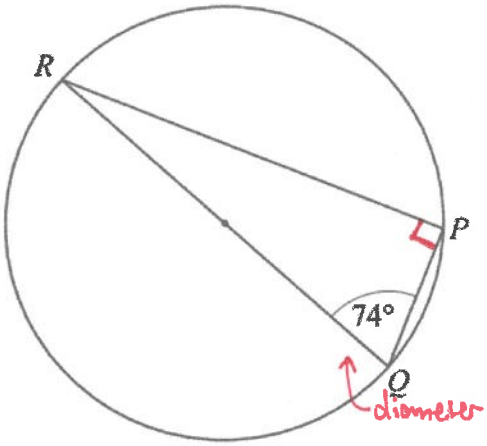
12



NOT TO SCALE

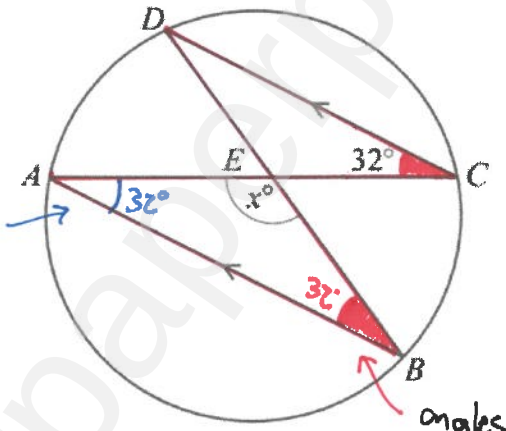
P, Q and R lie on a circle.
QR is a diameter.

$180 - (90 + 74) = 16°$

Find angle PRQ.
Give geometrical reasons for your answer.

Angle PRQ = 16° because angle in a semi-circle = 90° and angles in a triangle sum to 180°. [2]

13



NOT TO SCALE

alternate angles are equal

angles in the same segment are equal

A, B, C and D are points on a circle.
AB is parallel to DC and angle $ACD = 32°$.
Chords AC and DB intersect at E.

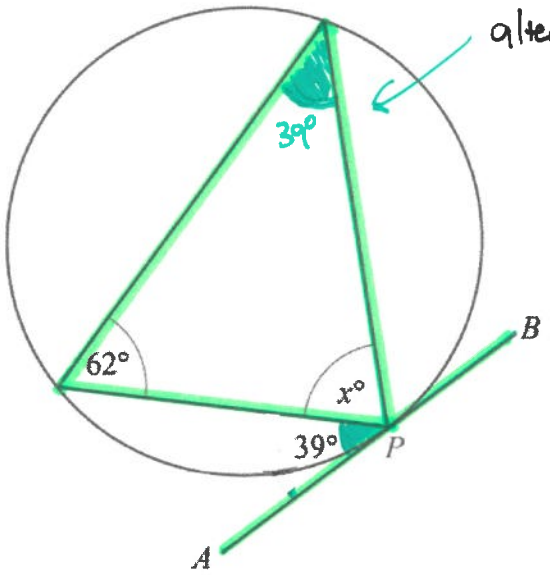
Find the value of x.

$180 - (32 + 32) = 116°$

angles in a triangle

x = 116° [2]

10



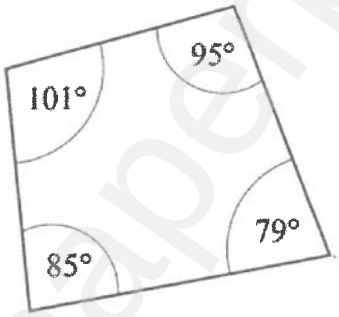
NOT TO SCALE

APB is a tangent to the circle at *P*.

Work out the value of *x*. $180 - (62 + 39) = 79°$

$x =$ 79° [2]

15



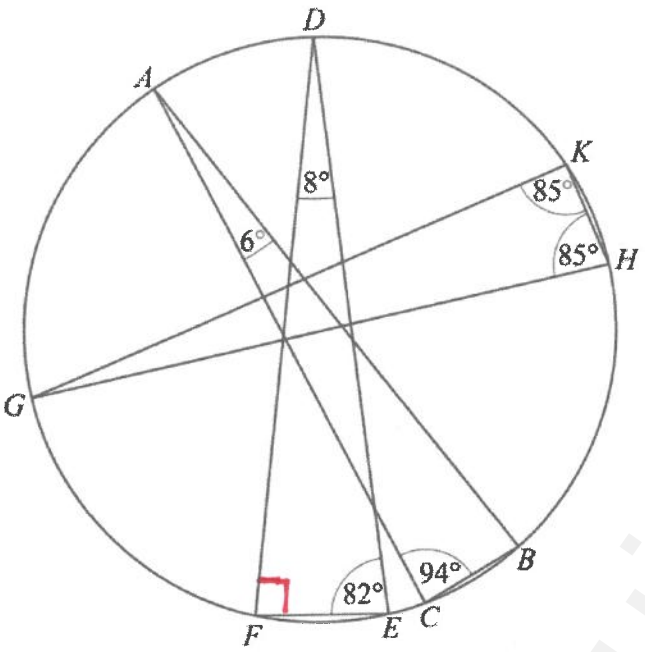
NOT TO SCALE

The diagram shows a quadrilateral.

Give a geometrical reason why this is a cyclic quadrilateral.

$85 + 95 = 180$ and $101 + 79 = 180$; opposite angles sum to 180° [1]

11 *ABC*, *DEF* and *GHK* are triangles with all vertices on the circumference of a circle.



NOT TO SCALE

From the list, draw a ring around the line that is a diameter of the circle.

AB *AC* (*DE*) *DF* *GH* *GK*

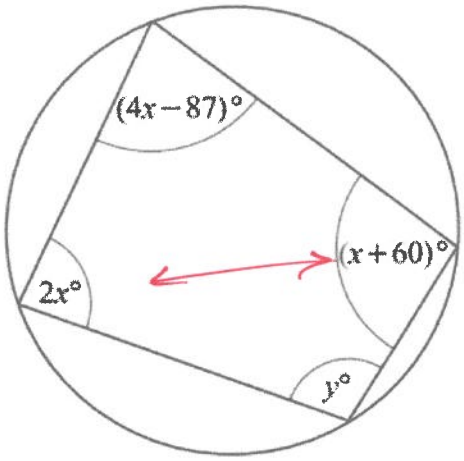
[1]

DEF: $8 + 82 = 90$

$180 - 90 = 90$

(angle in a semi-circle is 90°)

20



NOT TO SCALE

The diagram shows a cyclic quadrilateral.

Find the value of y.

$2x + x + 60 = 180$

$3x + 60 = 180$

$-60 \quad -60$

$3x = 120$

$\div 3 \quad \div 3$

$x = 40$

Find value of $4x - 87$ by sub. $x = 40$:

$4(40) - 87$

$= 160 - 87$

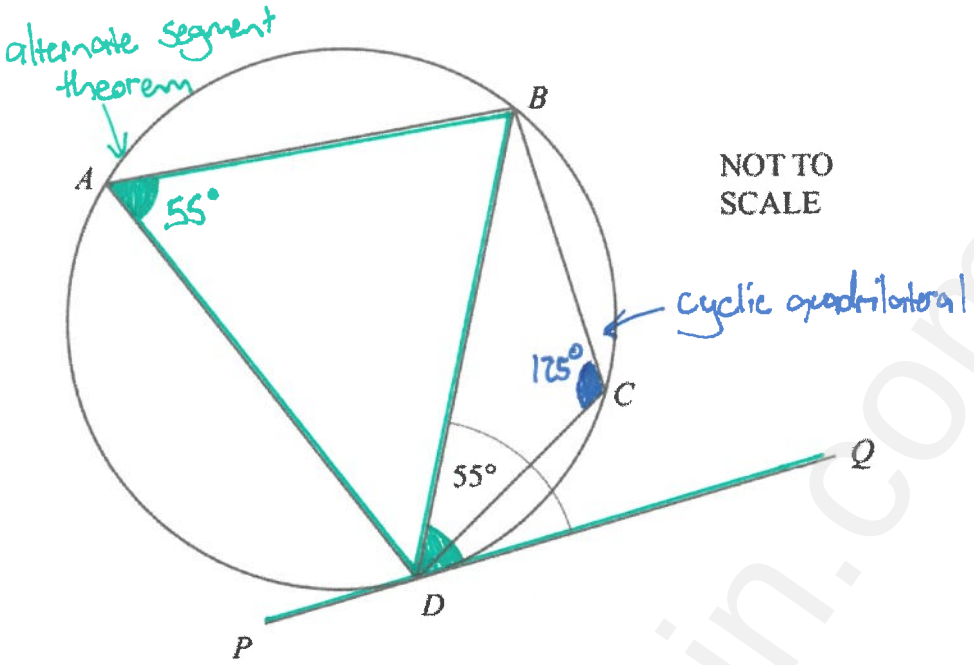
$= 73°$

Angle y:

$180 - 73 = 107°$

$y =$ 107° [4]

13



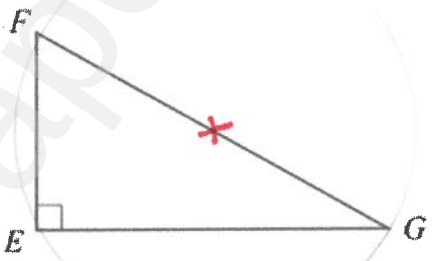
A, B, C and D are points on the circle.
PQ is a tangent to the circle at D.
Angle $BDQ = 55°$.

Complete these statements giving a reason for each answer.

(a) Angle BAD = 55° because angles in the alternate segment are equal [2]

(b) Angle BCD = 125° because opposite angles in a cyclic quadrilateral sum to 180°. (180 - 55 = 125) [2]

(b)



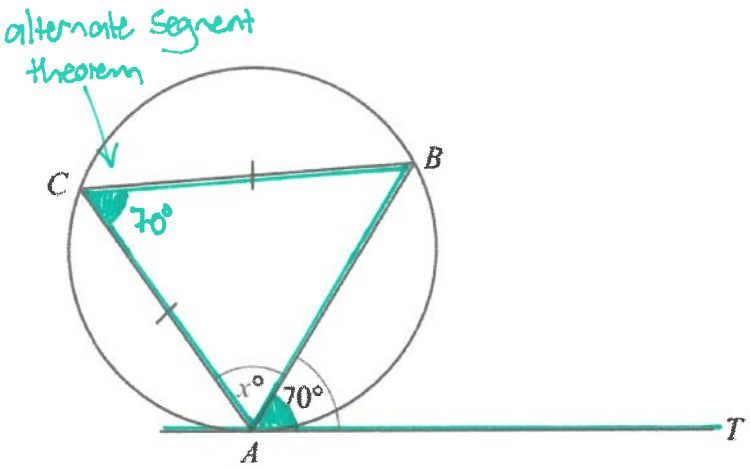
EFG is a right-angled triangle.
A circle can be drawn that passes through the three vertices of the triangle.

