14 y is directly proportional to the square of $(x + 3)$.
When $x = 2$, $y = 5$.

Find y when $x = 1$.

$y \propto (x+3)^2$

$y = k(x+3)^2$

$x=2,\ y=5:$

$5 = k(5)^2$

$5 = 25k$

$\div 25 \quad \div 25$

$k = \frac{1}{5}$

$y = \frac{1}{5}(x+3)^2$

$x=1:$ $y = \frac{1}{5}(4)^2$

$= \frac{1}{5} \times 16$

$= 3.2$

$y =$ ....3.2............ [3]

14 y is directly proportional to the square root of $(x - 3)$.
When $x = 28$, $y = 20$.

Find y when $x = 39$.

$y \propto \sqrt{x-3}$

$y = k\sqrt{x-3}$

$x=28,\ y=20:$

$20 = k\sqrt{25}$

$20 = 5k$

$\div 5 \quad \div 5$

$k = 4$

$y = 4\sqrt{x-3}$

$x=39:$

$y = 4\sqrt{36}$

$= 4 \times 6$

$= 24$

$y =$ ....24............ [3]

22 p is directly proportional to $(q + 2)^2$.
When $q = 1$, $p = 1$.

Find p when $q = 10$.

$p \propto (q+2)^2$

$p = k(q+2)^2$

$q=1,\ p=1:$

$1 = k(3)^2$

$1 = 9k$

$\div 9 \quad \div 9$

$k = \frac{1}{9}$

$p = \frac{1}{9}(q+2)^2$

$q=10:$

$p = \frac{1}{9}(12)^2$

$= \frac{1}{9} \times 144$

$= 16$

$p =$ ....16............ [3]

17 y is proportional to the square of $(x-7)$.
When $x = 12$, $y = 2$.

Find y when $x = 17$.

$y \propto (x-7)^2$

$y = k(x-7)^2$

$x=12,\ y=2:$

$2 = k(5)^2$

$2 = 25k$

$\div 25 \quad \div 25$

$k = \frac{2}{25}$

$y = \frac{2}{25}(x-7)^2$

$x=17:$

$y = \frac{2}{25}(10)^2$

$= \frac{2}{25} \times 100$

$= 8$

$y =$ ....8.......... [3]

11 y is directly proportional to the cube root of $(x+3)$.
When $x = 5$, $y = \frac{2}{3}$.

Find y when $x = 24$.

$y \propto \sqrt[3]{x+3}$

$y = k\sqrt[3]{x+3}$

$x=5,\ y=\frac{2}{3}:$

$\frac{2}{3} = k\sqrt[3]{8}$

$\frac{2}{3} = 2k$

$\div 2 \quad \div 2$

$k = \frac{1}{3}$

$y = \frac{1}{3}\sqrt[3]{x+3}$

$x=24:$

$y = \frac{1}{3}\sqrt[3]{27}$

$y = \frac{1}{3} \times 3$

$= 1$

$y =$ ....1.......... [3]

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16 y is inversely proportional to x^2.
When $x = 3$, $y = 2$.

Find y when $x = 2$.

$$y \propto \frac{1}{x^2}$$

$$y = \frac{k}{x^2}$$

$x=3, y=2:$

$$2 = \frac{k}{3^2}$$

$$2 = \frac{k}{9}$$

$\times 9$ $\times 9$

$k=18$

$$y = \frac{18}{x^2}$$

$x=2:$

$$y = \frac{18}{2^2}$$

$$= \frac{18}{4}$$

$$= 4.5$$

$y =$ 4.5 [3]

22 x is inversely proportional to the square root of w.
When $w = 16$, $x = 3$.

Find x in terms of w.

$$x \propto \frac{1}{\sqrt{w}}$$

$$x = \frac{k}{\sqrt{w}}$$

$w=16, x=3:$

$$3 = \frac{k}{\sqrt{16}}$$

$$3_{\times 4} = \frac{k}{4}_{\times 4}$$

$k=12$

$$x = \frac{12}{\sqrt{w}}$$

$x =$ $\frac{12}{\sqrt{w}}$ [2]

13 y varies inversely as $\sqrt{x}$.
When $x = 9$, $y = 2$.

