(b) Differentiate $18 + 5x - 2x^2$.

$5 - 4x$

$5 - 4x$ [2]

(c) Find the coordinates of the point on $y = 18 + 5x - 2x^2$ where the gradient is 17.

$5 - 4x = 17$

$-5 \qquad -5$

$-4x = 12$

$\div -4 \qquad \div -4$

$x = -3$

Sub. into y:

$y = 18 + 5(-3) - 2(-3)^2$

$= 18 - 15 - 2 \times 9$

$= 18 - 15 - 18$

$= -15$

(-3 , -15) [3]

12 **(a)** Find the gradient of the curve $y = 2x^3 - 7x + 4$ when $x = -2$.

$\frac{dy}{dx} = 6x^2 - 7$

$x = -2$:

$m = 6(-2)^2 - 7$

$= 6 \times 4 - 7$

$= 24 - 7$

$= 17$

17 [3]

(c) When $y = 2x^p + qx^2$, $\frac{dy}{dx} = 14x^6 + 6x$.

Find the value of p and the value of q.

$y = 2x^p + qx^2$

$\frac{dy}{dx} = 2px^{p-1} + 2qx$

$= 14x^6 + 6x$ (from above)

so:

$2p = 14$

$\div 2 \qquad \div 2$

$p = 7$

and:

$2q = 6$

$q = 3$

$p =$ 7

$q =$ 3 [2]

20 $y = 2x^k + ux^7$ and $\frac{dy}{dx} = 18x^{k-1} + 21x^6$

Find the value of k and the value of u.

$$y = 2x^k + ux^7$$

$$\frac{dy}{dx} = 2kx^{k-1} + 7ux^6$$

$$= 18x^{k-1} + 21x^6 \quad \text{(from above)}$$

So: $2k = 18$ $\div 2$ $\div 2$, $\underline{k = 9}$

and: $7u = 21$ $\div 7$ $\div 7$, $\underline{u = 3}$

$k =$ 9

$u =$ 3 [2]

25 The derivative of $2ax^7 + 3x^k$ is $42x^6 + 15x^{k-1}$.

Find the value of a and the value of k.

$$y = 2ax^7 + 3x^k$$

$$\frac{dy}{dx} = 14ax^6 + 3kx^{k-1}$$

$$= 42x^6 + 15x^{k-1} \quad \text{(from above)}$$

So: $14a = 42$ $\div 14$ $\div 14$, $\underline{a = 3}$

and: $3k = 15$ $\div 3$ $\div 3$, $\underline{k = 5}$

$a =$ 3

$k =$ 5 [2]

25 Find the x-coordinates of the points on the graph of $y = x^5 - 5x^4$ where the gradient is 0.

$$\frac{dy}{dx} = 5x^4 - 20x^3$$

$\frac{dy}{dx} = 0$: $5x^4 - 20x^3 = 0$

$$5x^3(x - 4) = 0$$

$5x^3 = 0$ or $x - 4 = 0$

$+4 \quad +4$

$\underline{x = 0}$ $\quad \underline{x = 4}$

$x = 0$ or $x = 4$ [4]

21 (a) Differentiate $6 + 4x - x^2$.

$$4 - 2x$$

$4 - 2x$ [2]

(b) Find the coordinates of the turning point of the graph of $y = 6 + 4x - x^2$.

Turning point is where gradient $= 0$, ie. $\frac{dy}{dx} = 0$:

$4 - 2x = 0$

$-4 \quad -4$

$-2x = -4$

$\div -2 \quad \div -2$

$\underline{x = 2}$

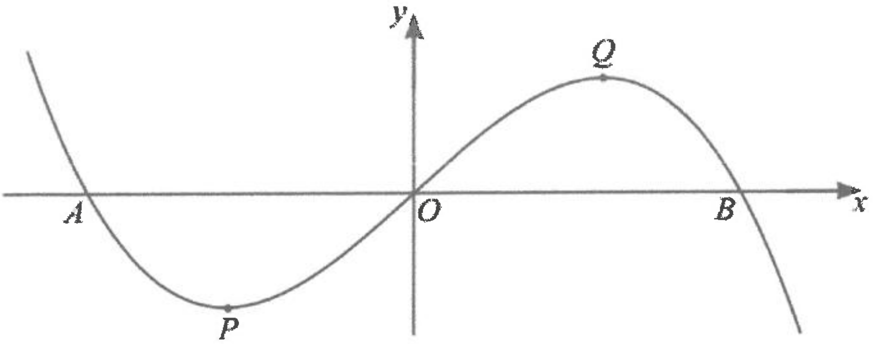
sub. into y:

$y = 6 + 4(2) - (2)^2$

$= 6 + 8 - 4$

$= \underline{10}$

(2 , 10) [2]



NOT TO SCALE

The diagram shows the graph of $y = 3x - x^3$.
The graph crosses the x-axis at A, at O and at B.
The turning points of the graph are at P and at Q.