On the diagram, mark the position of the centre of the circle with a cross.
Explain how you decide.

Angle in a semi-circle = 90°, so FG is the diameter.

[2]

9



NOT TO SCALE

A, B and C are points on a circle.
TA is a tangent to the circle at A.
$CA = CB$ and angle $BAT = 70°$.

Work out the value of x.

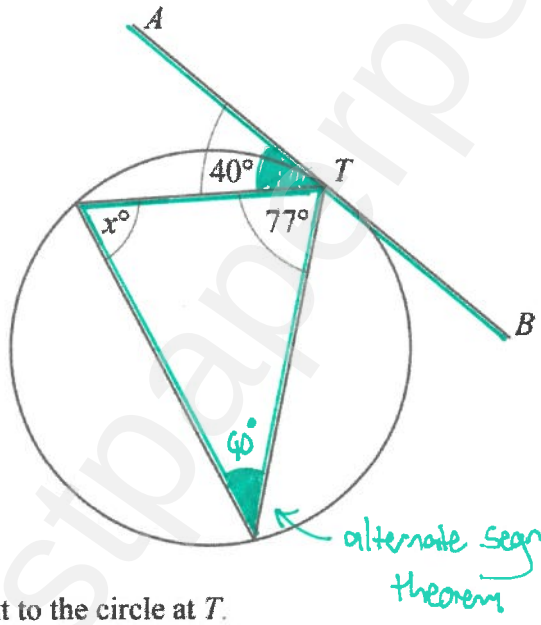
isosceles triangle:

$180 - 70 = 110$

$110 \div 2 = 55°$

$x =$ 55° [2]

13



NOT TO SCALE

AB is a tangent to the circle at T.

Find the value of x.

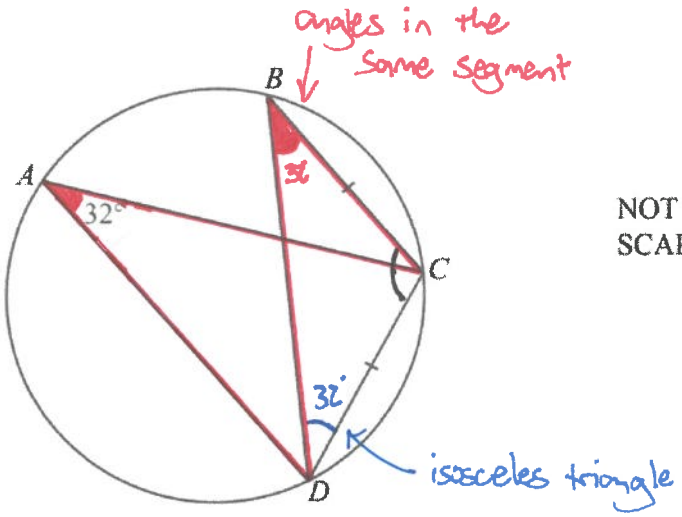
$180 - (40 + 77)$

$= 63°$

(angles in a triangle)

$x =$ 63° [2]

12 (a)



NOT TO SCALE

A, B, C, and D are points on a circle.
Angle $DAC = 32°$.
$BC = DC$.

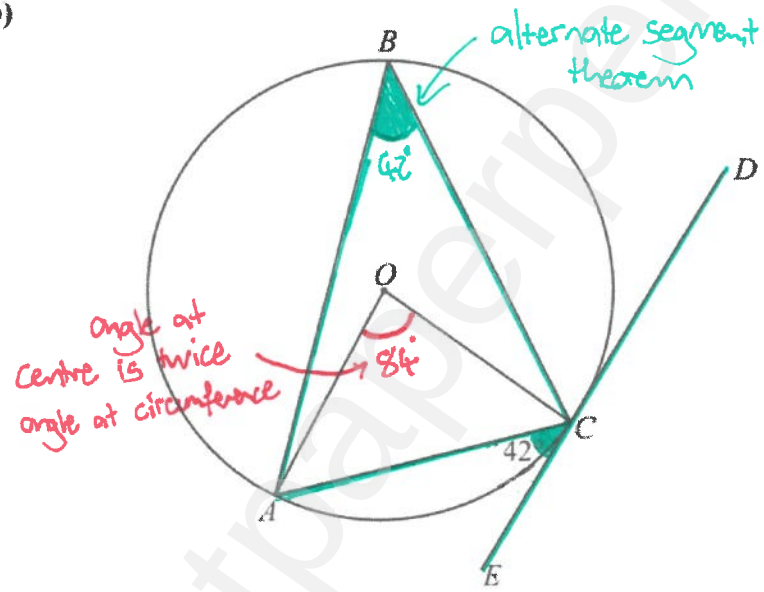
Find angle BCD.

$BCD = 180 - (32 + 32)$

$= 116$

Angle BCD = 116° [2]

(b)



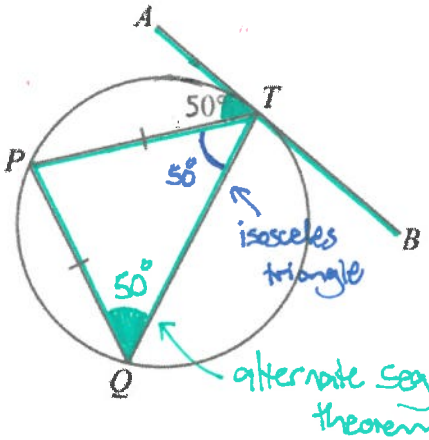
NOT TO SCALE

A, B and C are points on the circle centre O.
ECD is a tangent to the circle at C.
Angle $ACE = 42°$.

Find angle AOC.

Angle AOC = 84° [2]

19 (a)



NOT TO SCALE

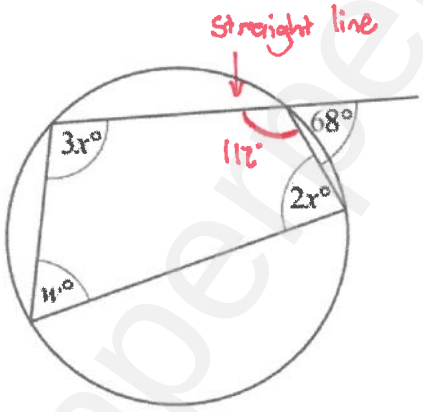
P, Q and T are points on a circle.
ATB is a tangent to the circle at T and $PT = PQ$.

Find angle TPQ.

$180 - (50 + 50) = 80°$

Angle TPQ = 80° [2]

(b)



NOT TO SCALE

The diagram shows a cyclic quadrilateral with an exterior angle of 68°.

Find the value of w and the value of x.

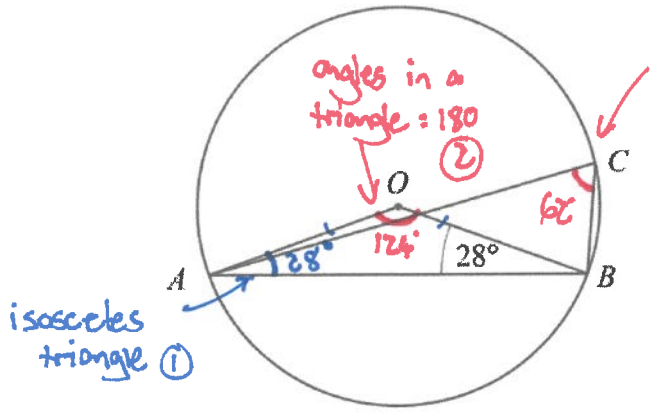
$w + 112 = 180$
$-112 \quad -112$
$w = 68$

$3x + 2x = 180$
$5x = 180$
$\div 5 \quad \div 5$
$x = 36$

w = 68

x = 36 [3]

17 (a)



angle at the centre is twice the angle at the circumference 3

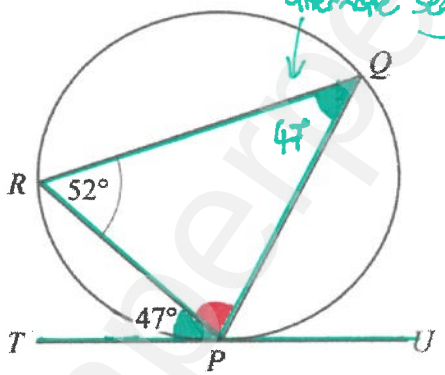
NOT TO SCALE

A, B and C are points on a circle, centre O.
Angle $OBA = 28°$.

Find angle ACB.

Angle ACB = 62° [2]

(b)



NOT TO SCALE

P, Q and R are points on a circle.
TU is a tangent to the circle at P.
Angle $TPR = 47°$ and angle $PRQ = 52°$.

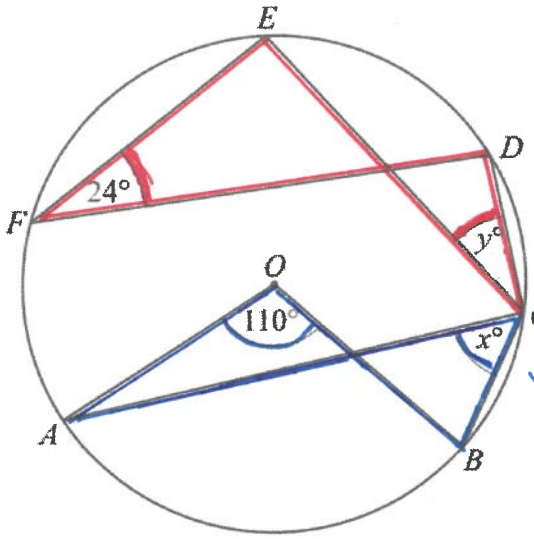
Find angle RPQ.