Find y in terms of x.

$$y \propto \frac{1}{\sqrt{x}}$$

$$y = \frac{k}{\sqrt{x}}$$

$x=9, y=2:$

$$2 = \frac{k}{\sqrt{9}}$$

$$2 = \frac{k}{3}$$

$\times 3$ $\times 3$

$k=6$

$$y = \frac{6}{\sqrt{x}}$$

$y =$ $\frac{6}{\sqrt{x}}$ [2]

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20 $y \propto \dfrac{1}{\sqrt{x}}$

When $y = 8$, $x = 4$.

Find y when $x = 49$.

$$y = \frac{k}{\sqrt{x}}$$

$y=8, x=4:$

$$8 = \frac{k}{\sqrt{4}}$$

$$8_{\times 2} = \frac{k}{2}_{\times 2}$$

$$\underline{k = 16}$$

$$y = \frac{16}{\sqrt{x}}$$

$x=49:$

$$y = \frac{16}{\sqrt{49}}$$

$$= \underline{\frac{16}{7}}$$

$y =$ $\dfrac{16}{7}$ [3]

19 m is inversely proportional to the square of $(t+2)$.

$m = 0.64$ when $t = 3$.

Find m when $t = 8$.

$$m \propto \frac{1}{(t+2)^2}$$

$$m = \frac{k}{(t+2)^2}$$

$m=0.64, t=3:$

$$0.64 = \frac{k}{5^2}$$

$$0.64_{\times 25} = \frac{k}{25}_{\times 25}$$

$$\underline{k = 16}$$

$$m = \frac{16}{(t+2)^2}$$

$t=8:$

$$m = \frac{16}{10^2} = \frac{16}{100} = \underline{0.16}$$

14 y varies inversely as $(x-3)^2$.

When $x = 1$, $y = 4$.

Find y in terms of x.

$$y \propto \frac{1}{(x-3)^2}$$

$$y = \frac{k}{(x-3)^2}$$

$x=1, y=4:$

$$4 = \frac{k}{(-2)^2}$$

$$4_{\times 4} = \frac{k}{4}_{\times 4}$$

$$\underline{k = 16}$$

$$y = \frac{16}{(x-3)^2}$$

$y =$ $\dfrac{16}{(x-3)^2}$ [2]

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19 y is inversely proportional to the square root of $(x+4)$.
When $x = 5$, $y = 2$.

Find y when $x = 77$.

$$y \propto \frac{1}{\sqrt{x+4}}$$

$$y = \frac{k}{\sqrt{x+4}}$$

$x=5,\ y=2:$

$$2 = \frac{k}{\sqrt{9}}$$

$$\underset{\times 3}{2} = \frac{k}{3} \times 3$$

$$k = 6$$

$$y = \frac{6}{\sqrt{x+4}}$$

$x=77:$

$$y = \frac{6}{\sqrt{81}} = \frac{6}{9} = \frac{2}{3}$$

$y =$ .... $\frac{2}{3}$ .... [3]

18 y is inversely proportional to the cube root of $(x + 5)$.
When $x = 3$, $y = 12$.

Find y when $x = 22$.

$$y \propto \frac{1}{\sqrt[3]{x+5}}$$

$$y = \frac{k}{\sqrt[3]{x+5}}$$

$x=3,\ y=12:$

$$12 = \frac{k}{\sqrt[3]{8}}$$

$$\underset{\times 2}{12} = \underset{\times 2}{\frac{k}{2}}$$

$$k = 24$$

$$y = \frac{24}{\sqrt[3]{x+5}}$$

$x=22:$

$$y = \frac{24}{\sqrt[3]{27}} = \frac{24}{3} = 8$$

$y =$ .... 8 .... [3]

24 y is inversely proportional to the cube of $(x - 1)$.
$y = 9.45$ when $x = 3$.