(b) (i) Differentiate $3x - x^3$.

$$\frac{dy}{dx} = 3 - 3x^2$$

$3 - 3x^2$ .... [2]

(ii) Find the coordinates of P and Q.

$\frac{dy}{dx} = 0$: $3 - 3x^2 = 0$
$-3 \qquad -3$

$-3x^2 = -3$
$\div -3 \qquad \div -3$

$x^2 = 1$

$x = \pm\sqrt{1}$

$\underline{x = -1}$ or $\underline{x = 1}$

sub. $x = 1$: $y = 3(1) - 1^3$
$= 3 - 1$
$= \underline{2}$ → $\underline{(1, 2)}$

sub. $x = -1$: $y = 3(-1) - (-1)^3$
$= -3 - -1$
$= -3 + 1$
$= \underline{-2}$ → $\underline{(-1, -2)}$

P (........1........ , ...2........)

Q (........-1........ , ...-2........) [4]

22 A curve has equation $y = x^n + qx^2 + 9x$.

$$\frac{dy}{dx} = 3x^2 - 12x + 9$$

(a) Find the value of n, and the value of q.

$y = x^n + qx^2 + 9x$

$\frac{dy}{dx} = nx^{n-1} + 2qx + 9$

$= 3x^2 - 12x + 9$ (from above)

so:

$n = 3$

and:

$2q = -12$
$\div 2 \quad \div 2$
$q = -6$

$n =$ ....3.... $q =$ ....-6.... [2]

$(y = x^3 - 6x^2 + 9x)$

(b) Work out the coordinates of the turning points of the curve.

$\frac{dy}{dx} = 0$: $3x^2 - 12x + 9 = 0$
$\div 3 \quad \div 3 \quad \div 3 \quad \div 3$

$x^2 - 4x + 3 = 0$

$(x-1)(x-3) = 0$

$x = 1$ or $x = 3$

Sub. $x = 1$ into y:

$y = (1)^3 - 6(1)^2 + 9(1)$
$= 1 - 6 + 9$
$= 4 \rightarrow (1, 4)$

Sub. $x = 3$ into y:

$y = (3)^3 - 6(3)^2 + 9(3)$
$= 27 - 6 \times 9 + 27$
$= 27 - 54 + 27$
$= 0 \rightarrow (3, 0)$

(....1.... , ...4....) and (....3.... , ...0....) [4]

21 A curve has equation $y = x^3 - 12x$.

(a) Find the gradient of the curve at the point $(1, -11)$.

$$\frac{dy}{dx} = 3x^2 - 12$$

Sub. $x = 1$:

$$m = 3(1)^2 - 12$$
$$= 3 - 12$$
$$= -9$$

-9 .... [3]

(b) Find the coordinates of the turning points of the curve.

$\frac{dy}{dx} = 0$:

$$3x^2 - 12 = 0$$
$+12 \quad +12$
$$3x^2 = 12$$
$\div 3 \quad \div 3$
$$x^2 = 4$$
$$x = \pm\sqrt{4}$$
$$\underline{x = -2} \text{ or } \underline{x = 2}$$

Sub. $x = 2$ into y: $y = (2)^3 - 12(2)$
$= 8 - 24$
$= \underline{-16} \rightarrow \underline{(2, -16)}$

Sub. $x = -2$ into y: $y = (-2)^3 - 12(-2)$
$= -8 - -24$
$= -8 + 24$
$= \underline{16} \rightarrow \underline{(-2, 16)}$

(..... 2 ..... , ... -16 .....) and (..... -2 ..... , ... 16 .....) [3]

24 A curve has equation $y = x^3 - 2x^2 + 5$.

Find the coordinates of its two stationary points.

$$\frac{dy}{dx} = 3x^2 - 4x$$

$\frac{dy}{dx} = 0$: $3x^2 - 4x = 0$
$$x(3x - 4) = 0$$
$\underline{x = 0}$ or $3x - 4 = 0$
$+4 \quad +4$
$$3x = 4$$
$\div 3 \quad \div 3$
$$\underline{x = \frac{4}{3}}$$

Sub. $x = 0$ into y: $y = 0^3 - 2(0)^2 + 5$
$= \underline{5} \rightarrow \underline{(0, 5)}$

Sub. $x = \frac{4}{3}$ into y: $y = \left(\frac{4}{3}\right)^3 - 2\left(\frac{4}{3}\right)^2 + 5$
$= \frac{64}{27} - \frac{32}{9} + 5$
$= \underline{\frac{103}{27}} \rightarrow \underline{\left(\frac{4}{3}, \frac{103}{27}\right)}$

(..... 0 ..... , ... 5 .....) and (..... $\frac{4}{3}$ ..... , ... $\frac{103}{27}$ .....) [5]

9 $f(x) = x^3 - 3x^2 - 4$

(a) Find the gradient of the graph of $y = f(x)$ where $x = 1$.