$180 - (52 + 47) = 81°$

↑ angles in a triangle

Angle RPQ = 81° [2]

10



NOT TO SCALE

y = 24°
angles in the same segment

x = 55°
angle at the centre is twice the angle at the circumference

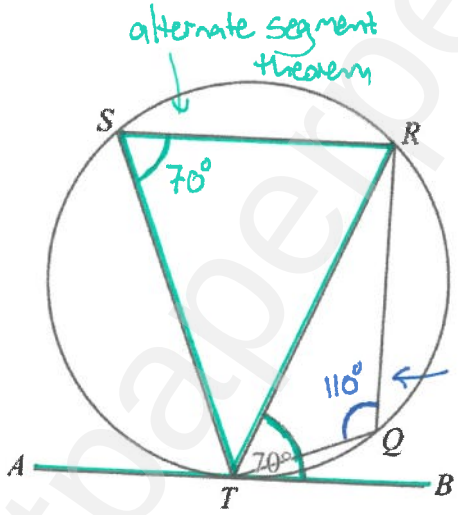
Points A, B, C, D, E and F lie on the circle, centre O.

Find the value of x and the value of y.

$x =$ 55° .....

$y =$ 24° ..... [2]

15



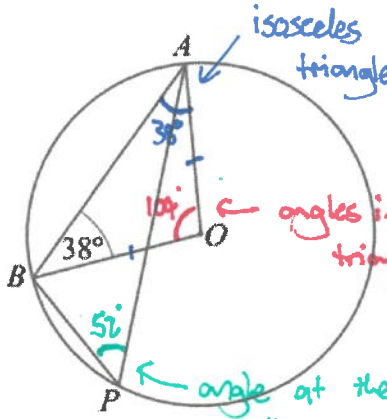
NOT TO SCALE

Points Q, R, S and T lie on the circle.
AB is a tangent to the circle at T.
Angle $RTB = 70°$.

Find angle RQT.

Angle $RQT =$ 110° ..... [2]

2 (a)



isosceles triangle (radius is equal) (1)

NOT TO SCALE

angles in a triangle (2)

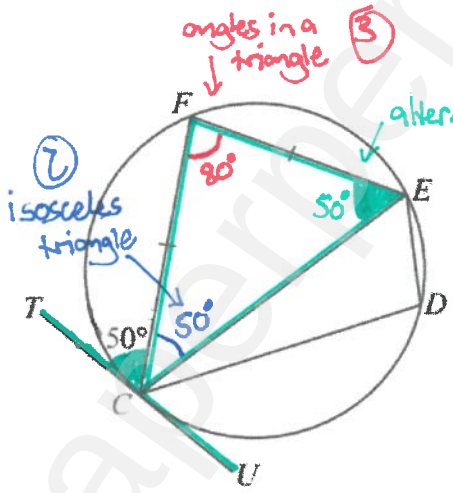
angle at the centre is twice the angle at the circumference (3)

A, B and P are points on a circle, centre O and angle $OBA = 38°$.

Find angle APB.

Angle APB = 52° [3]

(b)



angles in a triangle (3)

alternate segment theorem (1)

(2) isosceles triangle

NOT TO SCALE

$CDEF$ is a cyclic quadrilateral and $FC = FE$.
TU is a tangent to the circle at C and angle $TCF = 50°$.

Find

(i) angle EFC.

Angle EFC = 80° [2]

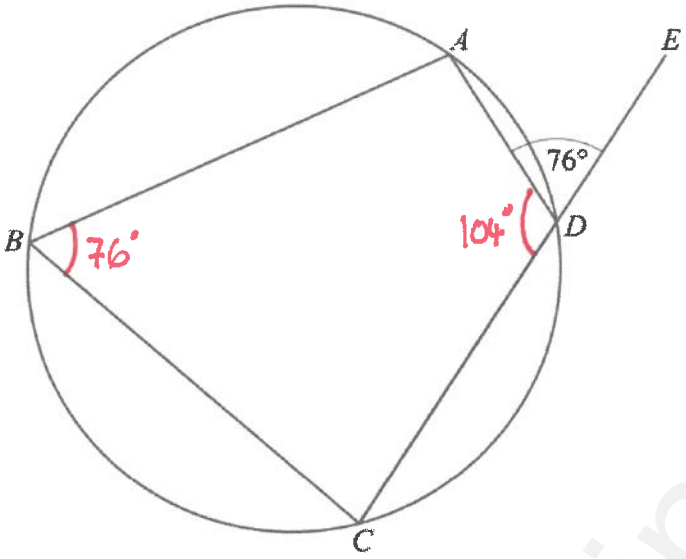
(ii) angle CDE.

100 − 80 = 100°

(opposite angles in a cyclic quadrilateral)

Angle CDE = 100° [1]

9 **(a)**



NOT TO SCALE

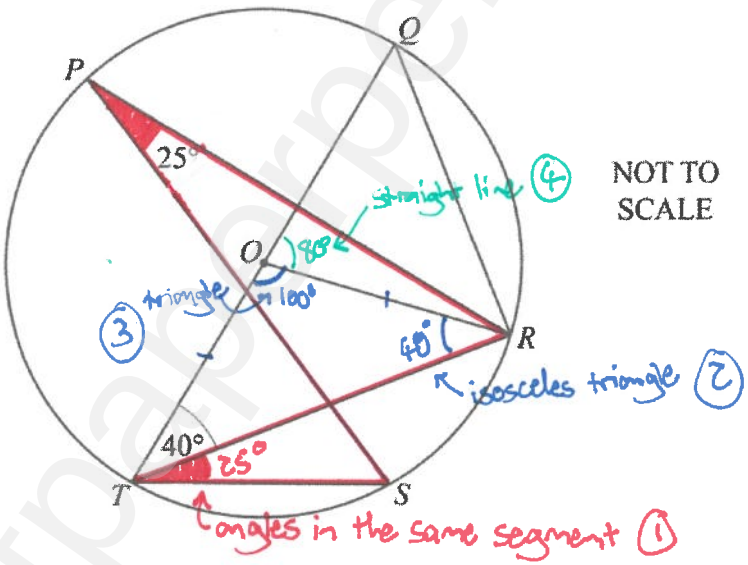
A, B, C, and D are points on a circle.
CDE is a straight line.

$180 - 104 = 76°$ (opposite angles in a cyclic quadrilateral)

Find angle ABC.

Angle ABC = 76° [1]

(b)



NOT TO SCALE

P, Q, R, S and T are points on the circle centre O.
TOQ is a straight line.

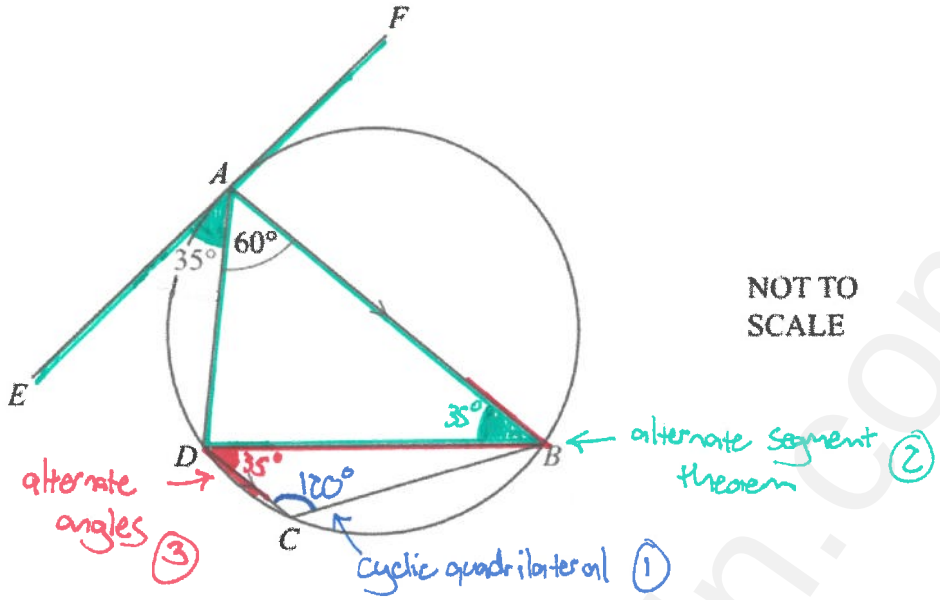
(i) Find angle STR.

Angle STR = 25° [1]

(ii) Find angle QOR.

Angle QOR = 80° [1]

13



NOT TO SCALE

A, B, C and D are points on a circle.
EF is a tangent to the circle at A.
AB is parallel to DC.

(a) Find angle DCB, giving a geometrical reason.

Angle DCB = 120° because opposite angles in a cyclic quadrilateral sum to 180°. [2]

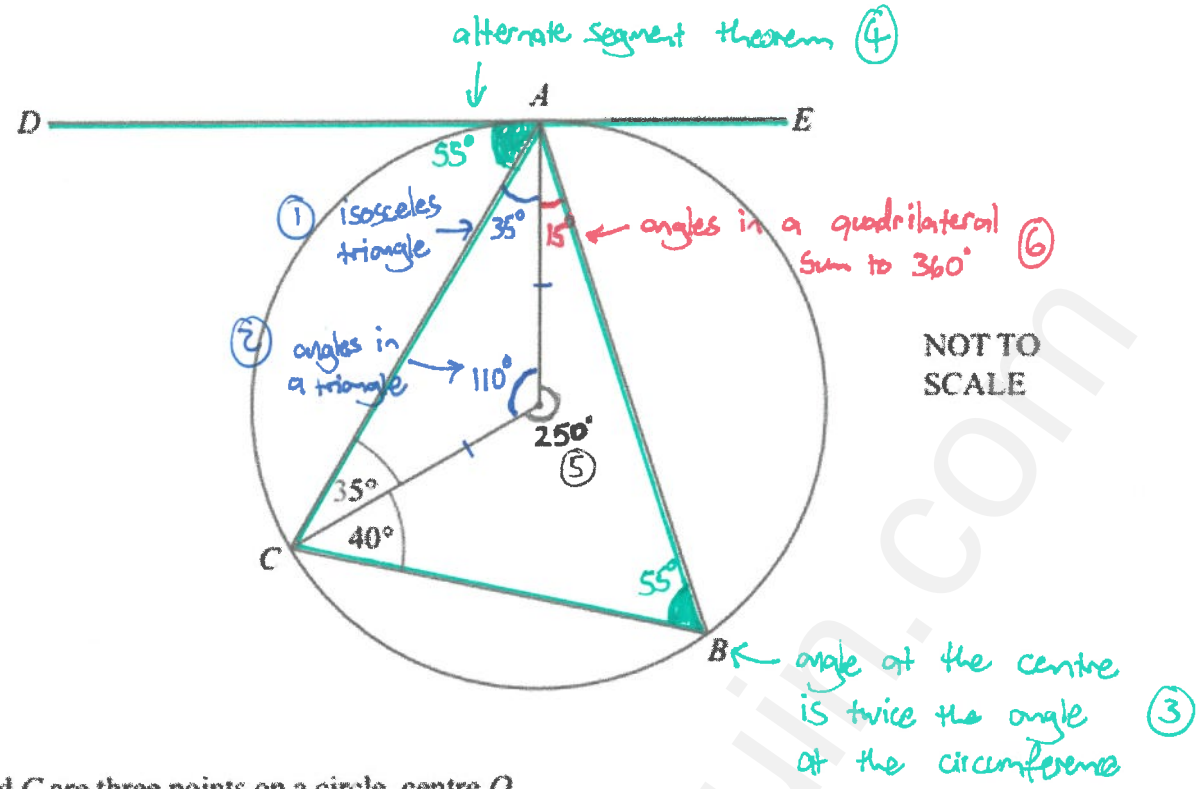
(b) Find angle DBC.

