21 Expand and simplify $(x+3)(x-5)(3x-1)$.

$(x+3)[3x^2-x-15x+5]$

$=(x+3)[3x^2-16x+5]$

$=3x^3-16x^2+5x+9x^2-48x+15$

$=\underline{\underline{3x^3-7x^2-43x+15}}$

.... [3]

20 Expand and simplify.

$(x-2)(2x+5)(x+3)$

$(x-2)[2x^2+6x+5x+15]$

$=(x-2)[2x^2+11x+15]$

$=2x^3+11x^2+15x-4x^2-22x-30$

$=\underline{\underline{2x^3+7x^2-7x-30}}$

.... [3]

22 **(a)** Expand and simplify.

$(2x-1)(x+4)(x-3)$

$(2x-1)[x^2-3x+4x-12]$

$=(2x-1)[x^2+x-12]$

$=2x^3+2x^2-24x-x^2-x+12$

$=\underline{\underline{2x^3+x^2-25x+12}}$

.... [3]

(c) Expand and simplify.

$$(y+3)(y-4)(2y-1)$$

$(y+3)[2y^2-y-8y+4]$

$=(y+3)[2y^2-9y+4]$

$=2y^3-9y^2+4y+6y^2-27y+12$

$=\underline{2y^3-3y^2-23y+12}$

.... [3]

(e) Expand and simplify.

$$(2x-3)(x+6)(x-4)$$

$(2x-3)[x^2-4x+6x-24]$

$=(2x-3)[x^2+2x-24]$

$=2x^3+4x^2-48x-3x^2-6x+72$

$=\underline{2x^3+x^2-54x+72}$

.... [3]

19 Expand and simplify.

$$(2x+3)(x-2)^2$$

$(2x+3)(x-2)(x-2)$

$=(2x+3)[x^2-2x-2x+4]$

$=(2x+3)[x^2-4x+4]$

$=2x^3-8x^2+8x+3x^2-12x+12$

$=\underline{2x^3-5x^2-4x+12}$

.... [3]

21 Expand and simplify.

$$(x-3)^2(2x+5)$$

$$(x-3)(x-3)(2x+5)$$

$$= (x-3)[2x^2+5x-6x-15]$$

$$= (x-3)[2x^2-x-15]$$

$$= 2x^3-x^2-15x-6x^2+3x+45$$

$$= \underline{\underline{2x^3-7x^2-12x+45}}$$

.... [3]

20 $(x+a)(x+2)(2x+3)$ is equivalent to $2x^3+bx^2+cx-18$.

Find the value of each of a, b and c.

$$(x+a)[2x^2+3x+4x+6]$$

$$= (x+a)[2x^2+7x+6]$$

$$= 2x^3+7x^2+6x+2ax^2+7ax+6a$$

$$= 2x^3+2ax^2+7x^2+7ax+6x+6a$$

$$= 2x^3+(2a+7)x^2+(7a+6)x+6a$$

$$= 2x^3+bx^2+cx-18 \quad \text{(from above)}$$

so: $6a = -18$ $\div 6 \quad \div 6$ $\underline{a = -3}$

$a=-3$: $7a+6=c$, $-21+6=c$, $\underline{-15=c}$

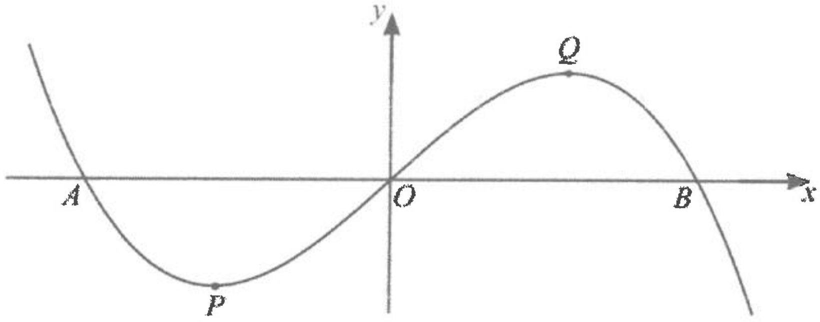
$a=-3$: $2a+7=b$, $-6+7=b$, $\underline{1=b}$

$a =$ .... -3 ....

