1 Nicola throws an ordinary fair six-sided dice. The random variable X is the number of throws that she takes to obtain a 6.

(a) Find $P(X < 8)$. [2]

$X \sim \text{Geo}\left(\frac{1}{6}\right)$

$P(X<8) = P(X \leqslant 7)$

$= 1 - q^7$

← probability of 7 failures

$= 1 - \left(\frac{5}{6}\right)^7$

$= \underline{0.721}$

(b) Find the probability that Nicola obtains a 6 for the second time on her 8th throw. [2]

Probability of rolling one 6 in the first seven throws:

$X \sim B\left(7, \frac{1}{6}\right)$

$P(X=1) = {}^7C_1 \times \left(\frac{1}{6}\right)^1 \times \left(\frac{5}{6}\right)^6$

$= 0.3907\ldots$ STO

Probability of a 6 on the 8^{th} throw: $P(6) = \frac{1}{6}$

Probability of both $= 0.3907\ldots \times \frac{1}{6}$

$= \underline{0.0651}$

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2 The random variable X takes the values $-2, -1, 0, 2, 3$. It is given that $P(X = x) = k(x^2 + 2)$, where k is a positive constant.

(a) Draw up the probability distribution table for X, giving the probabilities as numerical fractions. [3]

$x = -2$: $k((-2)^2 + 2) = 6k$ $x = 2$: $k(2^2 + 2) = 6k$

$x = -1$: $k((-1)^2 + 2) = 3k$ $x = 3$: $k(3^2 + 2) = 11k$

$x = 0$: $k(0^2 + 2) = 2k$

$\sum P = 1$: $6k + 3k + 2k + 6k + 11k = 1$

$28k = 1$

$k = \frac{1}{28}$

x	-2	-1	0	2	3
$P(X=x)$	$6/28$	$3/28$	$2/28$	$6/28$	$11/28$

(b) Find the value of Var(X). [3]

$E(X) = -2 \times \frac{6}{28} + -1 \times \frac{3}{28} + 0 + 2 \times \frac{6}{28} + 3 \times \frac{11}{28}$

$= \frac{15}{14}$

$Var(X) = (-2)^2 \times \frac{6}{28} + (-1)^2 \times \frac{3}{28} + 0 + 2^2 \times \frac{6}{28} + 3^2 \times \frac{11}{28} - (E(X))^2$

$= \frac{24}{28} + \frac{3}{28} + 0 + \frac{24}{28} + \frac{99}{28} - \left(\frac{15}{14}\right)^2$