$\frac{dy}{dx} = 3x^2 - 6x$

sub. $x=1$:

$m = 3(1)^2 - 6(1)$

$= 3 - 6 = -3$

-3 [3]

(b) Find the coordinates of the turning points of the graph of $y = f(x)$.

$\frac{dy}{dx} = 0$:

$3x^2 - 6x = 0$

$3x(x-2) = 0$

$3x = 0$ or $x - 2 = 0$ (+2 +2)

$x = 0$, $x = 2$

sub. $x=0$ into y:

$y = 0^3 - 3(0)^2 - 4$

$= -4 \rightarrow (0, -4)$

sub. $x=2$ into y:

$y = 2^3 - 3(2)^2 - 4$

$= 8 - 3 \times 4 - 4$

$= 8 - 12 - 4$

$= -8 \rightarrow (2, -8)$

(0 , -4) , (2 , -8) [4]

(c) A curve has equation $y = x^3 + ax + b$.
The stationary points of the curve have coordinates $(2, k)$ and $(-2, 10 - k)$.

Work out the value of a, the value of b and the value of k.

$$\frac{dy}{dx} = 3x^2 + a$$

$\frac{dy}{dx} = 0$: $3x^2 + a = 0$

$-a \quad -a$

$$3x^2 = -a$$

$\div 3 \quad \div 3$

$$x^2 = \frac{-a}{3}$$

$$x = \pm\sqrt{\frac{-a}{3}}$$

$x = \sqrt{\frac{-a}{3}}$ or $x = -\sqrt{\frac{-a}{3}}$

using x-coordinates from above:

$\sqrt{\frac{-a}{3}} = 2$ (or $-\sqrt{\frac{-a}{3}} = -2$)

square square

$$\frac{-a}{3} = 4$$

$\times 3 \quad \times 3$

$$-a = 12$$

$$a = -12$$

$y = x^3 - 12x + b$

sub. $(2, k)$:

$k = 2^3 - 12(2) + b$

$k = 8 - 24 + b$

$k = -16 + b$ ①

sub. $(-2, 10-k)$:

$10 - k = (-2)^3 - 12(-2) + b$

$10 - k = -8 + 24 + b$

$10 - k = 16 + b$

$-10 \quad -10$

$-k = 6 + b$ ②

① + ②:

$0 = -10 + 2b$

$+10 \quad +10$

$10 = 2b$

$\div 2 \quad \div 2$

$5 = b$

①:

$k = -16 + 5$

$= -11$

$a =$ ....-12.... , $b =$ ....5.... , $k =$ ....-6.... [6]

(c) (i) Find the coordinates of the turning points of the graph of $y = 10 + 9x^2 - 2x^3$.
You must show all your working.

$\frac{dy}{dx} = 18x - 6x^2$

$\frac{dy}{dx} = 0$: $18x - 6x^2 = 0$

$6x(3 - x) = 0$

$6x = 0$ or $3 - x = 0$

$\div 6 \quad \div 6 \qquad +x \quad +x$

$x = 0 \qquad 3 = x$

$x = 3$

Sub. $x = 0$ into y:

$y = 10 + 9(0)^2 - 2(0)^3$

$= 10 \rightarrow (0, 10)$

Sub. $x = 3$ into y:

$y = 10 + 9(3)^2 - 2(3)^3$

$= 10 + 9 \times 9 - 2 \times 27$

$= 10 + 81 - 54$

$= 37 \rightarrow (3, 37)$

(0 , 10) and (3 , 37) [5]

(ii) Determine whether each turning point is a maximum or a minimum.
Show how you decide.

$\frac{d^2y}{dx^2} = 18 - 12x$

Sub. $x = 0$: $18 - 12(0)$

$= 18$

$\frac{d^2y}{dx^2} > 0$, so $(0, 10)$ is a minimum point.

Sub. $x = 3$: $18 - 12(3)$

$= 18 - 36$

$= -18$

$\frac{d^2y}{dx^2} < 0$ so $(3, 37)$ is a maximum point.

