 - x = 2$
$y = 4$ $-x = -1$
$x = 1$

or: $(y + 5)(3 - x) = 18$
18×1
so: $y + 5 = 18$ and $3 - x = 1$
$y = 13$ $-x = -2$
$x = 2$

$x = \underline{\hspace{1cm} 1 \hspace{1cm}}$, $y = \underline{\hspace{1cm} 4 \hspace{1cm}}$ [2]

(or $x = 2$, $y = 13$)

20 Factorise completely.

(a) $2m + 3p - 8km - 12kp$

$$1(2m + 3p) - 4k(2m + 3p)$$

$$(1 - 4k)(2m + 3p)$$

.......$(1-4k)(2m+3p)$....... [2]

7 Factorise.

(a) $12ax - 2by + 3ay - 8bx$

Doesn't work if you try to factorise
this one as it is:

$$2(6ax - by) + 1(3ay - 8bx)$$

↖ ↗
Not the same!

Need to re-order expression first:

$$12ax - 8bx + 3ay - 2by$$

$$4x(3a - 2b) + y(3a - 2b)$$

$$(4x + y)(3a - 2b)$$

.......$(4x + y)(3a - 2b)$....... [2]

(b) $3ax + 2bx - 4by - 6ay$.

$$x(3a + 2b) - 2y(2b + 3a)$$

same as $3a + 2b$:

$$x(3a + 2b) - 2y(3a + 2b)$$

$$(x - 2y)(3a + 2b)$$

.......$(x - 2y)(3a + 2b)$....... [2]

(b) $15ab - 1 - 3a + 5b$

re-order:

$$15ab - 3a + 5b - 1$$

$$3a(5b - 1) + 1(5b - 1)$$

$$(3a + 1)(5b - 1)$$

.......$(3a + 1)(5b - 1)$....... [2]

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13 Factorise completely.

(b) $mt - n - m + nt$

re-order:

$$Mt - M + nt - n$$
$$M(t-1) + n(t-1)$$
$$(m+n)(t-1)$$

$\dots(m+n)(t-1)\dots$ [2]

(b) $1 + 4xy - 2x - 2y$

re-order:

$$4xy - 2x - 2y + 1$$
$$2x(2y-1) - 1(2y-1)$$
$$(2x-1)(2y-1)$$

$(2x-1)(2y-1)$ [2]

(ii) $2m - pk + 2k - pm$

re-order:

$$2m - pm + 2k - pk$$
$$m(2-p) + k(2-p)$$
$$(m+k)(2-p)$$

$(m+k)(2-p)$ [2]

20 Factorise.

$$3x + 8y - 6ax - 16ay$$

re-order:

$$3x - 6ax + 8y - 16ay$$
$$3x(1-2a) + 8y(1-2a)$$
$$(3x+8y)(1-2a)$$

$(3x+8y)(1-2a)$ [2]

12 Factorise.

$$\overset{a}{2x^2} \overset{b}{-3x} \overset{c}{-5}$$

$ac = -10$

Find two numbers which multiply to ac (-10) and add to give b (-3)

Two numbers: -5 and $+2$

Split the middle using these two numbers:

$$2x^2 + 2x - 5x - 5$$

Factorise:

$$2x(x+1) - 5(x+1)$$

$$(2x-5)(x+1)$$

.... [2]

(ii) $\overset{a}{5x^2} + \overset{b}{17x} - \overset{c}{40}$

$ac = -200$

Numbers which $\times$ to -200 and + to $\underline{17}$:

25 and -8

Split the middle:

$$5x^2 + 25x - 8x - 40$$

$$5x(x+5) - 8(x+5)$$

$$\underline{(5x-8)(x+5)}$$ [2]

(b) $\overset{a}{5x^2} \overset{b}{-6x} \overset{c}{-8}$

$ac = -40$

two numbers: $-10, 4$

Split the middle:

$$5x^2 - 10x + 4x - 8$$

$$5x(x-2) + 4(x-2)$$

$$\underline{(5x+4)(x-2)}$$ [2]

15 (a) Factorise fully.

$$6x^2 - 7x - 3$$

$ac = -18$

two numbers: $2, -9$

$$6x^2 + 2x - 9x - 3$$

$$2x(3x+1) - 3(3x+1)$$

$$\underline{(2x-3)(3x+1)}$$ [2]

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16 Factorise $6x^2 + 7x - 20$.

$ac = -120$

two numbers: $15, -8$

$6x^2 + 15x - 8x - 20$

$3x(2x + 5) - 4(2x + 5)$

$(3x - 4)(2x + 5)$ [2]

(b) (i) Factorise.
$$15x^2 - 2x - 8$$

$ac = -120$

two numbers: $10, -12$

$15x^2 + 10x - 12x - 8$

$5x(3x + 2) - 4(3x + 2)$

$(5x - 4)(3x + 2)$ [2]

(ii) Solve the equation.
$$15x^2 - 2x - 8 = 0$$

$(5x - 4)(3x + 2) = 0$

$5x - 4 = 0$ or $3x + 2 = 0$

$5x = 4$ $3x = -2$

$x = \frac{4}{5}$ $x = \frac{-2}{3}$

$x = \dfrac{4}{5}$ or $x = \dfrac{-2}{3}$ [1]