$180 - (120 + 35) = 25°$

angles in a triangle sum to 180°

Angle DBC = 25° [2]

10



A, B and C are three points on a circle, centre O.
DE is a tangent to the circle at A.
Angle $ACO = 35°$ and angle $BCO = 40°$.

Find

(a) angle AOC

Angle AOC = 110° [1]

(b) angle ABC

Angle ABC = 55° [1]

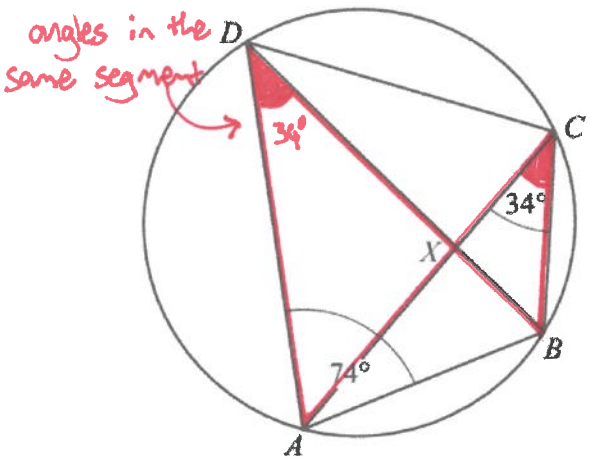
(c) angle DAC

Angle DAC = 55° [1]

(d) angle OAB.

Angle OAB = 15° [1]

17



NOT TO SCALE

The diagram shows a cyclic quadrilateral $ABCD$.
BD and AC intersect at X.

(a) Angle $BAD = 74°$ and angle $BCA = 34°$.

Find

(i) angle BDA

Angle BDA = 34° [1]

(ii) angle BCD

$180 - 74 = 106°$
(cyclic quadrilateral)

Angle BCD = 106° [1]

(iii) angle ABD.

$180 - (34 + 74) = 72°$
(angles in a triangle)

Angle ABD = 72° [1]

(b) In the diagram, triangle ADX is similar to triangle BCX.
$BC = 4.5$ cm, $AD = 9$ cm and $CX = 3.3$ cm.

Work out XD.

XD is similar to XC.

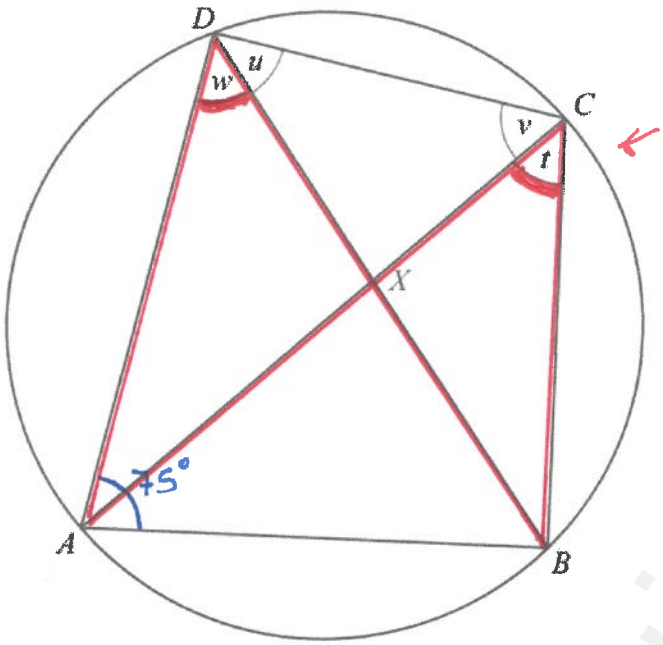
Use AD and BC to find scale factor.

$S.F. = \frac{9}{4.5} = 2$

$XD = 3.3 \times 2$
$= 6.6$

XD = 6.6 cm [2]

10



← angles in the same segment

NOT TO SCALE

ABCD is a cyclic quadrilateral and the diagonals *AC* and *BD* intersect at *X*.

(a) Complete the statement using two of *t*, *u*, *v* and *w*.

Angle ..t.. is equal to angle ..w..

[1]

(b) Angle *DAB* = 75°.

Find angle *DCB*.

180 − 75 = 105°

(cyclic quadrilateral)

Angle *DCB* = ..105°.. [1]

(c) *AB* = 8 cm, *AX* = 6 cm, *BX* = 4 cm and *DC* = 5 cm.

Work out *CX*.

CX is similar to BX.

Use AB and DC to find scale factor:

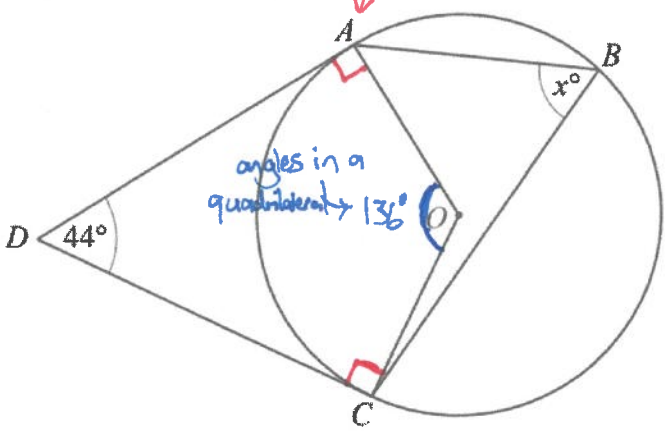
$S.F. = \frac{8}{5}$

$CX = 4 \div \frac{8}{5} = 2.5\text{cm}$

↑ divide because going from big triangle to small triangle.

CX = ..2.5.. cm [2]

13



angle between tangent and radius = 90°

angles in a quadrilateral → 136°

NOT TO SCALE

A, B and C are points on a circle, centre O.
DA and DC are tangents.
Angle $ADC = 44°$.

Work out the value of x.

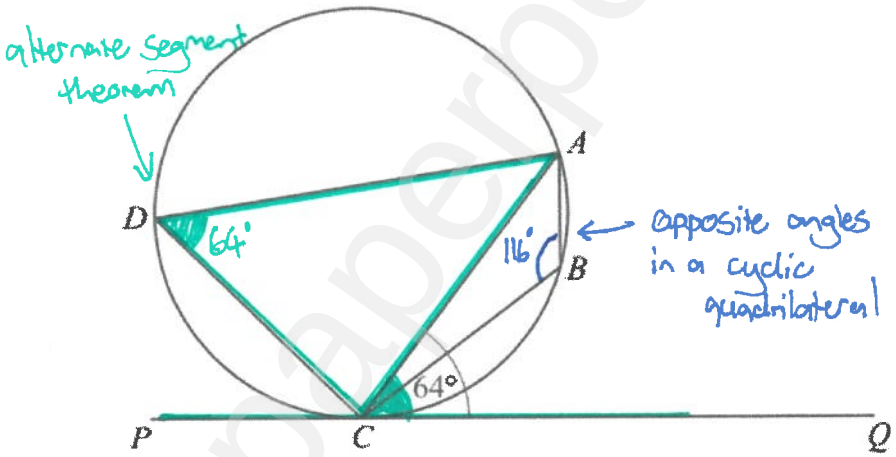
$x = 136 \div 2$

$= 68°$

(angle at the centre is twice the angle at the circumference.)

$x =$ 68° [3]

15



alternate segment theorem

opposite angles in a cyclic quadrilateral

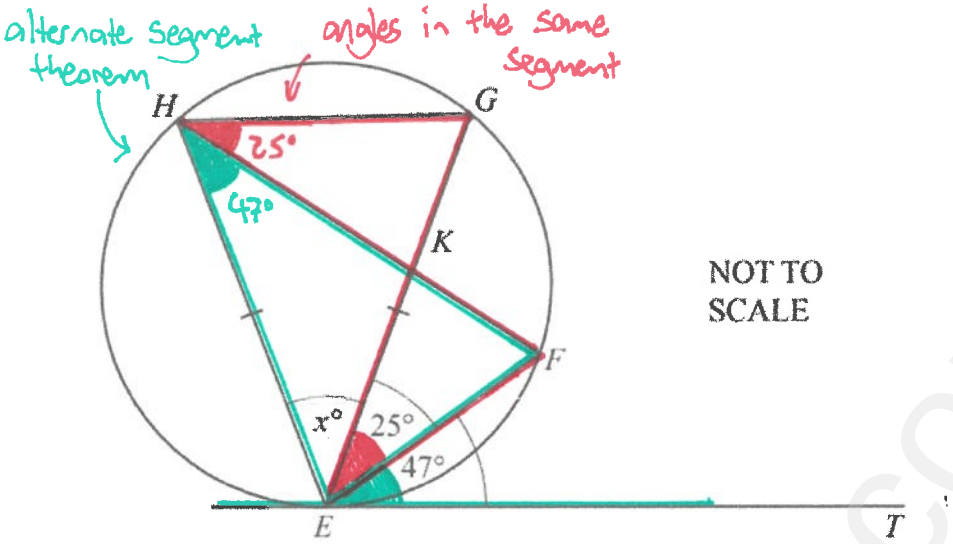
NOT TO SCALE

A, B, C and D lie on the circle.
PCQ is a tangent to the circle at C.
Angle $ACQ = 64°$.

Work out angle ABC, giving reasons for your answer.