[3]

11 A curve has equation $y = x^3 - 3x + 4$.

(a) Work out the coordinates of the two stationary points.

$$\frac{dy}{dx} = 3x^2 - 3$$

$\frac{dy}{dx} = 0$: $3x^2 - 3 = 0$

$+3 \quad +3$

$3x^2 = 3$

$\div 3 \quad \div 3$

$x^2 = 1$

$x = \pm\sqrt{1}$

$\underline{x = -1}$ or $\underline{x = 1}$

Sub. $x = 1$ into y:

$y = (1)^3 - 3(1) + 4$

$= 1 - 3 + 4$

$= \underline{2}$ $\rightarrow$ $\underline{(1, 2)}$

Sub. $x = -1$ into y:

$y = (-1)^3 - 3(-1) + 4$

$= -1 + 3 + 4$

$= \underline{6}$ $\rightarrow$ $\underline{(-1, 6)}$

(.......1......... , ...2..........)

(.......-1......... , ...6..........) [5]

(b) Determine whether each stationary point is a maximum or a minimum.
Give reasons for your answers.

$$\frac{d^2y}{dx^2} = 6x$$

Sub. $x = 1$: $6(1) = 6$

$\frac{d^2y}{dx^2} > 0$, so

$(1, 2)$ is a minimum

Sub. $x = -1$: $6(-1) = -6$

$\frac{d^2y}{dx^2} < 0$, so

$(-1, 6)$ is a maximum

[3]

(c) A curve has equation $y = 2x^3 - 4x^2 + 6$.

(i) Find $\frac{dy}{dx}$, the derived function of y.

$$\frac{dy}{dx} = 6x^2 - 8x$$

$6x^2 - 8x$ [2]

(ii) Calculate the gradient of the curve $y = 2x^3 - 4x^2 + 6$ at $x = 4$.

Sub. $x = 4$:

$$m = 6(4)^2 - 8(4)$$
$$= 6 \times 16 - 32$$
$$= 96 - 32$$
$$= 64$$

64 [2]

(iii) Find the coordinates of the two stationary points on the curve.

$\frac{dy}{dx} = 0$:

$$6x^2 - 8x = 0$$
$$2x(3x - 4) = 0$$

$2x = 0$ or $3x - 4 = 0$

$\div 2 \quad \div 2 \qquad +4 \quad +4$

$\underline{x = 0} \qquad 3x = 4$

$\div 3 \quad \div 3$

$\underline{x = \frac{4}{3}}$

Sub. $x = 0$ into y:

$$y = 2(0)^3 - 4(0)^2 + 6$$
$$= \underline{6} \rightarrow \underline{(0, 6)}$$

Sub. $x = \frac{4}{3}$ into y:

$$y = 2\left(\frac{4}{3}\right)^3 - 4\left(\frac{4}{3}\right)^2 + 6$$
$$= 2 \times \frac{64}{27} - 4 \times \frac{16}{9} + 6$$
$$= \underline{\frac{98}{27}} \rightarrow \underline{\left(\frac{4}{3}, \frac{98}{27}\right)}$$

(0 , 6) and ($4/3$, $98/27$) [4]

(b) A curve has the equation $y = x^3 + 8x^2 + 5x$.

(i) Work out the coordinates of the two turning points.

$$\frac{dy}{dx} = 3x^2 + 16x + 5$$

$\frac{dy}{dx} = 0$: $3x^2 + 16x + 5 = 0$

ac: 15

two numbers: 1, 15

$\rightarrow 3x^2 + 1x + 15x + 5 = 0$

$x(3x+1) + 5(3x+1) = 0$

$(x+5)(3x+1) = 0$

$x + 5 = 0$ or $3x + 1 = 0$

$-5 \quad -5 \qquad -1 \quad -1$

$x = -5$ $\qquad 3x = -1$

$\div 3 \quad \div 3$

$x = \frac{-1}{3}$

Sub. $x = -5$ into y:

$y = (-5)^3 + 8(-5)^2 + 5(-5)$

$= -125 + 8 \times 25 + -25$

$= -125 + 200 - 25$

$= 50 \rightarrow (-5, 50)$

Sub. $x = \frac{-1}{3}$ into y:

$y = \left(\frac{-1}{3}\right)^3 + 8\left(\frac{-1}{3}\right)^2 + 5\left(\frac{-1}{3}\right)$

$= \frac{-1}{27} + 8 \times \frac{1}{9} - \frac{5}{3}$

$= \frac{-1}{27} + \frac{8}{9} - \frac{5}{3}$

$= \frac{-22}{27} \rightarrow \left(\frac{-1}{3}, \frac{-22}{27}\right)$

(-5 , 50) and ($\frac{-1}{3}$, $\frac{-22}{27}$) [6]

(ii) Determine whether each of the turning points is a maximum or a minimum. Give reasons for your answers.

$$\frac{d^2y}{dx^2} = 6x + 16$$

Sub. $x = -5$:

$6(-5) + 16$

$= -30 + 16$

$= -14$

$\frac{d^2y}{dx^2} < 0$, so

$(-5, 50)$ is a maximum

Sub. $x = \frac{-1}{3}$:

$6\left(\frac{-1}{3}\right) + 16$

$= -2 + 16$

$= 14$

$\frac{d^2y}{dx^2} > 0$, so

$\left(\frac{-1}{3}, \frac{-22}{27}\right)$ is a minimum

[3]

12 **(a)** A curve has equation $y = 4x^3 - 3x + 3$.

