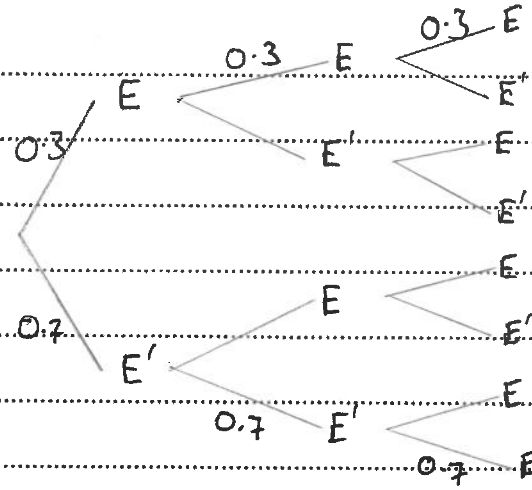
1 30% of the residents of Wimfield own an electric car. Three residents are chosen at random.

(a) Find the probability that either all three own an electric car or none of them owns an electric car. [2]



$E \rightarrow 0{\cdot}3 \times 0{\cdot}3 \times 0{\cdot}3 = 0{\cdot}027$

$E' \rightarrow 0{\cdot}7 \times 0{\cdot}7 \times 0{\cdot}7 = 0{\cdot}343$

$0{\cdot}027 + 0{\cdot}343 = 0{\cdot}37$

A random sample of 125 of the residents of Wimfield is selected.

(b) Use a suitable approximation to find the probability that more than 45 of these residents own an electric car. [5]

$E \sim B(125, 0{\cdot}3)$

$\mu = 125 \times 0{\cdot}3$ $\qquad\qquad \sigma^2 = 37{\cdot}5 \times 0{\cdot}7$

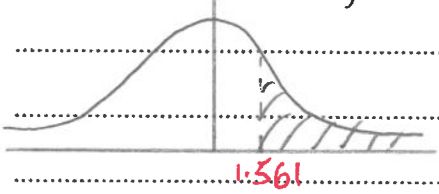
$= 37{\cdot}5$ $\qquad\qquad = 26{\cdot}25$

$E \sim N(37{\cdot}5, 26{\cdot}25)$

$P(E > 45) \rightarrow P(E > 45{\cdot}5)$ (continuity correction)

$P\left(Z > \dfrac{45{\cdot}5 - 37{\cdot}5}{\sqrt{26{\cdot}25}}\right)$

$= P(Z > 1{\cdot}561)$



$= 1 - \Phi(1{\cdot}561)$

$= 1 - 0{\cdot}9407$

$= 0{\cdot}0593$

2 A red fair six-sided dice has faces labelled 1, 1, 1, 2, 2, 2. A blue fair six-sided dice has faces labelled 1, 1, 2, 2, 3, 3. Both dice are thrown. The random variable X is the product of the scores on the two dice.

(a) Draw up the probability distribution table for X. [3]

Possibility Space Diagram:

Red

Blue	1	1	1	2	2	2
1	1	1	1	2	2	2
1	1	1	1	2	2	2
2	2	2	2	4	4	4
2	2	2	2	4	4	4
3	3	3	3	6	6	6
3	3	3	3	6	6	6

x	1	2	3	4	6
$P(X=x)$	$6/36$	$12/36$	$6/36$	$6/36$	$6/36$
	$1/6$	$2/6$	$1/6$	$1/6$	$1/6$

(b) Find E(X). [1]

$E(x) = 1 \times \frac{1}{6} + 2 \times \frac{2}{6} + 3 \times \frac{1}{6} + 4 \times \frac{1}{6} + 6 \times \frac{1}{6}$

$= \frac{1}{6} + \frac{4}{6} + \frac{3}{6} + \frac{4}{6} + \frac{6}{6}$

$= \frac{18}{6}$

$= \underline{3}$

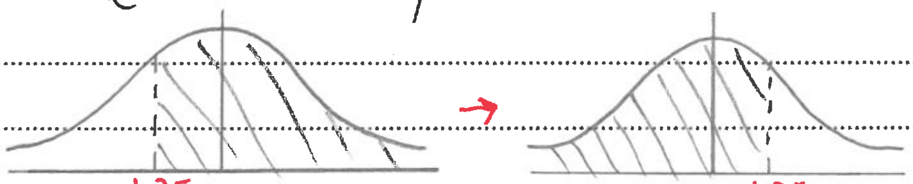
3 In Molimba, the heights, in cm, of adult males are normally distributed with mean 176 cm and standard deviation 4.8 cm.

