1 At a college, the students choose exactly one of tennis, hockey or netball to play. The table shows the numbers of students in Year 1 and Year 2 at the college playing each of these sports.

	Tennis	Hockey	Netball
Year 1	16	22	12
Year 2	24	18	28

One student is chosen at random from the 120 students. Events X and N are defined as follows:

X: the student is in Year 1

N: the student plays netball.

(a) Find $P(X|N)$. [1]

$P(X \cap N) = P(X/N) \times P(N)$ $\rightarrow$ $P(X/N) = \frac{12}{40}$

$P(X/N) = \frac{P(X \cap N)}{P(N)}$

(b) Find $P(N|X)$. [1]

$P(N|X) = \frac{P(N \cap X)}{P(X)} = \frac{12}{50}$

(c) Determine whether or not X and N are independent events. [1]

If independent, $P(X) \times P(N) = P(X \cap N)$

$P(X) = \frac{50}{120}$ $\quad P(N) = \frac{40}{120}$ $\quad P(X \cap N) = \frac{12}{120}$

$P(X) \times P(N) = \frac{50}{120} \times \frac{40}{120} = \frac{5}{36}$ $\quad \frac{5}{36} \neq \frac{12}{120}$ so not independent.

One of the students who plays netball takes 8 shots at goal. On each shot, the probability that she will succeed is 0.15, independently of all other shots.

(d) Find the probability that she succeeds on fewer than 3 of these shots. [3]

$X \sim B(8, 0.15)$

$P(X < 3) = P(0) + P(1) + P(2)$

$= {}^8C_0 \times 0.15^0 \times 0.85^8 + {}^8C_1 \times 0.15^1 \times 0.85^7 + {}^8C_2 \times 0.15^2 \times 0.85^6$

$= 0.895$

2 (a) Find the number of different arrangements of the 9 letters in the word ALGEBRAIC. [1]

AALGEBRIC

$$\frac{9!}{2!} = 181\,440$$

2 As

(b) Find the number of different arrangements of the 9 letters in the word ALGEBRAIC in which there are no more than two letters between the two As. [3]

Two letters between As: A __ __ A

Pick two of remaining letters to go between As and permutate: $^7P_2$

Treat A__A as one object and permutate with remaining 5 letters: __ __ __ __ __ __

A__A

$\rightarrow {}^7P_2 \times 6!$

One letter between As: A __ A

$\rightarrow {}^7P_1$

A_A is one object, permutate with remaining 6 letters:

__ __ __ __ __ __ __

A_A

$\rightarrow {}^7P_1 \times 7!$

No letters between As:

__ __ __ __ __ __ __ __

AA

one object

$8!$

Total:

${}^7P_2 \times 6!$

$+ {}^7P_1 \times 7!$

$+ 8!$

$= 105\,840$

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3 A fair coin and an ordinary fair six-sided dice are thrown at the same time. The random variable X is defined as follows.

- If the coin shows a tail, X is twice the score on the dice.
- If the coin shows a head, X is the score on the dice if the score is even and X is 0 otherwise.

(a) Draw up the probability distribution table for X. [3]

	1	2	3	4	5	6
H	0	2	0	4	0	6
T	2	4	6	8	10	12

← possibility space diagram

x	0	2	4	6	8	10	12
$P(X=x)$	$3/12$	$2/12$	$2/12$	$2/12$	$1/12$	$1/12$	$1/12$

(b) Find $\text{Var}(X)$. [3]

$E(X) = 0\times\frac{3}{12} + 2\times\frac{2}{12} + 4\times\frac{2}{12} + 6\times\frac{2}{12} + 8\times\frac{1}{12} + 10\times\frac{1}{12} + 12\times\frac{1}{12}$

$= 0 + \frac{4}{12} + \frac{8}{12} + \frac{12}{12} + \frac{8}{12} + \frac{10}{12} + \frac{12}{12}$

$= 4.5$

$\text{Var}(X) = 0^2\times\frac{3}{12} + 2^2\times\frac{2}{12} + 4^2\times\frac{2}{12} + 6^2\times\frac{2}{12} + 8^2\times\frac{1}{12} + 10^2\times\frac{1}{12} + 12^2\times\frac{1}{12} - (E(X))^2$