(i) Find the coordinates of the two stationary points.

$\frac{dy}{dx} = 12x^2 - 3$

$\frac{dy}{dx} = 0$: $12x^2 - 3 = 0$

$+3 \quad +3$

$12x^2 = 3$

$\div 12 \quad \div 12$

$x^2 = \frac{1}{4}$

$x = \pm\sqrt{\frac{1}{4}}$

$\underline{x = \frac{1}{2}}$ or $\underline{x = \frac{-1}{2}}$

Sub. $x = \frac{1}{2}$: $y = 4\left(\frac{1}{2}\right)^3 - 3\left(\frac{1}{2}\right) + 3$

$= 4 \times \frac{1}{8} - \frac{3}{2} + 3$

$= \frac{1}{2} - \frac{3}{2} + 3$

$= \underline{2}$ $\rightarrow$ $\underline{\left(\frac{1}{2}, 2\right)}$

Sub. $x = \frac{-1}{2}$: $y = 4\left(\frac{-1}{2}\right)^3 - 3\left(\frac{-1}{2}\right) + 3$

$= 4 \times \frac{-1}{8} + \frac{3}{2} + 3$

$= \underline{4}$ $\rightarrow$ $\underline{\left(\frac{-1}{2}, 4\right)}$

($\frac{1}{2}$, 2) and ($\frac{-1}{2}$, 4) [5]

(ii) Determine whether each of the stationary points is a maximum or a minimum. Give reasons for your answers.

$\frac{d^2y}{dx^2} = 24x$

Sub. $x = \frac{1}{2}$:

$24\left(\frac{1}{2}\right) = 12$

$\frac{d^2y}{dx^2} > 0$, so

$\left(\frac{1}{2}, 2\right)$ is a minimum

Sub. $x = \frac{-1}{2}$:

$24\left(\frac{-1}{2}\right) = -12$

$\frac{d^2y}{dx^2} < 0$, so

$\left(\frac{-1}{2}, 4\right)$ is a maximum

[3]

10 **(a)** $y = x^4 - 4x^3$

(i) Find the value of y when $x = -1$.

$y = (-1)^4 - 4(-1)^3$

$= 1 - 4 \times -1$

$= 1 + 4$

$= 5$

$y =$ 5 [2]

(ii) Find the two stationary points on the graph of $y = x^4 - 4x^3$.

$\frac{dy}{dx} = 4x^3 - 12x^2$

$\frac{dy}{dx} = 0$: $4x^3 - 12x^2 = 0$

$4x^2(x - 3) = 0$

$4x^2 = 0$ or $x - 3 = 0$

$\div 4$ $\div 4$ $+3$ $+3$

$x^2 = 0$ $x = 3$

$x = 0$

Sub. $x = 0$ into y:

$y = 0^4 - 4(0)^3$

$= 0 \rightarrow (0, 0)$

Sub. $x = 3$ into y:

$y = 3^4 - 4(3)^3$

$= 81 - 4 \times 27$

$= 81 - 108$

$= -27 \rightarrow (3, -27)$

$(0, 0)$ and $(3, -27)$

(b) $y = x^p + 2x^q$

$\frac{dy}{dx} = 11x^{10} + 10x^4$, where $\frac{dy}{dx}$ is the derived function.

Find the value of p and the value of q.

$y = x^p + 2x^q$

$\frac{dy}{dx} = px^{p-1} + 2qx^{q-1}$

$= 11x^{10} + 10x^4$ (from above)

So: $p = 11$ and: $2q = 10$

$\div 2$ $\div 2$

$q = 5$

$p =$ 11

$q =$ 5 [2]

12 A curve has equation $y = x^3 - kx^2 + 1$.
When $x = 2$, the gradient of the curve is 6.

(a) Show that $k = 1.5$.

$\frac{dy}{dx} = 3x^2 - 2kx$

$\frac{dy}{dx} = 6$ when $x = 2$:

$6 = 3(2)^2 - 2k \times 2$

$6 = 3 \times 4 - 4k$

$6 = 12 - 4k$

$-12 \quad -12$

$-6 = -4k$

$\div -4 \quad \div -4$

$\frac{6}{4} = k$

$\frac{3}{2} = k$

$\underline{k = 1.5}$

[5]

(b) Find the coordinates of the two stationary points of $y = x^3 - 1.5x^2 + 1$.
You must show all your working.