(a) Find the probability that a randomly chosen adult male in Molimba has a height greater than 170 cm. [3]

$P(H > 170)$

$= P\left(Z > \dfrac{170 - 176}{4.8}\right)$

$= P(Z > -1.25)$



$= \Phi(1.25)$

$= 0.8944$

60% of adult males in Molimba have a height between 170 cm and k cm, where k is greater than 170.

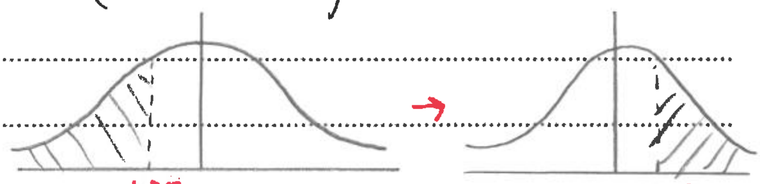
(b) Find the value of k, giving your answer correct to 1 decimal place. [4]

$P(170 < H < k) = 0.6$

$P\left(\dfrac{170 - 176}{4.8} < Z < \dfrac{k - 176}{4.8}\right) = 0.6$

$P\left(-1.25 < Z < \dfrac{k - 176}{4.8}\right) = 0.6$

$P\left(Z < \dfrac{k-176}{4.8}\right) - P(Z < -1.25) = 0.6$



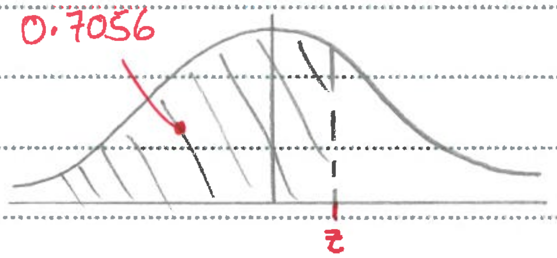
$P\left(Z < \dfrac{k - 176}{4.8}\right) - (1 - \Phi(1.25)) = 0.6$

$P\left(Z < \dfrac{k - 176}{4.8}\right) - 0.1056 = 0.6$

$P\left(Z < \dfrac{k - 176}{4.8}\right) = 0.7056$

continued ...