17 Solve.
$$(5x - 3)(2x + 7) = 0$$

$5x - 3 = 0$ or $2x + 7 = 0$

$5x = 3$ $2x = -7$

$x = \frac{3}{5}$ $x = \frac{-7}{2}$

$x = \dfrac{3}{5}$ or $x = \dfrac{-7}{2}$ [1]

(e) Solve by factorisation. $\quad$ a $\quad$ b $\quad$ c
$$4x^2 + 8x - 5 = 0$$

$ac = -20$
two numbers: $10, -2$

$4x^2 + 10x - 2x - 5 = 0$
$2x(2x + 5) - 1(2x + 5) = 0$
$(2x - 1)(2x + 5) = 0$
$2x - 1 = 0 \quad$ or $\quad 2x + 5 = 0$
$\qquad 2x = 1 \qquad\qquad 2x = -5$

$x = \dfrac{1}{2}$ or $x = \dfrac{-5}{2}$ [3]

---

11 Solve.
$$6x^2 - 5x - 6 = 0$$

$ac = -36$
two numbers: $-9, 4$
$6x^2 - 9x + 4x - 6 = 0$
$3x(2x - 3) + 2(2x - 3) = 0$

$(3x + 2)(2x - 3) = 0$
$3x + 2 = 0 \quad$ or $\quad 2x - 3 = 0$
$\quad 3x = -2 \qquad\qquad 2x = 3$
$\quad x = -\dfrac{2}{3} \qquad\qquad x = 3\tfrac{1}{2}$

$x = \dfrac{-2}{3}$ or $x = \dfrac{3}{2}$ [3]

---

11 Solve.

(a) $\quad 4x^2 - 5x - 6 = 0$

$ac = -24$
two numbers: $-8, 3$
$4x^2 - 8x + 3x - 6 = 0$
$4x(x - 2) + 3(x - 2) = 0$
$(4x + 3)(x - 2) = 0$
$4x + 3 = 0 \qquad x - 2 = 0$
$\quad 4x = -3 \qquad\qquad x = 2$
$\quad x = \dfrac{-3}{4}$

$x = \dfrac{-3}{4}$ or $x = 2$ [3]

9 (a) Solve.
$$5x^2 = 12 - 17x$$

Set $= 0$:

$$5x^2 + 17x - 12 = 0$$

$ac = -60$

two numbers: $20, -3$

$$5x^2 + 20x - 3x - 12 = 0$$
$$5x(x + 4) - 3(x + 4) = 0$$
$$(5x - 3)(x + 4) = 0$$

$5x - 3 = 0$ or $x + 4 = 0$
$5x = 3$ $x = -4$
$x = \dfrac{3}{5}$

$x = $ ......$\dfrac{3}{5}$...... or $x = $ ......-4...... [4]

(b) $ax^2 + a = b$ where a and b are integers.

One solution of this equation is $x = 6$.

Write down the other solution.

$$ax^2 + a = b$$
$$ax^2 = b - a$$
$$x^2 = \frac{b - a}{a}$$

If $x = 6$:

$$36 = \frac{b - a}{a}$$

So $x^2 = 36$

only other possible solution is $x = -6$

$x = $ ....-6................. [1]

Factorise.
(ii) $7(a + 2b)^2 + 4a(a + 2b)$

Factorise by taking $(a + 2b)$ out in front:

$$(a + 2b)\left[7(a + 2b) + 4a\right]$$
$$(a + 2b)\left[7a + 14b + 4a\right]$$
$$(a + 2b)(11a + 14b)$$

$\underline{(a + 2b)(11a + 14b)}$ [2]

23 Solve $\frac{4}{x+1} + \frac{2}{2x-5} = 3$.

You must show all your working.

$$\frac{4}{x+1} \times \frac{(2x-5)}{(2x-5)} + \frac{2}{2x-5} \times \frac{(x+1)}{(x+1)} = 3$$

$$\frac{4(2x-5) + 2(x+1)}{(x+1)(2x-5)} = 3$$

$$\frac{8x - 20 + 2x + 2}{(x+1)(2x-5)} = 3$$

$$\frac{10x - 18}{(x+1)(2x-5)} = 3$$

$\times (x+1)(2x+5)$

$$10x - 18 = 3(x+1)(2x-5)$$

$10x - 18 = 3[2x^2 - 5x + 2x - 5]$

$10x - 18 = 3(2x^2 - 3x - 5)$

$10x - 18 = 6x^2 - 9x - 15$

set $= 0$:

$6x^2 - 19x + 3 = 0$

$ac = 18$, two numbers: $-18, -1$

$6x^2 - 18x - 1x + 3 = 0$

$6x(x-3) - 1(x-3) = 0$

$(6x-1)(x-3) = 0$

$6x - 1 = 0$ or $x - 3 = 0$

$6x = 1$ $x = 3$

$x = \underline{\quad \frac{1}{6} \quad}$ or $x = \underline{\quad 3 \quad}$ [7]

---

15 Factorise.

$5x^2 - xy - 4y^2$

$\overset{a}{5}x^2 \overset{b}{-} yx \overset{c}{-} 4y^2$

$ac = -20y^2$

Two numbers that multiply to $-20y^2$

and add to $-y$:

$-5y$ and $+4y$

split middle:

$5x^2 - 5yx + 4yx - 4y^2$

$5x(x-y) + 4y(x-y)$

$\longrightarrow (5x + 4y)(x - y)$

$\underline{\quad\quad\quad\quad\quad\quad\quad}$ [2]