$\frac{dy}{dx} = 3x^2 - 3x$

$\frac{dy}{dx} = 0$: $3x^2 - 3x = 0$

$3x(x - 1) = 0$

$3x = 0$ or $x - 1 = 0$

$\div 3 \quad \div 3 \qquad +1 \quad +1$

$\underline{x = 0} \qquad \underline{x = 1}$

Sub. $x = 0$ into y:

$y = 0^3 - 1.5(0)^2 + 1$

$= \underline{1} \rightarrow \underline{(0, 1)}$

Sub. $x = 1$ into y:

$y = 1^3 - 1.5(1)^2 + 1$

$= 1 - 1.5 + 1$

$= \underline{0.5} \rightarrow \underline{(1, 0.5)}$

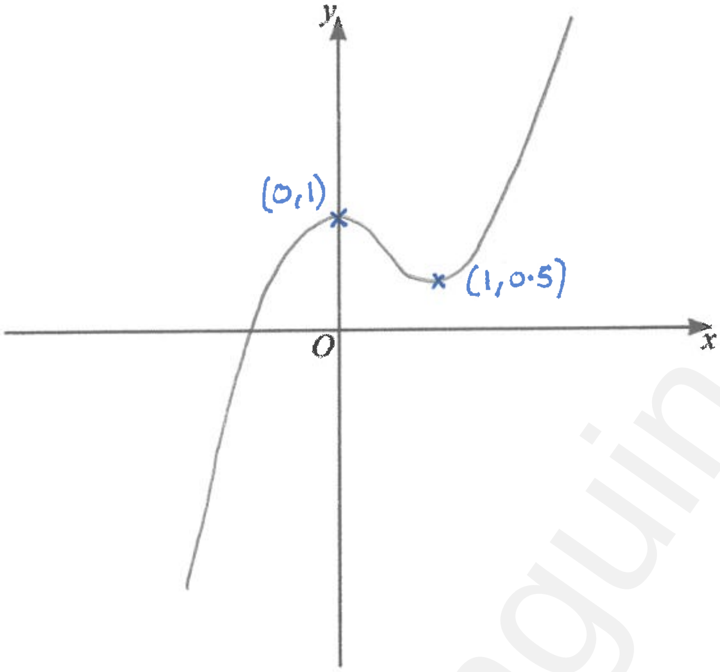
(0 , 1) and (1 , 0.5) [4]

positive cubic so looks like:

(as opposed to if $-x^3$)

(c) Sketch the curve $y = x^3 - 1.5x^2 + 1$.

(continued from previous page)



[2]

12 The equation of a curve is $y = x^4 - 8x^2 + 5$.

(a) Find the derivative, $\left(\frac{dy}{dx}\right)$, of $y = x^4 - 8x^2 + 5$.

$$\frac{dy}{dx} = 4x^3 - 16x$$

$4x^3 - 16x$ [2]

(b) Find the coordinates of the three turning points.
You must show all your working.

$$\frac{dy}{dx} = 0: \quad 4x^3 - 16x = 0$$

$$4x(x^2 - 4) = 0$$

$$4x(x+2)(x-2) = 0$$

$4x = 0$ or $x + 2 = 0$ or $x - 2 = 0$

$\div 4 \quad \div 4 \qquad -2 \quad -2 \qquad +2 \quad +2$

$x = 0 \qquad x = -2 \qquad x = 2$

Sub. $x = 0$ into y: $y = 0^4 - 8(0)^2 + 5$

$= 5 \rightarrow (0, 5)$

Sub. $x = 2$ into y: $y = 2^4 - 8(2)^2 + 5$

$= 16 - 32 + 5$

$= -11 \rightarrow (2, -11)$

Sub. $x = -2$ into y: $y = (-2)^4 - 8(-2)^2 + 5$

$= 16 - 32 + 5$

$= -11 \rightarrow (-2, -11)$

(0 , 5) and (2 , -11) and (-2 , -11) [4]

(c) Determine which one of these turning points is a maximum.
Justify your answer.

$$\frac{d^2y}{dx^2} = 12x^2 - 16$$

Sub. $x = 0$:

$12(0)^2 - 16$

$= 0 - 16$

$= -16$

$\frac{d^2y}{dx^2} < 0$, so $(0, 5)$ is a maximum

[2]

22 A curve has equation $y = x^3 + x^2 - x$.

The curve has a stationary point at $\left(\frac{1}{3}, -\frac{5}{27}\right)$.