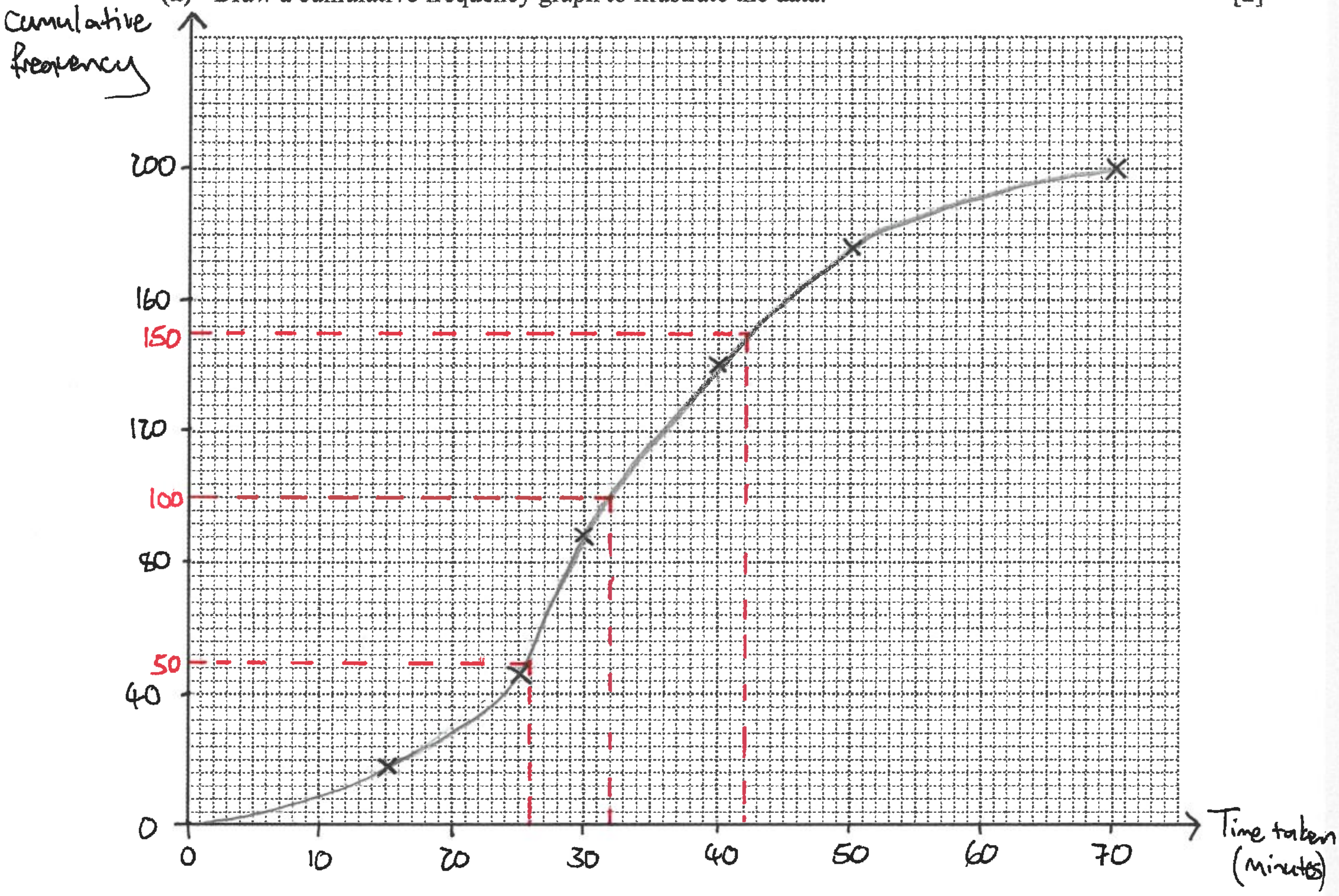
$= \frac{150}{28} - \frac{225}{196}$

$= \frac{825}{196}$

3 The time taken, in minutes, to walk to school was recorded for 200 pupils at a certain school. These times are summarised in the following table.

Time taken (t minutes)	$t \leqslant 15$	$t \leqslant 25$	$t \leqslant 30$	$t \leqslant 40$	$t \leqslant 50$	$t \leqslant 70$
Cumulative frequency	18	46	88	140	176	200

(a) Draw a cumulative frequency graph to illustrate the data. [2]



(b) Use your graph to estimate the median and the interquartile range of the data. [3]

Median: $0.5 \times 200 = 100$

Median $= 32$

Q_3: $0.75 \times 200 = 150$

$Q_3 = 42$

Q_1: $0.25 \times 200 = 50$

$Q_1 = 26$

IQR $= 42 - 26$

$= 16$

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(c) Calculate an estimate for the mean value of the times taken by the 200 pupils to walk to school. [3]

Mid-point (t)	Frequency (f)	$f \times x$
7.5	18	135
20	28	560
27.5	42	1155
35	52	1820
45	36	1620
60	24	1440
	$\Sigma f = 200$	$\Sigma fx = 6730$