Angle $ABC =$ 116° because ADC = 64° from the alternate segment theorem. ABC = 180 − 64 = 116 because opposite angles in a cyclic quadrilateral sum to 180°. [3]

15



Points E, F, G and H lie on the circle and $EG = EH$.
HF and EG intersect at K.
ET is a tangent to the circle at E.
Angle $FET = 47°$ and angle $FEG = 25°$.

Find the value of x.

$E\hat{H}G = 47 + 25$

$= 72°$

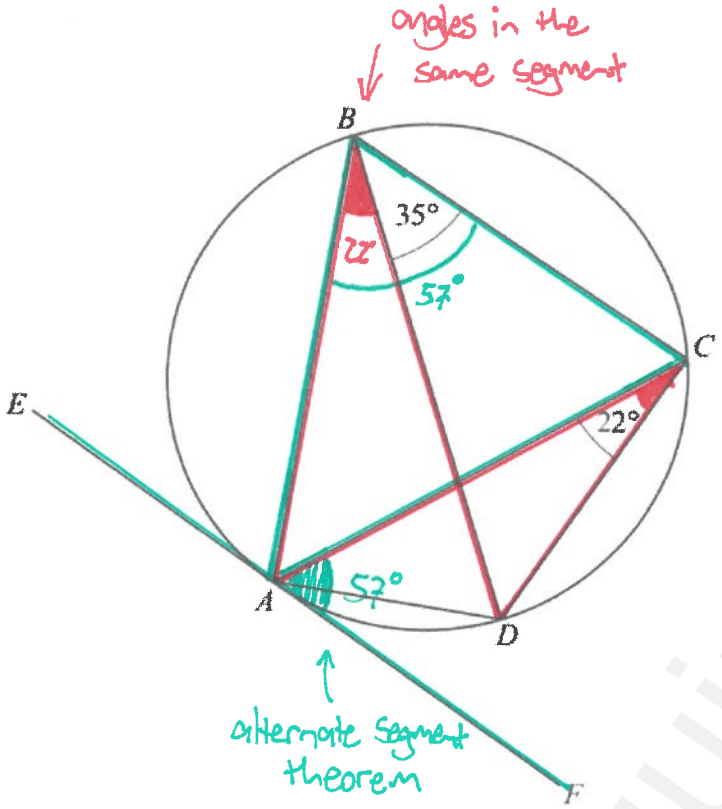
$E\hat{G}H = 72°$ (isosceles triangle)

$x = 180 - (72 + 72)$

$= \underline{36°}$ (angles in a triangle)

$x =$ 36° [2]

12



NOT TO SCALE

A, B, C and D are points on the circle.
EF is a tangent to the circle at A.
Angle $DBC = 35°$ and angle $ACD = 22°$.

Find

(a) angle ABD

Angle ABD = 22° [1]

(b) angle ADC

cyclic quadrilateral

$ABC = 22 + 35$
$= 57°$

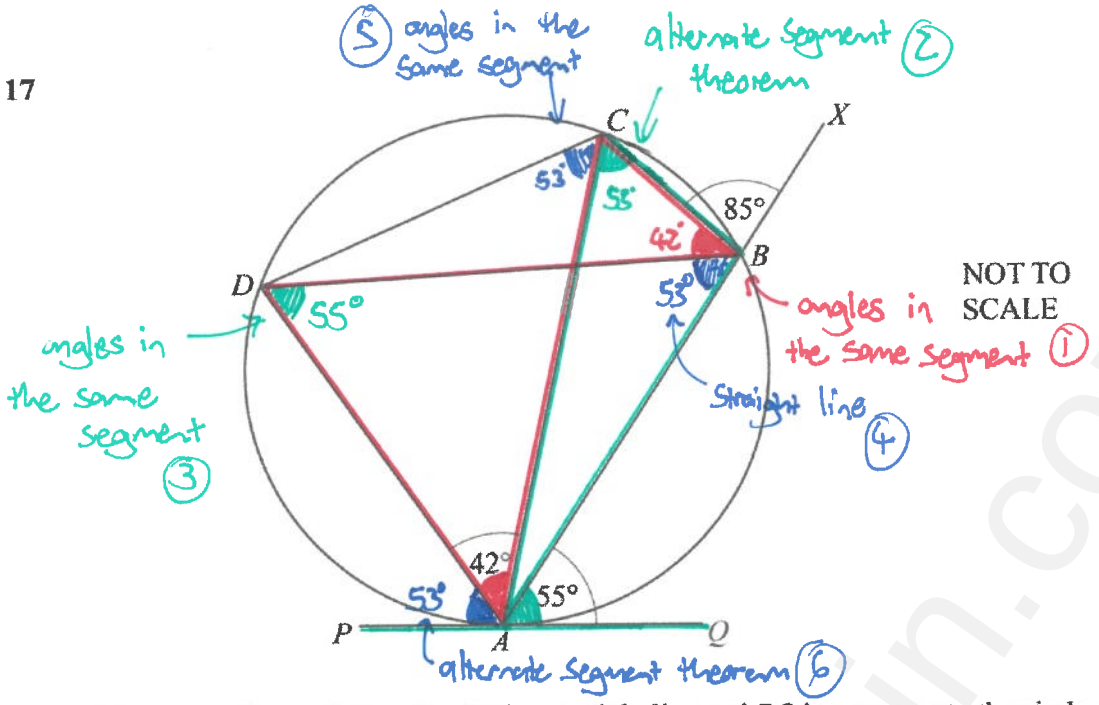
$ADC = 180 - 57$
$= 123°$

Angle ADC = 123° [1]

(c) angle CAF.

Angle CAF = 57° [1]

17



ABCD is a cyclic quadrilateral, *ABX* is a straight line and *PQ* is a tangent to the circle at *A*.
Angle *CBX* = 85°, angle *BAQ* = 55° and angle *CAD* = 42°.

Find

(a) angle *CBD*

Angle *CBD* = 47° [1]

(b) angle *ACB*

Angle *ACB* = 55° [1]

(c) angle *ADC*

Angle *ADC* = 55° [1]

(d) angle *BCD*

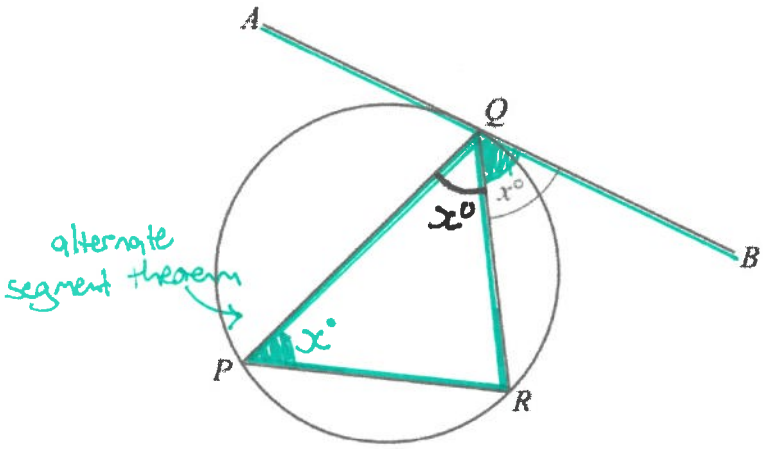
53 + 55 = 108°

Angle *BCD* = 108° [2]

(e) angle *PAD*.

Angle *PAD* = 53° [1]

15



NOT TO SCALE

P, R and Q are points on the circle.
AB is a tangent to the circle at Q.
QR bisects angle PQB.
Angle $BQR = x°$ and $x < 60$.

Use this information to show that triangle PQR is an isosceles triangle.
Give a geometrical reason for each step of your work.

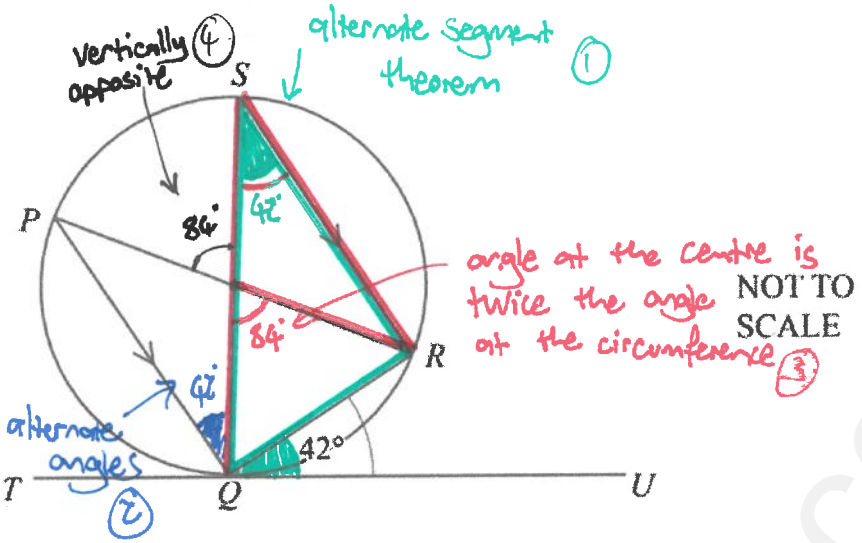
QR bisects $P\hat{Q}B$, so angle $P\hat{Q}R = x$.

$R\hat{P}Q = B\hat{Q}R$ using the alternate segment theorem.

$P\hat{Q}R = R\hat{P}Q = x$, so the triangle is isosceles.

[3]

(b)



P, Q, R and S are points on the circle and TQU is a tangent to the circle at Q.
PR and SQ intersect at the centre of the circle, O, and PQ is parallel to SR.
Angle $RQU = 42°$.

Calculate

(i) angle QSR

Angle QSR = 42° [1]

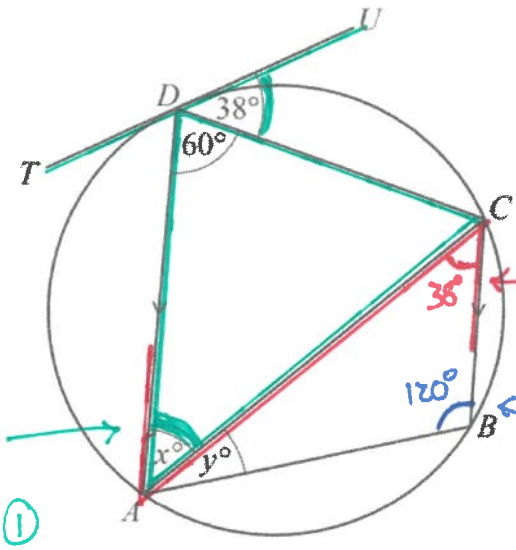
(ii) angle PQS

Angle PQS = 42° [1]

(iii) angle POS.