(a) Find the coordinates of the other stationary point.

$$\frac{dy}{dx} = 3x^2 + 2x - 1$$

$\frac{dy}{dx} = 0$: $3x^2 + 2x - 1 = 0$

ac: -3

two numbers: $3, -1$

$\rightarrow 3x^2 + 3x - 1x - 1 = 0$

$3x(x+1) - 1(x+1) = 0$

$(3x-1)(x+1) = 0$

$3x - 1 = 0$ or $x + 1 = 0$

$+1 \quad +1 \qquad -1 \quad -1$

$3x = 1 \qquad x = -1$

$\div 3 \quad \div 3$

$x = \frac{1}{3}$

not the point we're interested in

Sub. $x = -1$ into y:

$y = (-1)^3 + (-1)^2 - (-1)$

$= -1 + 1 + 1$

$= 1$

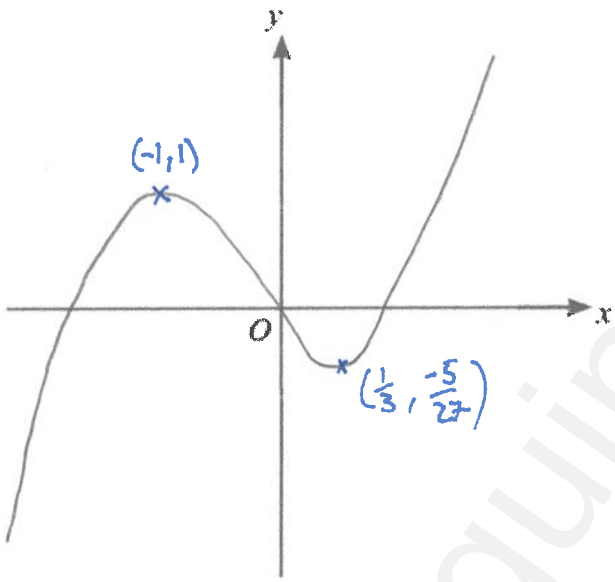
$(-1, 1)$

(..........-1.........., ...1..........) [5]

positive cubic 

(continued from previous page)

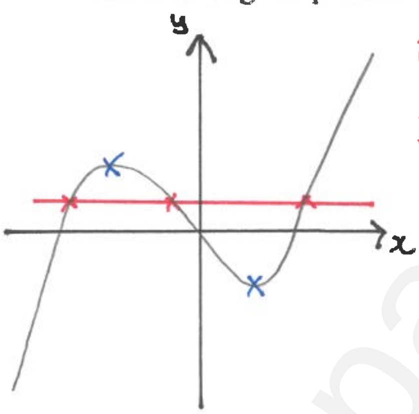
(b) By sketching the graph of $y = x^3 + x^2 - x$, determine whether the stationary point $\left(\frac{1}{3}, -\frac{5}{27}\right)$ is a maximum or a minimum.



$\left(\frac{1}{3}, -\frac{5}{27}\right)$ is a ......Minimum...... [2]

(c) The equation $x^3 + x^2 - x = k$ has fewer than 3 solutions.

Find the range of possible values for k.



if k was anything between $y = \frac{-5}{27}$ and $y = 1$, the equation would have three solutions e.g.

So line must be below (or equal to) $y = \frac{-5}{27}$ or above $y = 1$

$\rightarrow \underline{\underline{k \leq \frac{-5}{27}}}$ or $\underline{\underline{k \geq 1}}$

(b) **(i)** Find the derivative, $\frac{dy}{dx}$, of $y = 5 + 8x - \frac{4}{3}x^3$.

$\frac{dy}{dx} = 8 - 4x^2$

$8 - 4x^2$ [2]

(ii) Find the gradient of $y = 5 + 8x - \frac{4}{3}x^3$ at $x = -1$.

Sub. $x=-1$:

$m = 8 - 4(-1)^2$

$= 8 - 4 \times 1$

$= \underline{4}$

4 [2]

(iii) A tangent is drawn to the graph of $y = 5 + 8x - \frac{4}{3}x^3$.

The gradient of the tangent is -28.

Find the coordinates of the two possible points where this tangent meets the graph.