$b =$ .... 1 ....

$c =$ .... -15 .... [3]

21



NOT TO SCALE

The diagram shows the graph of $y = 3x - x^3$.
The graph crosses the x-axis at A, at O and at B.
The turning points of the graph are at P and at Q.

(a) Find the x-coordinate of A and the x-coordinate of B.
Give your answers as exact values.

Find roots by setting $y = 0$:

$$3x - x^3 = 0$$

$$x(3 - x^2) = 0$$

$\underline{x = 0}$ or $3 - x^2 = 0$

$+x^2 \quad +x^2$

$$3 = x^2$$

$$\underline{x = \pm\sqrt{3}}$$

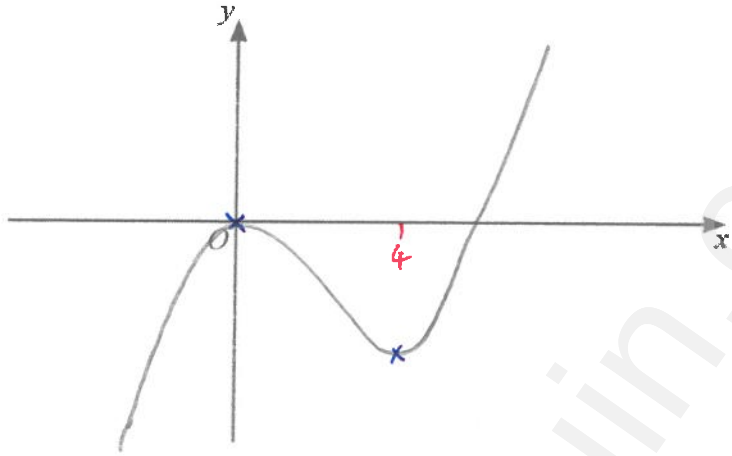
x-coordinate of A ....$-\sqrt{3}$........

x-coordinate of B ....$\sqrt{3}$........

[3]

21 The graph of a cubic function has two turning points.
When $x < 0$ and when $x > 4$ the gradient of the graph is positive. → turning points at $x=0$ and $x=4$
When $0 < x < 4$ the gradient of the graph is negative.
The graph passes through the origin.

Sketch the graph.



[2]

9 $$f(x) = x(x-1)(x-2)$$

(a) Find the coordinates of the points where the graph of $y = f(x)$ crosses the x-axis.

Sub $y=0$:

$x(x-1)(x-2)=0$

$x=0$ or $x-1=0$ or $x-2=0$

$+1 \quad +1 \qquad +2 \quad +2$

$x=1 \qquad x=2$

(0 , 0)

(1 , 0)

(2 , 0) [2]

(b) Show that $f(x) = x^3 - 3x^2 + 2x$.