$= \frac{2}{3} + \frac{8}{3} + 6 + \frac{16}{3} + \frac{25}{3} + 12 - (4.5)^2$

$= 14.75$

4 The heights, in metres, of white pine trees are normally distributed with mean 19.8 and standard deviation 2.4 .

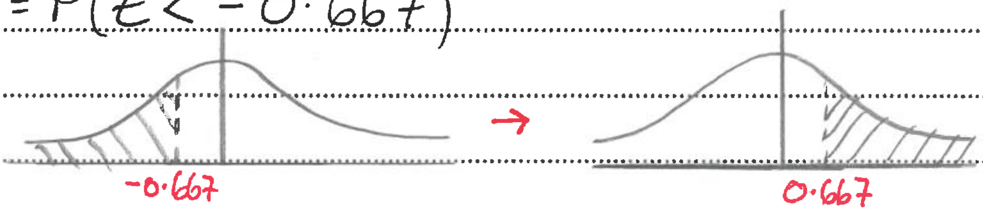
In a certain forest there are 450 white pine trees.

(a) How many of these trees would you expect to have height less than 18.2 metres? [4]

$P(H < 18.2)$

$= P\left(Z < \frac{18.2 - 19.8}{2.4}\right)$

$= P(Z < -0.667)$



$= 1 - \Phi(0.667)$

$= 1 - 0.7477$

$= \underline{0.2523}$

Expected number of trees $= 0.2523 \times 450$

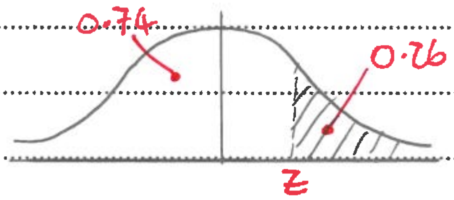
$= \underline{114}$ (to the nearest whole number)

The heights, in metres, of red pine trees are normally distributed with mean 23.4 and standard deviation σ. It is known that 26% of red pine trees have height greater than 25.5 metres.

(b) Find the value of σ. [3]

$P(R > 25.5) = 0.26$

$P\left(Z > \frac{25.5 - 23.4}{\sigma}\right) = 0.26$



$0.74 = \Phi(0.643)$

$\rightarrow \frac{25.5 - 23.4}{\sigma} = 0.643$

$\frac{2.1}{\sigma} = 0.643$

$2.1 = 0.643\sigma$

$\underline{\sigma = 3.27}$

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5 In a class of 21 students, there are 10 violinists, 6 guitarists and 5 pianists. A group of 7 is to be chosen from these 21 students. The group will consist of 4 violinists, 2 guitarists and 1 pianist.

(a) In how many ways can the group of 7 be chosen? [2]

$$\underbrace{{}^{10}C_4}_{\text{4 violinists from 10}} \times \underbrace{{}^{6}C_2}_{\text{guitarists}} \times \underbrace{{}^{5}C_1}_{\text{pianist}} = \underline{\underline{15\,750}}$$

On another occasion a group of 5 will be chosen from the 21 students. The group must contain at least 2 violinists, at least 1 guitarist and at most 1 pianist.

V G P

(b) In how many ways can the group of 5 be chosen? [4]

4V, 1G, 0P : ${}^{10}C_4 \times {}^{6}C_1 = 1260$

3V, 2G, 0P : ${}^{10}C_3 \times {}^{6}C_2 = 1800$

2V, 3G, 0P : ${}^{10}C_2 \times {}^{6}C_3 = 900$

3V, 1G, 1P : ${}^{10}C_3 \times {}^{6}C_1 \times {}^{5}C_1 = 3600$

2V, 2G, 1P : ${}^{10}C_2 \times {}^{6}C_2 \times {}^{5}C_1 = 3375$

Sum of these = $\underline{\underline{10\,935}}$

6 Teams of 15 runners took part in a charity run last Saturday. The times taken, in minutes, to complete the course by the runners from the Falcons and the runners from the Kites are shown in the table.

Falcons	38	39	42	44	46	48	50	51	52	56	58	59	64	69	76
Kites	32	40	40	45	47	48	52	54	58	59	59	60	61	63	65

(a) Draw a back-to-back stem-and-leaf diagram to represent this information, with the Falcons on the left-hand side. [4]

Falcons		Kites
9 8	3	2
8 6 4 (Q_1) 2	4	0 0 5 7 8
9 (Q_3) 8 6 2 1 (Q_2) 0	5	2 4 8 9 9
9 4	6	0 1 3 5
6	7	

Key: 8|3|2 means 38 minutes for Falcons and 32 minutes for Kites

(b) Find the median and the interquartile range of the times for the Falcons. [3]

$Q_1: \frac{15+1}{4} = 4^{th}$ $\quad Q_2: \frac{15+1}{2} = 8^{th}$ $\quad Q_3: \frac{3(15+1)}{4} = 12^{th}$

$Q_1 = 44$ $\quad Q_2 = 51$ $\quad Q_3 = 59$

Median = 51 minutes $\quad$ IQR = 59 − 44

= 15 minutes

Let x and y denote the times, in minutes, of a runner from the Falcons and a runner from the Kites respectively.