Angle POS = 84° [1]

18



NOT TO SCALE

A, *B*, *C* and *D* are points on a circle.
TU is a tangent to the circle at *D*.
DA is parallel to *CB*.

Find the value of x and the value of y.

$x = 38°$ (alternate segment theorem)

$y = 180 - (120 + 38)$

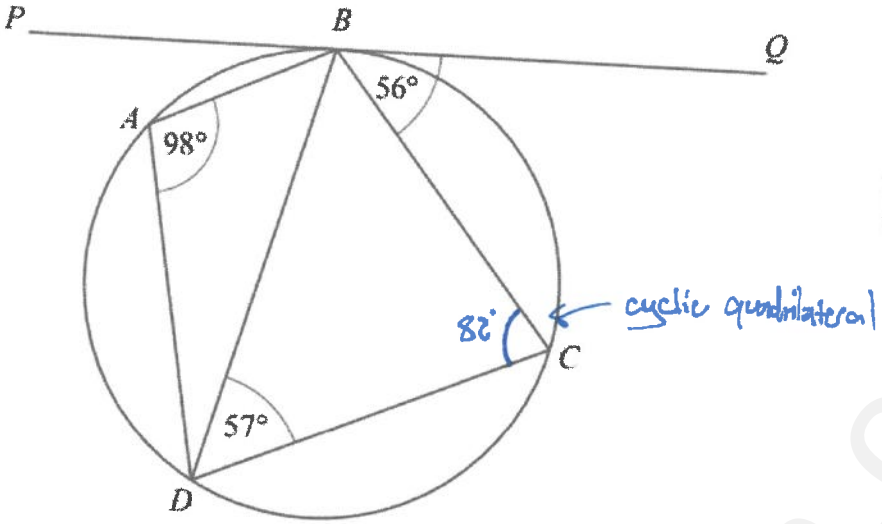
$= 22°$

(angles in a triangle)

$x =$ 38° .....

$y =$ 22° ..... [3]

9



NOT TO SCALE

A, B, C and D are points on the circle.
PBQ is a straight line.

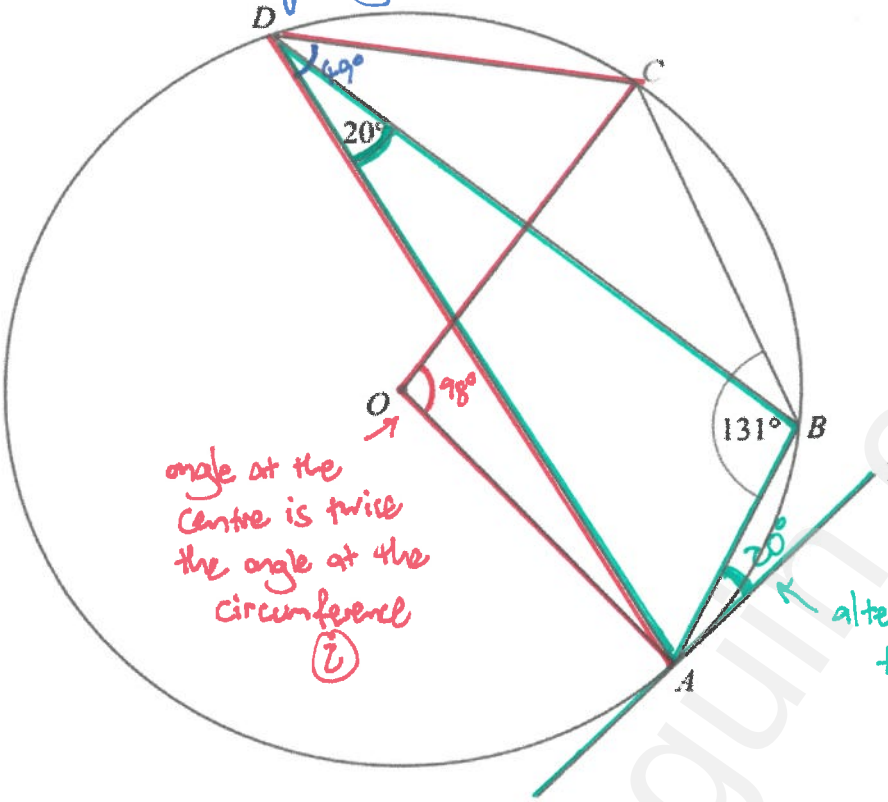
(a) Find angle DCB, giving a reason for your answer.

Angle DCB = 82° because opposite angles in a cyclic quadrilateral sum to 180°. [2]

(b) Is PBQ a tangent to the circle?
Give a reason for your answer.

No because if it was, angle BDC (57°) would be equal to angle QBC (56°) from the alternate segment theorem. [1]

20



opposite angles in a cyclic quadrilateral (1)

angle at the centre is twice the angle at the circumference (2)

alternate segment theorem (3)

A, B, C and D lie on the circle, centre O.
TA is a tangent to the circle at A.
Angle $ABC = 131°$ and angle $ADB = 20°$.

Find

(a) angle ADC,

$180 - 131 = 49°$ (cyclic quadrilateral)

Angle $ADC =$ 49° [1]

(b) angle AOC,

Angle $AOC =$ 98° [1]

(c) angle BAT,

Angle $BAT =$ 20° [1]

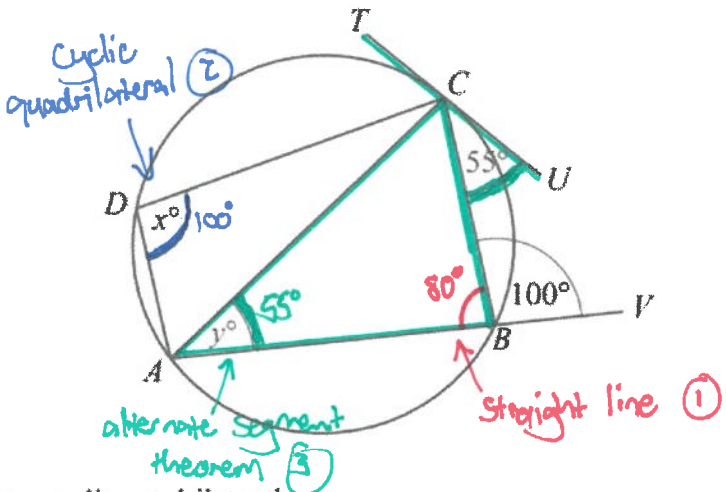
(d) angle OAB.

$90 - 20 = 70°$

angle between tangent and radius = 90°

Angle $OAB =$ 70° [1]

12



NOT TO SCALE

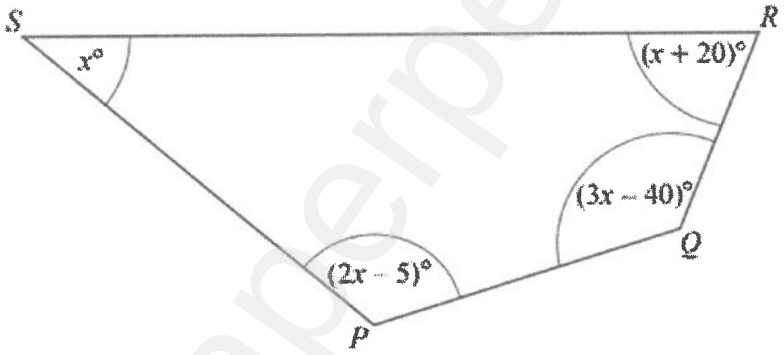
ABCD is a cyclic quadrilateral.
ABV is a straight line and *TU* is a tangent to the circle at *C*.

Find the value of *x* and the value of *y*.

$x =$ 100°

$y =$ 55° [2]

(d)



NOT TO SCALE

Show that *PQRS* is a cyclic quadrilateral.

(1) Find x by using the fact that the interior angles sum to 360:

$x + x + 20 + 3x - 40 + 2x - 5 = 360$

$7x - 25 = 360$

$+25 \quad +25$

$7x = 385$

$\div 7 \quad \div 7$

$x = 55$

(2) Show that opposite angles sum to 180:

x: 55°

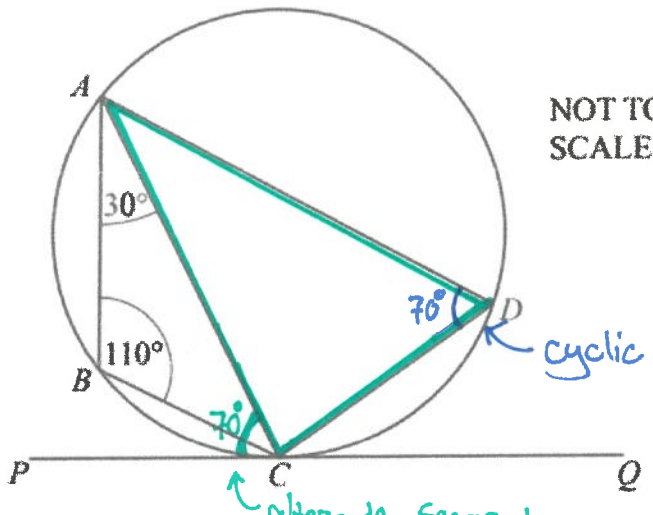
$3x-40$: $3(55)-40 = 125$ } $55+125=180$

$x+20$: $55+20 = 75$

$2x-5$: $2(55)-5 = 105$ } $75+105=180$

[5]

12



NOT TO SCALE

cyclic quadrilateral

alternate segment theorem

The points A, B, C and D lie on a circle.
PCQ is a tangent to the circle at C.
Angle $ABC = 110°$ and angle $BAC = 30°$.