$x(x-1)(x-2)$

$= x[x^2 - 2x - x + 2]$

$= x[x^2 - 3x + 2]$

$= x^3 - 3x^2 + 2x$

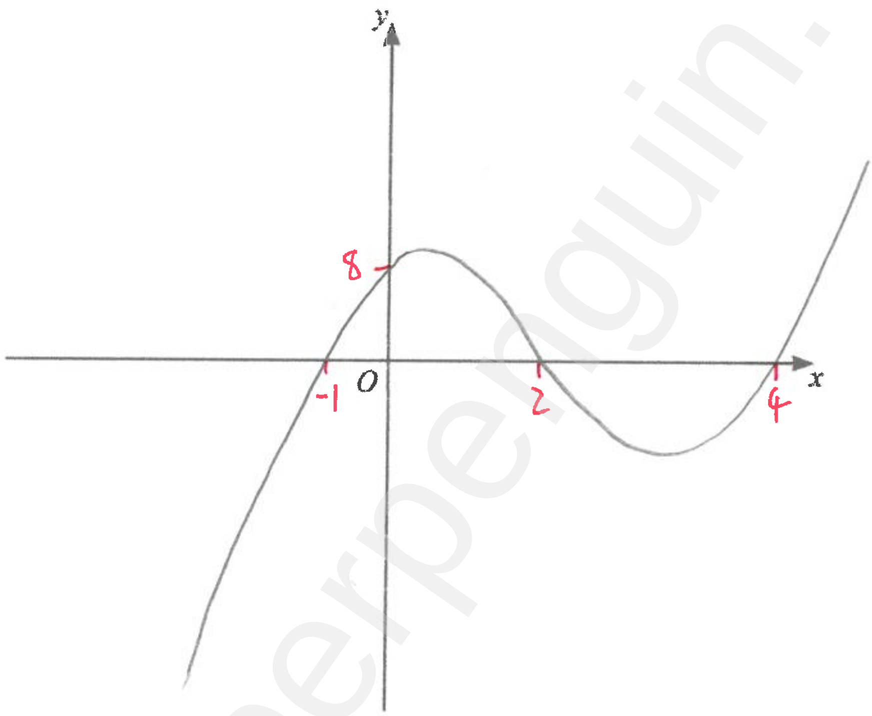
[2]

8 (a) (i) Show that the equation $y = (x-4)(x+1)(x-2)$ can be written as $y = x^3 - 5x^2 + 2x + 8$.

$$y = (x-4)\left[x^2 - 2x + x - 2\right]$$
$$= (x-4)\left[x^2 - x - 2\right]$$
$$= x^3 - x^2 - 2x - 4x^2 + 4x + 8$$
$$= \underline{x^3 - 5x^2 + 2x + 8}$$

[2]

(ii) On the diagram, sketch the graph of $y = x^3 - 5x^2 + 2x + 8$, indicating the values where the graph crosses the axes.



[4]

Roots: set $y=0$ in factorised form:

$$(x-4)(x+1)(x-2) = 0$$

$x-4=0$ or $x+1=0$ or $x-2=0$

$+4 \quad +4 \qquad -1 \quad -1 \qquad +2 \quad +2$

$\underline{x=4} \qquad \underline{x=-1} \qquad \underline{x=2}$

y-intercept: Set $x=0$:

$$y = 0^3 - 5(0)^2 + 2(0) + 8$$
$$y = 8$$

Shape: positive x^3, so

as opposed to

$$y = x^3 - 5x^2 + 2x + 8$$

9 (a) Sketch the graph of $y = (x+1)(3-x)(3+x)$, indicating the coordinates of the points where the graph crosses the x-axis and the y-axis.

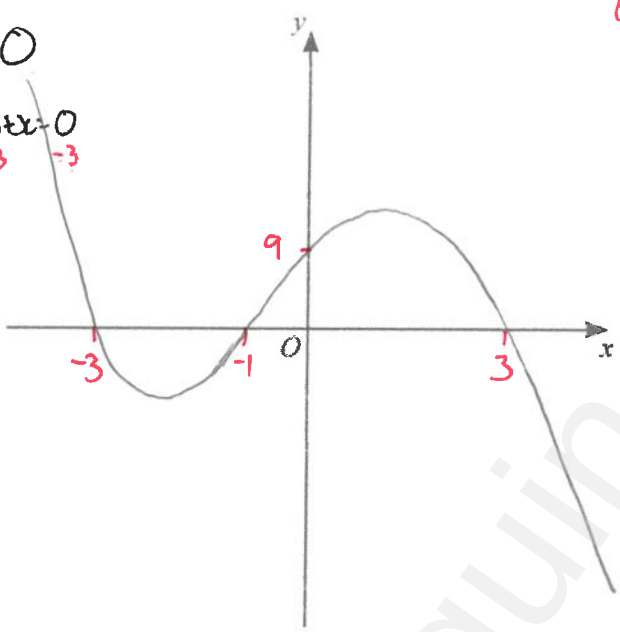
roots: set $y=0$:

$(x+1)(3-x)(3+x) = 0$

$x+1=0$ or $3-x=0$ or $3+x=0$

$-1 \quad -1 \qquad +x \quad +x \qquad -3 \quad -3$

$\underline{x=-1}$ $\underline{x=3}$ $\underline{x=-3}$



y-intercept: set $x=0$ in expanded form from part(b):

$y = 9 + 9(0) + 0^2 - 0^3$

$\underline{y=9}$

Shape:

$y = 9 + 9x - x^2 - x^3$

negative x^3, so:

(b) (i) Show that $y = (x+1)(3-x)(3+x)$ can be written as $y = 9 + 9x - x^2 - x^3$.

$y = (x+1)[9 + 3x - 3x - x^2]$

$= (x+1)[9 - x^2]$

$= 9x - x^3 + 9 - x^2$

$= \underline{9 + 9x - x^2 - x^3}$ [2]

9 **(a)** **(i)** The equation $y = x^3 - 4x^2 + 4x$ can be written as $y = x(x-a)^2$.

Find the value of a.

$y = x[x^2 - 4x + 4]$

$= x(x-2)(x-2)$

$= x(x-2)^2$

$a =$ ...... 2 ...... [2]

(ii) On the axes, sketch the graph of $y = x^3 - 4x^2 + 4x$, indicating the values where the graph meets the axes.

y-intercept:

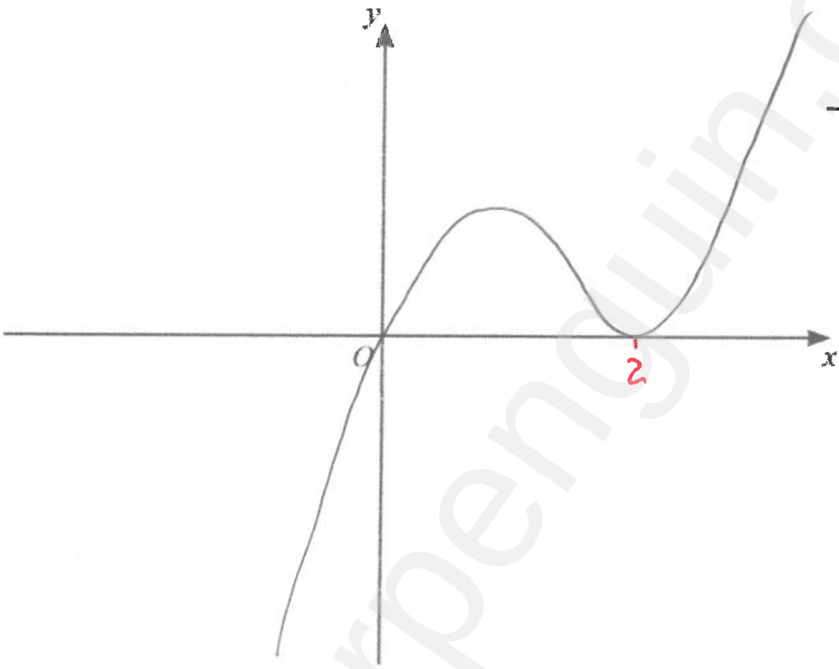
$y = 0^3 - 4(0)^2 + 4(0)$

$y = 0$

Shape:

$y = x^3 - 4x^2 + 4x$

positive cubic



roots:

$x(x-2)^2 = 0$

$x = 0$ or $x - 2 = 0$

$x = 2$

this is a repeated root (because $(x-2)^2$) so it touches axis here, but doesn't cross

[4]

11 The graph of $y = (x-3)(x+b)(x+2)$ intersects the y-axis at -30.

(a) Find the value of b.