$$P\left(Z < \frac{k-176}{4.8}\right) = 0.7056$$



$$0.7056 = \Phi(0.541)$$

$$\frac{k-176}{4.8} = 0.541$$

$$k - 176 = 2.5968$$

$$k = \underline{178.6\text{ cm}} \ (1\text{dp})$$

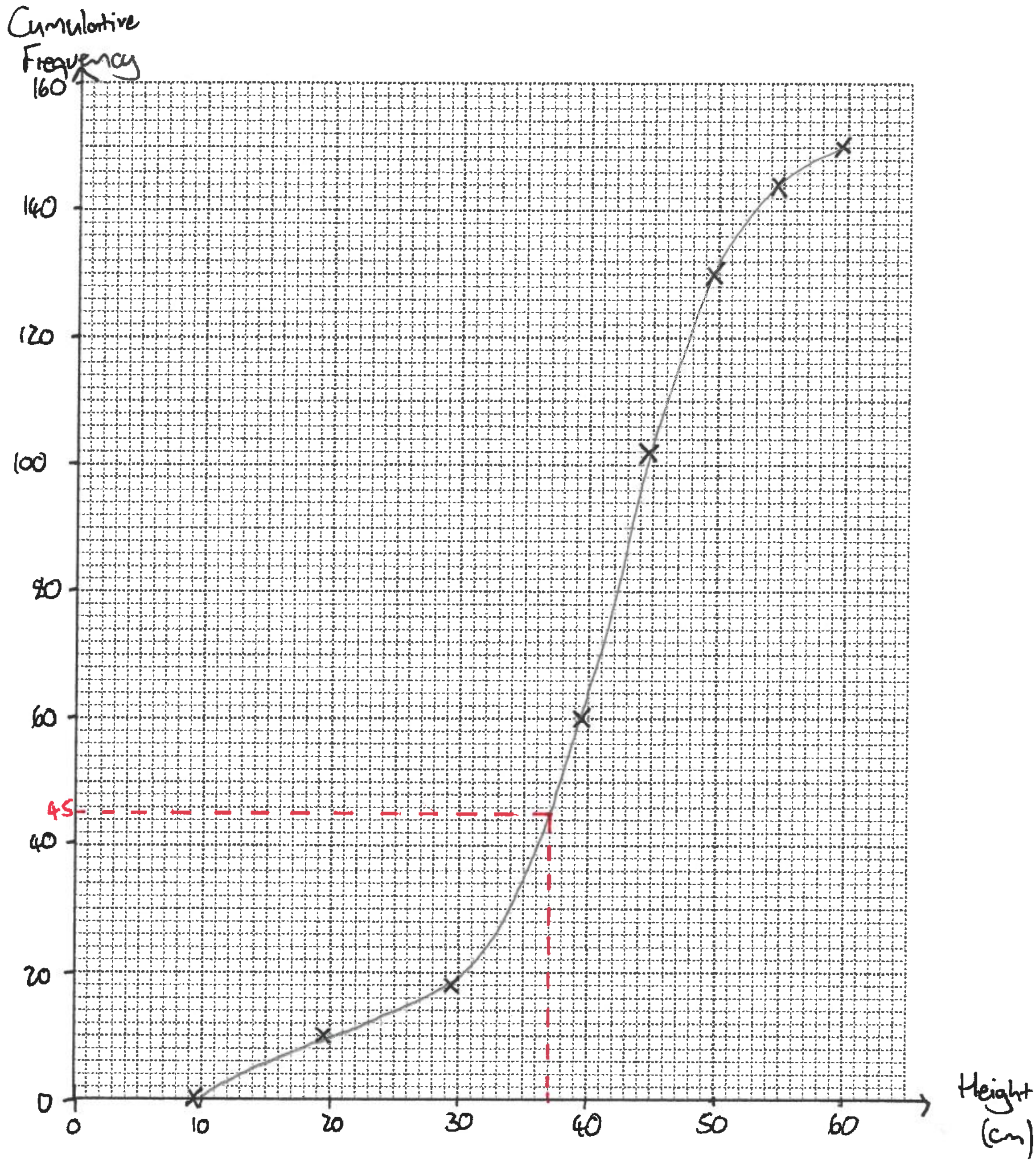
4 On a certain day, the heights of 150 sunflower plants grown by children at a local school are measured, correct to the nearest cm. These heights are summarised in the following table.

9.5 19.5 29.5 39.5 44.5 49.5 54.5 59.5

Height (cm)	10–19	20–29	30–39	40–44	45–49	50–54	55–59
Frequency	10	18	32	42	28	14	6

cf 10 28 60 102 130 144 150

(a) Draw a cumulative frequency graph to illustrate the data. [4]



(b) Use your graph to estimate the 30th percentile of the heights of the sunflower plants. [2]

$0.3 \times 150 = 45$

45^{th} percentile = 37 cm

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(c) Calculate estimates for the mean and the standard deviation of the heights of the 150 sunflower plants. [5]

Mid-point (x)	Frequency (f)	$f \times x$
14.5	10	145
24.5	18	441
34.5	32	1104
42	42	1764
47	28	1316
52	14	728
57	6	342

$\Sigma f = 150$ $\Sigma fx = 5840$

$\bar{x} = \dfrac{\Sigma fx}{\Sigma f}$

$= \dfrac{5840}{150}$

$= 38.933...$ STO

$= 38.9\text{cm}$

$\text{Var} = \dfrac{14.5^2 \times 10 + 24.5^2 \times 18 + 34.5^2 \times 32 + 42^2 \times 42 + 47^2 \times 28 + 52^2 \times 14 + 57^2 \times 6}{150} - (38.93..)^2$

$= \dfrac{244\,285}{150} - 38.93..^2$

$= 112.76...$

$\sigma = \sqrt{\text{Var}}$

$= 10.6$

5 A factory produces chocolates. 30% of the chocolates are wrapped in gold foil, 25% are wrapped in red foil and the remainder are unwrapped.

Indigo chooses 8 chocolates at random from the production line.