Find

(a) angle ADC,

$180 - 110 = 70°$

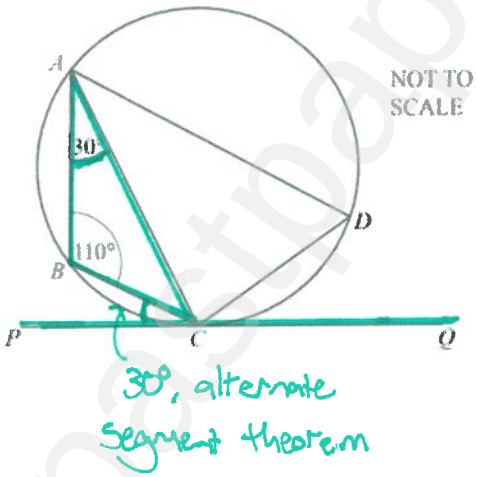
(cyclic quadrilateral)

Angle ADC = 70° [1]

(b) angle ACP,

Angle ACP = 70° [1]

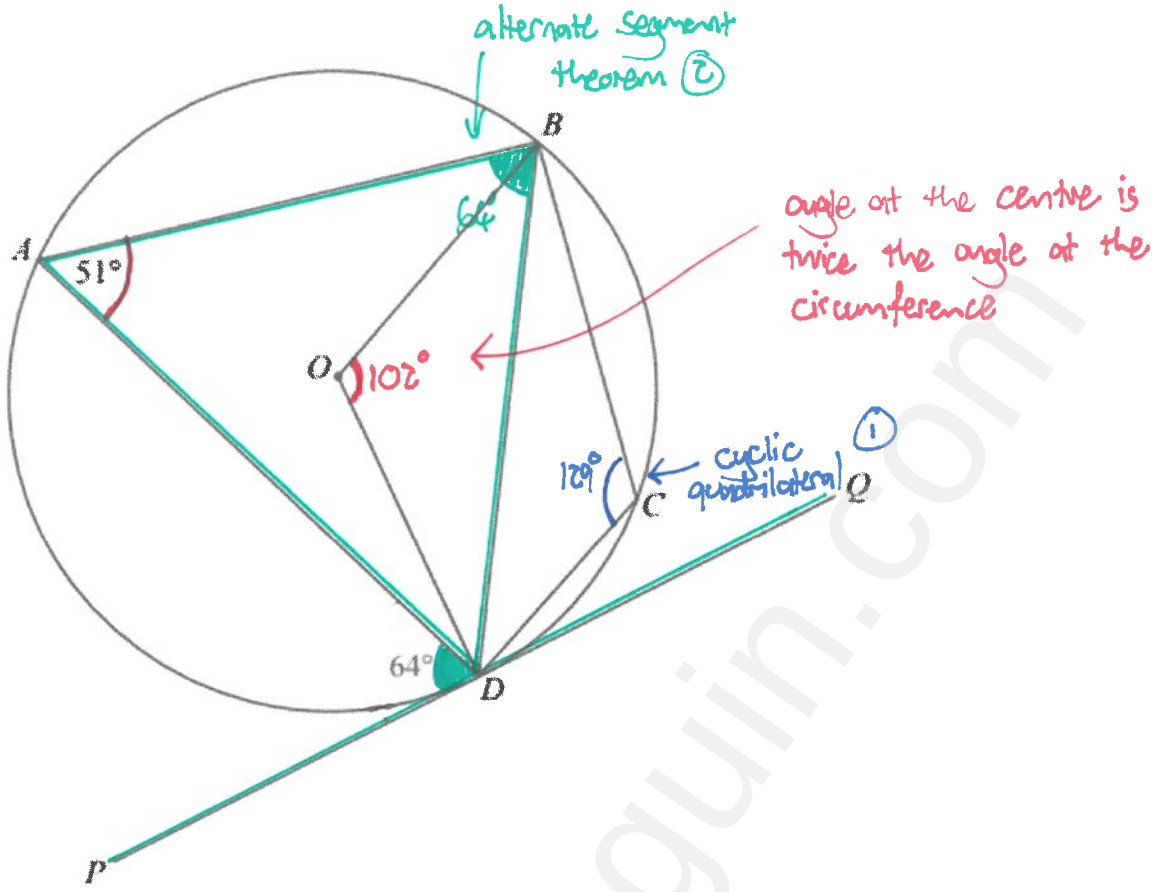
(c) angle PCB.



30°, alternate segment theorem

Angle PCB = 30° [1]

10



A, B, C and D are points on the circle centre O.
PDQ is a tangent to the circle at D.
Angle $BAD = 51°$ and angle $PDA = 64°$.

Find

(a) angle BCD

$180 - 51 = 129°$
(cyclic quadrilateral)

Angle BCD = ... 129° ..... [1]

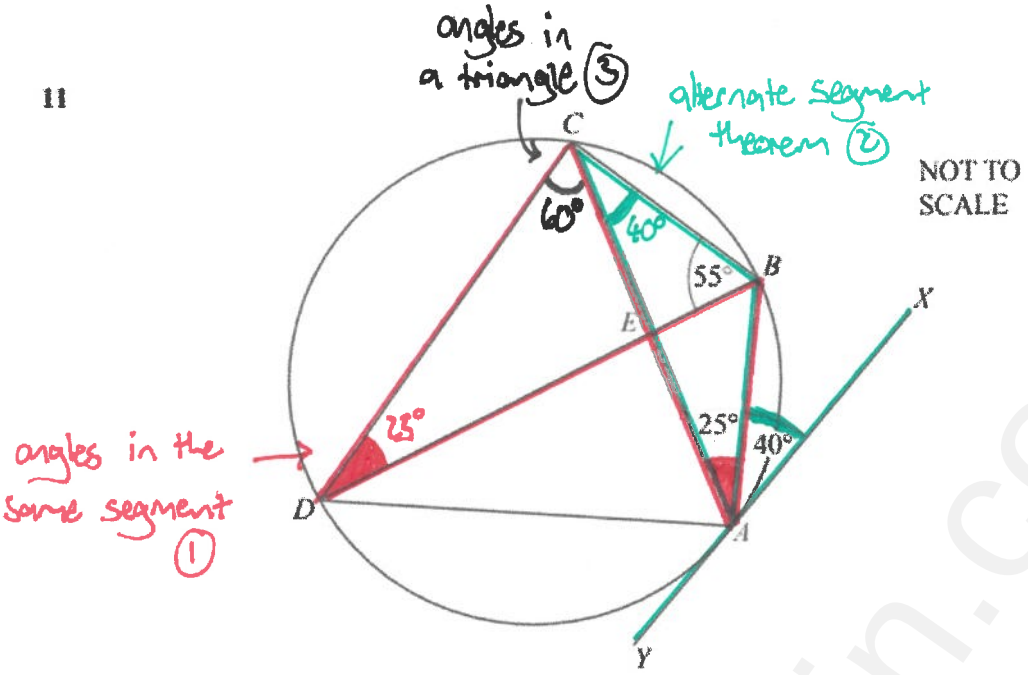
(b) angle ABD

Angle ABD = ... 64° ..... [1]

(c) the obtuse angle BOD.

Angle BOD = ... 102° ..... [1]

11



A, B, C and D are four points on a circle.
AC and BD meet at E.
XAY is a tangent to the circle at A.

Find

(a) angle CDB,

Angle CDB = 25° [1]

(b) angle ACB,

Angle ACB = 40° [1]

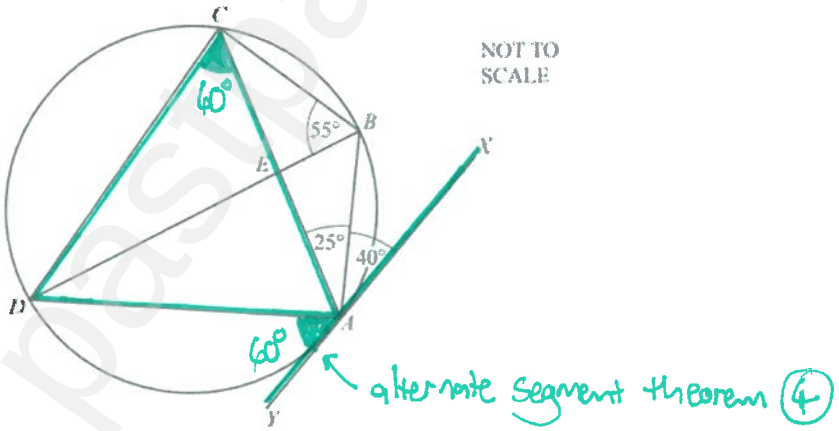
(c) angle DCE,

$180 - (25 + 55 + 40) = 60°$

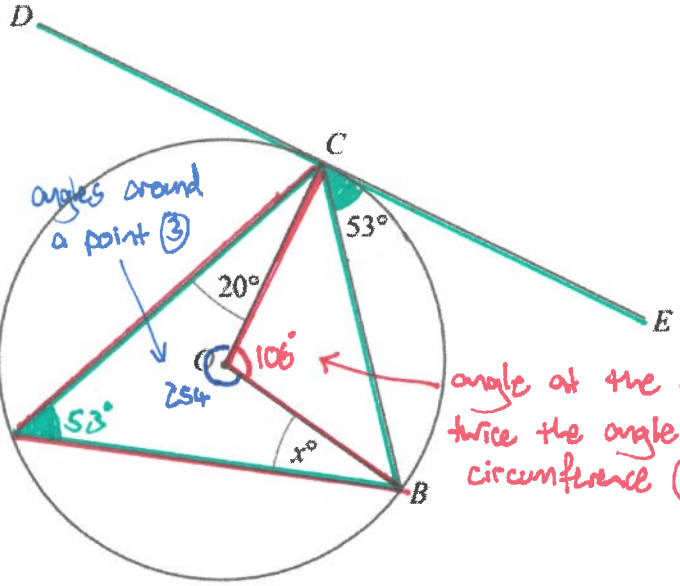
Angle DCE = 60° [1]

(d) angle YAD.

Angle YAD = 60° [1]



17



NOT TO SCALE

angles around a point (3)

alternate segment theorem (1)

angle at the centre is twice the angle at the circumference (2)

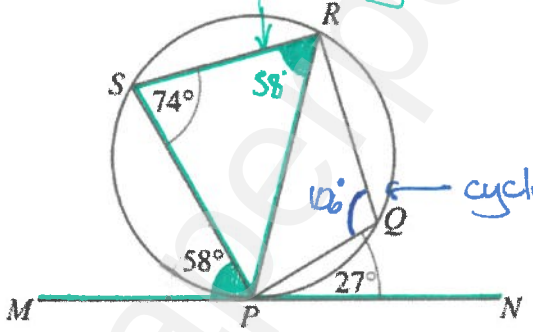
A, B and C are points on the circumference of a circle, centre O.
Tangent DE touches the circle at C.
Angle $BCE = 53°$ and angle $ACO = 20°$.

Find the value of x. $360 - (53 + 20 + 254) = 33°$

↑ angles in a quadrilateral

$x =$ 33° [3]

(b)



alternate segment theorem (1)

NOT TO SCALE

cyclic quadrilateral (2)

P, Q, R and S lie on a circle.
MPN is a tangent to the circle at P.
Angle $MPS = 58°$, angle $PSR = 74°$ and angle $QPN = 27°$.