Find y when $x = 4$.

$$y \propto \frac{1}{(x-1)^3}$$

$$y = \frac{k}{(x-1)^3}$$

$y=9.45,\ x=3:$

$$9.45 = \frac{k}{2^3}$$

$$\underset{\times 8}{9.45} = \frac{k}{8} \times 8$$

$$k = 75.6$$

$$y = \frac{75.6}{(x-1)^3}$$

$x=4:$

$$y = \frac{75.6}{3^3} = \frac{75.6}{27} = 2.8$$

$y =$ .... 2.8 .... [3]

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22 y is inversely proportional to the square of $(x+3)$.
When $x = 5$, $y = 0.375$.

Find y in terms of x.

$$y \propto \frac{1}{(x+3)^2}$$

$$y = \frac{k}{(x+3)^2}$$

$x=5,\ y=0.375$:

$$0.375 = \frac{k}{8^2}$$

$$0.375 = \frac{k}{64}$$

$\times 64$ $\times 64$

$$k = 24$$

$$y = \frac{24}{(x+3)^2}$$

$y =$ $\frac{24}{(x+3)^2}$ [2]

20 $y \propto \frac{1}{\sqrt{x}}$

(a) When $x = 9$, $y = 2$.

Find the value of y when $x = 36$.

$$y = \frac{k}{\sqrt{x}}$$

$x=9,\ y=2$:

$$2 = \frac{k}{\sqrt{9}}$$

$$2 = \frac{k}{3}$$

$\times 3$ $\times 3$

$$k = 6$$

$$y = \frac{6}{\sqrt{x}}$$

$x=36$:

$$y = \frac{6}{\sqrt{36}} = \frac{6}{6} = 1$$

$y =$ 1 [3]

(b) When x is increased by a factor of 4, the value of y changes by a factor of p.

Find the value of p.

Increased by a factor of 4:

$$x \rightarrow 4x$$

Sub. $4x$:

$$y = \frac{k}{\sqrt{4x}}$$

$$y = \frac{k}{2\sqrt{x}}$$

$$y = \frac{1}{2} \times \frac{k}{\sqrt{x}}$$

So y has changed by a factor of $\frac{1}{2}$

$p =$ $\frac{1}{2}$ [1]

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7 y varies inversely as x.
When $x = 3$, $y = 16$.

Find x when $y = 6$.

$y \propto \frac{1}{x}$

$y = \frac{k}{x}$

$x=3,\ y=16:$

$\underset{\times 3}{16} = \frac{k}{3} \times 3$

$\underline{k = 48}$

$y = \frac{48}{x}$

$y=6:$

$\underset{\times x}{6} = \frac{48}{x} \times x$

$\underset{\div 6}{6x} = \underset{\div 6}{48}$

$x = 8$

$x =$ ....8.......... [3]

21 The force of attraction, F Newtons, between two magnets is inversely proportional to the square of the distance, d cm, between the magnets.

When $d = 1.5$, $F = 48$.

(a) Find an expression for F in terms of d.

$F \propto \frac{1}{d^2}$

$F = \frac{k}{d^2}$

$d=1.5,\ F=48:$

$48 = \frac{k}{1.5^2}$

$\underset{\times 2.25}{48} = \frac{k}{2.25} \times 2.25$

$\underline{108 = k}$

$F = \frac{108}{d^2}$

$F =$ .... $\frac{108}{d^2}$ .... [2]

(b) When the distance between the two magnets is doubled the new force is n times the original force.

Work out the value of n.

distance is doubled:

$d \to 2d$

sub. 2d:

$F = \frac{108}{(2d)^2}$

$F = \frac{108}{4d^2}$

$F = \frac{1}{4} \times \frac{108}{d^2}$

so F is now $\frac{1}{4}$ of the original force

$n =$ .... $\frac{1}{4}$ .... [1]

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12 y is inversely proportional to the square root of x.
v is directly proportional to y^2.
When $x = 9$, $y = 2$ and $v = 12$.

Find v in terms of x.
Give your answer in its simplest form.