$\frac{dy}{dx} = -28$:

$8 - 4x^2 = -28$

$-8 \qquad -8$

$-4x^2 = -36$

$\div -4 \qquad \div -4$

$x^2 = 9$

$x = \pm\sqrt{9}$

$\underline{x=-3}$ or $\underline{x=3}$

Sub. $x=3$ into y:

$y = 5 + 8(3) - \frac{4}{3}(3)^3$

$= 5 + 24 - \frac{4}{3} \times 27$

$= 29 - 36$

$= \underline{-7} \rightarrow \underline{(3,-7)}$

Sub. $x=-3$ into y:

$y = 5 + 8(-3) - \frac{4}{3}(-3)^3$

$= 5 - 24 - \frac{4}{3} \times -27$

$= -19 + 36$

$= \underline{17} \rightarrow \underline{(-3,17)}$

(3 , -7)

(-3 , 17) [5]

(iii) Find the equation of the tangent to the graph of $y = 18 - 3x - x^2$ at $x = 4$.
Give your answer in the form $y = mx + c$.

$\frac{dy}{dx} = -3 - 2x$

Sub. $x = 4$:

$m = -3 - 2(4)$
$= -3 - 8$
$= -11$

gradient $= -11$

Find y-coordinate by substituting $x=4$ into y:

$y = 18 - 3(4) - (4)^2$
$= 18 - 12 - 16$
$= -10 \rightarrow (4, -10)$

Sub. gradient into $y = mx + c$:

$y = -11x + c$

Sub. $(4, -10)$:

$-10 = -11(4) + c$
$-10 = -44 + c$
$+44 \quad +44$
$34 = c$

$y = -11x + 34$

$y =$ $-11x + 34$ [6]

(b) Find the equation of the tangent to the graph of $y = x^3 - 4x^2 + 4x$ at $x = 4$.
Give your answer in the form $y = mx + c$.

$\frac{dy}{dx} = 3x^2 - 8x + 4$

Sub. $x = 4$:

$m = 3(4)^2 - 8(4) + 4$

$= 3 \times 16 - 32 + 4$

$= 48 - 28$

$= \underline{20}$

gradient $= 20$

Find y-coordinate by substituting $x = 4$ into y:

$y = 4^3 - 4(4)^2 + 4(4)$

$= 64 - 64 + 16$

$= \underline{16} \rightarrow \underline{(4, 16)}$

Sub. gradient into $y = mx + c$:

$y = 20x + c$

Sub. $(4, 16)$:

$16 = 20(4) + c$

$16 = 80 + c$

$-80 \quad -80$

$\underline{-64 = c}$

$\underline{y = 20x - 64}$

$y =$ $20x - 64$ [7]

(b) The graph of $y = x^3 - 5x^2 + 2x + 8$ has two tangents with a gradient of 10. $y = 10x + c$

Find the equations of these two tangents.
You must show all your working and give your answers in the form $y = mx + c$.

$\frac{dy}{dx} = 3x^2 - 10x + 2$

gradient = 10:

$10 = 3x^2 - 10x + 2$

$-10 \qquad -10$

$3x^2 - 10x - 8 = 0$

ac: -24

two numbers: $-12, 2$

$\rightarrow 3x^2 - 12x + 2x - 8 = 0$

$3x(x-4) + 2(x-4) = 0$

$(3x+2)(x-4) = 0$

$3x + 2 = 0$ or $x - 4 = 0$

$-2 \quad -2 \qquad +4 \quad +4$

$3x = -2 \qquad \underline{x = 4}$

$\div 3 \quad \div 3$

$\underline{x = \frac{-2}{3}}$

Find y-coordinates:

Sub. $x = 4$: $y = 4^3 - 5(4)^2 + 2(4) + 8$

$= 64 - 5 \times 16 + 8 + 8$

$= 64 - 80 + 16$

$= \underline{0} \rightarrow \underline{(4, 0)}$

Sub. $x = \frac{-2}{3}$: $y = \left(\frac{-2}{3}\right)^3 - 5\left(\frac{-2}{3}\right)^2 + 2\left(\frac{-2}{3}\right) + 8$

$= \frac{-8}{27} - 5 \times \frac{4}{9} - \frac{4}{3} + 8$

$= \frac{-8}{27} - \frac{20}{9} - \frac{4}{3} + 8$

$= \underline{\frac{112}{27}} \rightarrow \underline{\left(\frac{-2}{3}, \frac{112}{27}\right)}$

Sub. into $y = 10x + c$:

Sub. $(4, 0)$: $0 = 10(4) + c$

$0 = 40 + c$

$-40 \quad -40$

$\underline{-40 = c} \rightarrow \underline{y = 10x - 40}$

Sub. $\left(\frac{-2}{3}, \frac{112}{27}\right)$:

$\frac{112}{27} = 10\left(\frac{-2}{3}\right) + c$

$\frac{112}{27} = \frac{-20}{3} + c$

$+\frac{20}{3} \quad +\frac{20}{3}$

$\underline{\frac{292}{27} = c} \rightarrow \underline{y = 10x + \frac{292}{27}}$

$\underline{y = 10x - 40}$ and $\underline{y = 10x + \frac{292}{27}}$