(i) Find angle PRS.

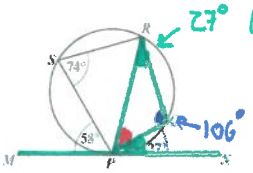
Angle $PRS =$ 58° [1]

(ii) Find angle PQR.

$180 - 74 = 106°$ (cyclic quadrilateral)

Angle $PQR =$ 106° [1]

(iii) Find angle RPQ.

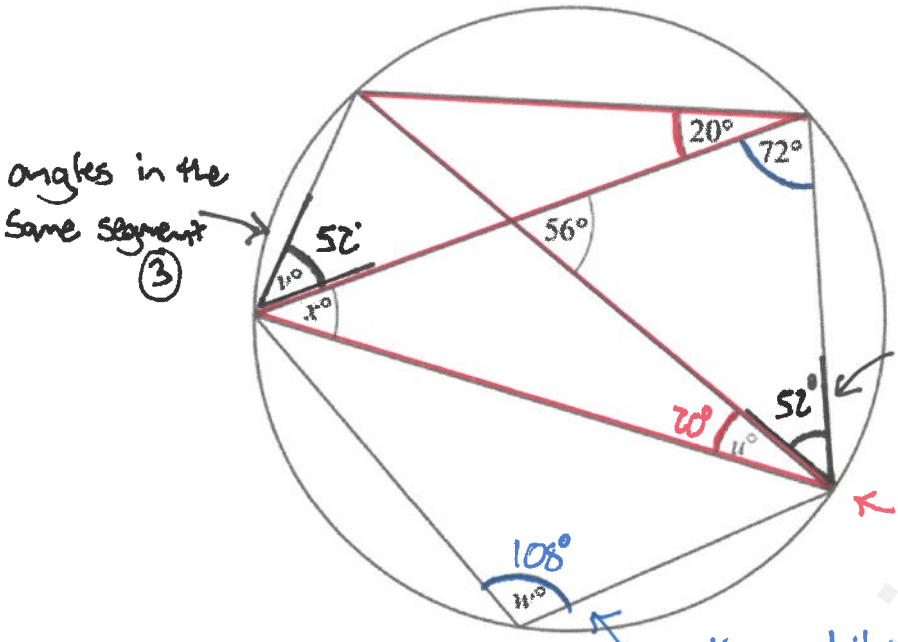


27° (alternate segment theorem)

$180 - (27 + 106) = 47°$ (angles in a triangle)

Angle $RPQ =$ 47° [2]

16



NOT TO SCALE

angles in the same segment (3)

angles in a triangle (2)
180 - (56 + 72) = 52°

angles in the same segment (1)

cyclic quadrilateral (4)
180 - 72 = 108°

The diagram shows a circle and eight chords.

Calculate the values of u, v, w and x.

u = 20°

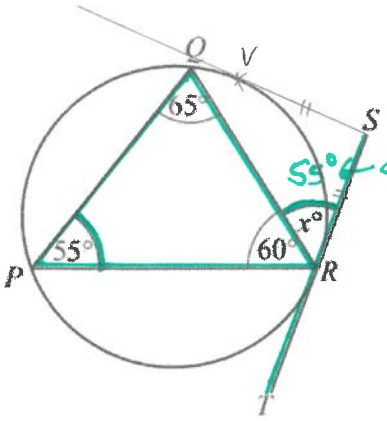
v = 52°

w = 108°

x = 36° [4]

$x = 180 - (72 + 52 + 20)$
$= 36°$
(angles in a triangle = 180°)

14



alternate segment theorem

NOT TO SCALE

P, Q and R are points on a circle.
ST is a tangent to the circle at R.

(a) Write down the value of x.
Give a geometrical reason for your answer.

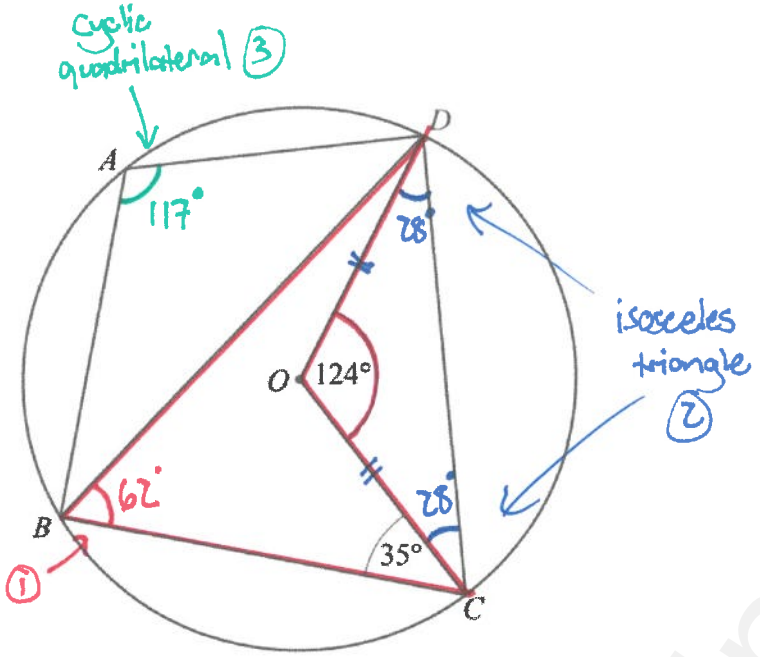
$x =$ 55° because angles in the alternate segment are equal. [2]

(b) Another tangent from the point S touches the circle at V.

Give a geometrical reason why triangle SVR is isosceles.

Tangents from an external point (s) are equal in length, so VS = RS, meaning SVR is isosceles. [1]

5 (a)



A, B, C and D are points on a circle, centre O.
Angle $COD = 124°$ and angle $BCO = 35°$.

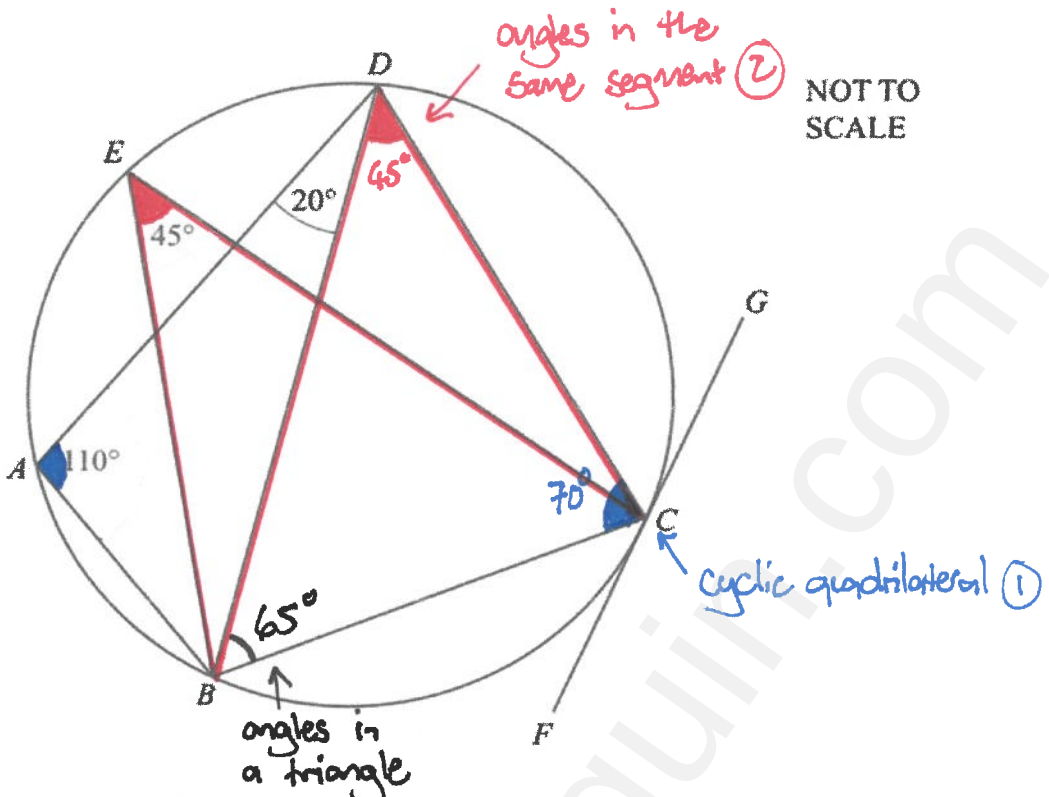
(i) Work out angle CBD.
Give a geometrical reason for your answer.

Angle CBD = 62° because the angle at the centre (124°) is twice the angle at the circumference [2]

(ii) Work out angle BAD.
Give a geometrical reason for each step of your working.

Angle BAD = 117° because $O\hat{C}D = 28°$ (base angles in an isosceles triangle are equal), $B\hat{C}D = 35+28 = 63°$, $B\hat{A}D = 180-63 = 117°$ (opposite angles in a cyclic quadrilateral sum to 180°). [4]

14



A, B, C, D and E lie on a circle.
FG is a tangent to the circle at C.
Angle $BAD = 110°$, angle $ADB = 20°$ and angle $BEC = 45°$.

(a) Find angle BCD.
Give a geometrical reason for your answer.

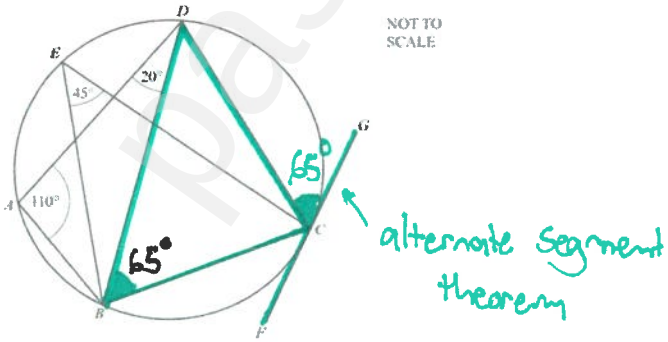
Angle BCD = 70° because opposite angles in a cyclic quadrilateral sum to 180° [2]

(b) **(i)** Find angle DBC.

180 − (45 + 70) = 65°
(angles in a triangle)

Angle DBC = 65° [2]

(ii) Find angle DCG.



Angle DCG = ... [1]