$y \propto \frac{1}{\sqrt{x}}$

$y = \frac{k}{\sqrt{x}}$

$x=9, y=2:$

$2 = \frac{k}{\sqrt{9}}$

$2 = \frac{k}{3}$ $\times 3$ $\times 3$

$k = 6$

$y = \frac{6}{\sqrt{x}}$ ①

$v \propto y^2$

$v = ky^2$

$y=2, v=12:$

$12 = k \times 2^2$

$12 = k \times 4$ $\div 4$ $\div 4$

$k = 3$

$v = 3y^2$ ②

Sub. ① into ②:

$v = 3\left(\frac{6}{\sqrt{x}}\right)^2$

$= 3 \times \frac{36}{x}$

$v = \frac{108}{x}$

$v = \frac{108}{x}$ [4]

18 (a) y is directly proportional to the cube root of $(x + 1)$.
When $x = 7$, $y = 1$.

Find the value of y when $x = 124$.

$y \propto \sqrt[3]{x+1}$

$y = k\sqrt[3]{x+1}$

$x=7, y=1:$

$1 = k\sqrt[3]{8}$

$1 = 2k$ $\div 2$ $\div 2$

$k = \frac{1}{2}$

$y = \frac{1}{2}\sqrt[3]{x+1}$

$x = 124:$

$y = \frac{1}{2}\sqrt[3]{125}$

$= \frac{1}{2} \times 5$

$= 2.5$

$y = 2.5$ [3]

(b) F is inversely proportional to the square of d.

Explain what happens to F when d is halved.

$F \propto \frac{1}{d^2}$

$F = \frac{k}{d^2}$

$d \to \frac{d}{2}:$ $F = \frac{k}{\left(\frac{d}{2}\right)^2}$

$F = \frac{k}{\frac{d^2}{4}}$ $\times 4$ $\times 4$

$F = \frac{4k}{d^2}$

$F = 4 \times \frac{k}{d^2}$

[1]

→ So when d is halved, F increases by a factor of 4.

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25 w is proportional to the square root of y.
y is inversely proportional to x.
When $x = 4$, $y = 16$ and $w = 8$.

Find w in terms of x.

$w \propto \sqrt{y}$

$w = k\sqrt{y}$

$w=8,\ y=16:$

$8 = k\sqrt{16}$

$8 = 4k$

$\div 4 \quad \div 4$

$k = 2$

$w = 2\sqrt{y}$ ①

$y \propto \frac{1}{x}$

$y = \frac{K}{x}$

$x=4,\ y=16:$

$16 = \frac{K}{4}$

$\times 4 \quad \times 4$

$64 = K$

$y = \frac{64}{x}$ ②

Sub. ② into ①:

$w = 2\sqrt{\frac{64}{x}}$

$= 2 \times \frac{\sqrt{64}}{\sqrt{x}}$

$= 2 \times \frac{8}{\sqrt{x}}$

$w =$ .... $\frac{16}{\sqrt{x}}$ .... [3]

23 y is inversely proportional to $\sqrt{x}$ and x is directly proportional to w^2.
When $w = 12$, $y = 12$.

Find y in terms of w.

$y \propto \frac{1}{\sqrt{x}}$

$y = \frac{k}{\sqrt{x}}$ ①

$x \propto w^2$

$x = Kw^2$ ②

Sub. ② into ①:

$y = \frac{k}{\sqrt{Kw^2}}$

$y = \frac{k}{\sqrt{K}} \times \frac{1}{\sqrt{w^2}}$

can use a single letter to represent this constant: C

$y = \frac{C}{w}$

$w=12,\ y=12:$

$12 = \frac{C}{12}$

$\times 12 \quad \times 12$

$C = 144$

$y = \frac{144}{w}$

(e) The energy of a moving object is directly proportional to the square of its speed.
The speed of the object is increased by 30%. → 100% + 30% = 130%

Calculate the percentage increase in the energy of the object.

$E \propto v^2$

$E = kv^2$

$v \rightarrow 1.3v$:

$E = k \times (1.3v)^2$

$= k \times 1.69v^2$

$= 1.69 \times kv^2$

so E has increased by 69% (100% + 69% = 169%)

.... 69 .... % [2]