$y = (x-3)[x^2 + 2x + bx + 2b]$

$y = x^3 + 2x^2 + bx^2 + 2bx - 3x^2 - 6x - 3bx - 6b$

Set $x = 0$ and the only term left is $-6b$:

$y = -6b$

$-30 = -6b$

$\div -6 \quad \div -6$

$b =$ ...... 5 ...... [2]

(b) When $x > 0$ the graph crosses the x-axis once.

Write down the coordinates of this point.

$(x-3)(x+5)(x+2) = 0$

$x - 3 = 0$ $x + 5 = 0$ $x + 2 = 0$

$+3 \quad +3$ $-5 \quad -5$ $-2 \quad -2$

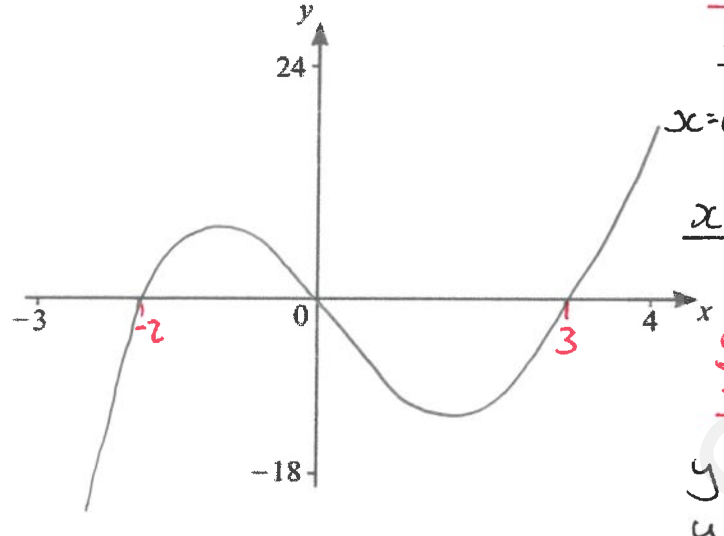
$x = 3$ ✓ $x = -5$ ✗ $x = -2$ ✗

(...... 3 ...... , ... 0 ......) [1]

Shape:

$y = x^3 - x^2 - 6x$

positive x^3 so



$\mathrm{f}(x) = x(x+2)(x-3)$

Roots:

$x(x+2)(x-3) = 0$

$x = 0$ or $x + 2 = 0$ or $x - 3 = 0$

$\quad -2 \quad -2 \qquad +3 \quad +3$

$\underline{x = 0} \quad \underline{x = -2} \quad \underline{x = 3}$

y-intercept: use expanded form in part (b):

$y = 0^3 - 0^2 - 6(0)$

$\underline{y = 0}$

(a) On the diagram, sketch the graph of $y = \mathrm{f}(x)$ for $-3 \leqslant x \leqslant 4$.
Show the values of the intersections with the axes. [3]

(b) Expand and simplify.

$x(x+2)(x-3)$

$= x[x^2 - 3x + 2x - 6]$

$= x[x^2 - x - 6]$

$= x^3 - x^2 - 6x$

$x^3 - x^2 - 6x$ [3]

(c) A is the point $(1, -6)$.
The tangent to the graph of $y = \mathrm{f}(x)$ at A meets the y-axis at B.

Find the coordinates of B.

(requires Differentiation)

$\frac{dy}{dx} = 3x^2 - 2x - 6$

sub. $x = 1$ to find gradient at A:

$m = 3(1)^2 - 2(1) - 6$

$= 3 - 2 - 6$

$= \underline{-5}$

$y = -5x + c$

sub. $(1, -6)$:

$-6 = -5(1) + c$

$-6 = -5 + c$

$+5 \qquad +5$

$\underline{-1 = c}$

$y = -5x - 1$

crosses y-axis at $(0, -1)$

(0 , –1) [5]