It is given that

$$\Sigma x = 792, \qquad \Sigma x^2 = 43\,504, \qquad \Sigma y = 783, \qquad \Sigma y^2 = 42\,223.$$

(c) Find the mean and the standard deviation of the times taken by all 30 runners from the two teams. [3]

$$\text{mean} = \frac{\Sigma x + \Sigma y}{n_x + n_y}$$

$$= \frac{792 + 783}{15 + 15}$$

$$= \frac{1575}{30}$$

$$= 52.5$$

$$\sigma = \sqrt{\frac{\Sigma x^2 + \Sigma y^2}{n_x + n_y} - (\text{mean})^2}$$

$$= \sqrt{\frac{43504 + 42223}{15 + 15} - 52.5^2}$$

$$= \sqrt{\frac{85727}{30} - 2756.25}$$

$$= 10.1$$

7 In a game, players attempt to score a goal by kicking a ball into a net. The probability that Leno scores a goal is 0.4 on any attempt, independently of all other attempts. The random variable X denotes the number of attempts that it takes Leno to score a goal.

$X \sim \text{Geo}(0.4)$

(a) Find $P(X = 5)$. [1]

$P(X=5) = 0.4 \times 0.6^4$

$= 0.05184$

(b) Find $P(3 \leqslant X \leqslant 7)$. [2]

$P(3 \leqslant X \leqslant 7) = P(X \leqslant 7) - P(X \leqslant 2)$

$= (1 - q^7) - (1 - q^2)$

7 failures   2 failures

$= (1 - 0.6^7) - (1 - 0.6^2)$

$= 0.332$

(c) Find the probability that Leno scores his second goal on or before his 5th attempt. [3]

Second goal on 5th attempt:

$\underbrace{{}^4C_1 \times 0.4^1 \times 0.6^3}_{\text{one goal in first four attempts (binomial)}} \times \overbrace{0.4}^{\text{goal on fifth attempt}} = 0.13824$

Second goal on 4th attempt:

${}^3C_1 \times 0.4^1 \times 0.6^2 \times 0.4 = 0.1728$

one goal in first three attempts

Second goal on 3rd attempt:

${}^2C_1 \times 0.4^1 \times 0.6^1 \times 0.4 = 0.192$

Second goal on 2nd attempt:

$0.4 \times 0.4 = 0.16$

$0.13824 + 0.1728 + 0.192 + 0.16 = 0.66304$

Leno has 75 attempts to score a goal.

(d) Use a suitable approximation to find the probability that Leno scores more than 28 goals but fewer than 35 goals. [5]

$G \sim B(75, 0.4)$

$\mu = 75 \times 0.4$ $\sigma^2 = 30 \times 0.6$

$= 30$ $= 18$

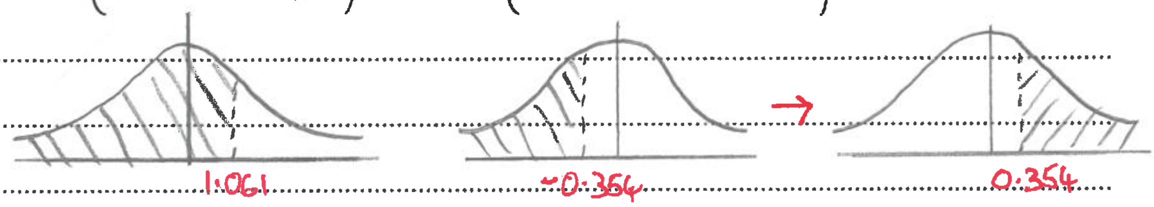
$G \sim N(30, 18)$

$P(28 < G < 35) \rightarrow P(28.5 < G < 34.5)$ (Continuity Correction)

$P\left(\frac{28.5 - 30}{\sqrt{18}} < Z < \frac{34.5 - 30}{\sqrt{18}}\right)$

$= P(-0.354 < Z < 1.061)$

$= P(Z < 1.061) - P(Z < -0.354)$



$= \Phi(1.061) - (1 - \Phi(0.354))$

$= 0.8556 - (1 - 0.6383)$

$= 0.8556 - 0.3617$

$= \underline{0.4939}$