(a) Find the probability that she obtains no more than 2 chocolates that are wrapped in red foil. [3]

$R \sim B(8, 0.25)$

$P(R \leqslant 2) = P(0) + P(1) + P(2)$

$= {}^8C_0 \times 0.25^0 \times 0.75^8 + {}^8C_1 \times 0.25^1 \times 0.75^7 + {}^8C_2 \times 0.25^2 \times 0.75^6$

$= \underline{0.679}$

Jake chooses chocolates one at a time at random from the production line.

(b) Find the probability that the first time he obtains a chocolate that is wrapped in red foil is before the 7th choice. [2]

$R \sim Geo(0.25)$

$P(R < 7) = P(R \leqslant 6)$

$= 1 - q^6$

probability of 6 failures

$= 1 - 0.75^6$

$= \underline{0.822}$

Keifa chooses chocolates one at a time at random from the production line.

(c) Find the probability that the second chocolate chosen is the first one wrapped in gold foil given that the fifth chocolate chosen is the first unwrapped chocolate. [5]

$P(X)$

$P(Y)$ $\hookrightarrow P(\text{unwrapped}) = 0.45$

$$P(X \cap Y) = P(X|Y) \times P(Y)$$

$$P(X|Y) = \frac{P(X \cap Y)}{P(Y)}$$

$P(X \cap Y)$: Probability that the second chocolate is the first gold one, and the fifth is the first unwrapped.

This means the first chocolate must be red (not gold or unwrapped)
Second is gold
Third is wrapped (red or gold)
Fourth is wrapped (red or gold)
Fifth is unwrapped

$$\rightarrow \underset{R}{0.25} \times \underset{G}{0.3} \times \underset{R/G}{0.55} \times \underset{R/G}{0.55} \times \underset{U}{0.45} = \underset{STO}{0.0102\ldots}$$

$P(Y)$: $U \sim \text{Geo}(0.45)$

$$P(U=5) = 0.45 \times 0.55^4$$
$$= 0.0411\ldots \text{ STO}$$

$$P(X|Y) = \frac{0.0102\ldots}{0.0411\ldots}$$

$$= \frac{30}{121}$$

6 (a) Find the number of different arrangements of the 9 letters in the word HAPPINESS. [1]

HAPPINESS

$$\frac{9!}{2! \times 2!} = \underline{90\,720}$$

two Ps → two Ss

(b) Find the number of different arrangements of the 9 letters in the word HAPPINESS in which the first and last letters are not the same as each other. [3]

Find cases where first and last letters are the same:

P _ _ _ _ _ _ _ P

fixed fixed

$$\frac{7!}{2!} = 2520$$

two Ss →

or: S _ _ _ _ _ _ _ S

fixed fixed

$$\frac{7!}{2!} = 2520$$

two Ps →

→ All possibilities subtract these cases:

$$90\,720 - 2520 - 2520 = \underline{85\,680}$$

(c) Find the number of different arrangements of the 9 letters in the word HAPPINESS in which the two Ps are together and there are exactly two letters between the two Ss. [4]

① With Ps together between the two Ss:

One object → SPPS

$6! = 720$

② With Ps together, but not between the two Ss:

S __ __ S

Pick two of HAINE to go between Ss and permutate: ${}^5P_2$

Treat S _ _ S and P P as single objects and permutate with remaining 3 letters:

S__S PP $= {}^5P_2 \times 5!$

Sum both cases: $720 + {}^5P_2 \times 5! = 3120$

The 9 letters in the word HAPPINESS are divided at random into a group of 5 and a group of 4.

(d) Find the probability that both Ps are in one group and both Ss are in the other group. [3]

Ps in group of 5, Ss in group of 4:

$\underbrace{{}^2C_2 \times {}^5C_3}_{} \times \underbrace{{}^2C_2 \times {}^2C_2}_{} = 10$

both Ps, three from HAINE, both Ss, remaining two from HAINE

Ss in group of 5, Ps in group of 4:

$\underbrace{{}^2C_2 \times {}^5C_3}_{} \times \underbrace{{}^2C_2 \times {}^2C_2}_{} = 10$

Ss, HAINE, Ps, HAINE

No restrictions: ${}^9C_5 \times {}^4C_4 = 126$

five from HAPPINESS for first group; remaining four letters for second group

$\rightarrow \frac{20}{126} = \frac{10}{63}$