$$\bar{t} = \frac{6730}{200}$$

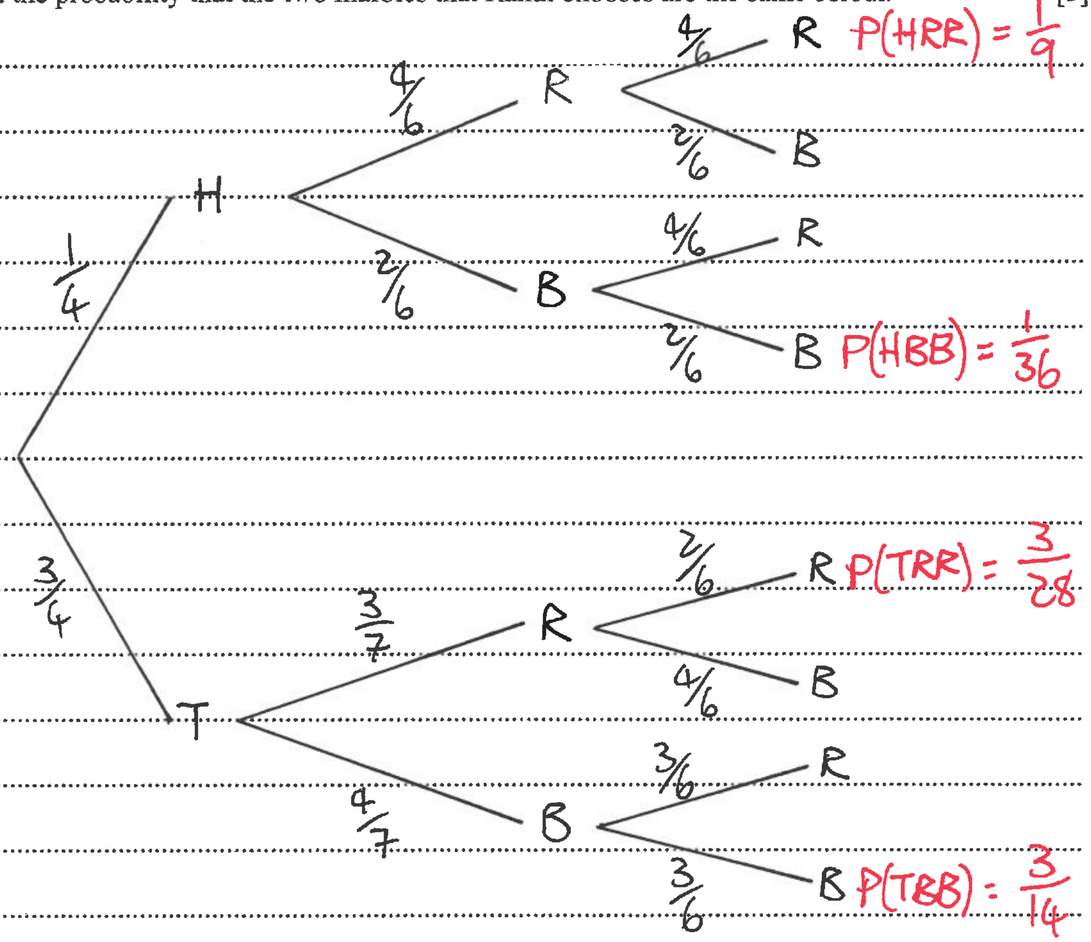
$$= \underline{33.65 \text{ minutes}}$$

4 Rahul has two bags, X and Y. Bag X contains 4 red marbles and 2 blue marbles. Bag Y contains 3 red marbles and 4 blue marbles. Rahul also has a coin which is biased so that the probability of obtaining a head when it is thrown is $\frac{1}{4}$.

Rahul throws the coin.

- If he obtains a head, he chooses at random a marble from bag X. He notes the colour and replaces the marble in bag X. He then chooses at random a second marble from bag X.
- If he obtains a tail, he chooses at random a marble from bag Y. He notes the colour and discards the marble. He then chooses at random a second marble from bag Y.

(a) Find the probability that the two marbles that Rahul chooses are the same colour. [3]



$P(\text{both same colour}) = \frac{1}{9} + \frac{1}{36} + \frac{3}{28} + \frac{3}{14}$

$= \frac{29}{63}$

(b) Find the probability that the two marbles that Rahul chooses are both from bag Y given that both marbles are blue. [3]

This is just another way of saying the probability of Tails.

$P(T \cap BB) = P(T|BB) \times P(BB)$

$P(T|BB) = \dfrac{P(T \cap BB)}{P(BB)}$

$P(T \cap BB) = \dfrac{3}{14}$

$P(BB) = \dfrac{1}{36} + \dfrac{3}{14}$

$= \dfrac{61}{252}$

$P(T|BB) = \dfrac{3/14}{61/252}$

$= \dfrac{54}{61}$

5 The weights of the green apples sold by a shop are normally distributed with mean 90 grams and standard deviation 8 grams.

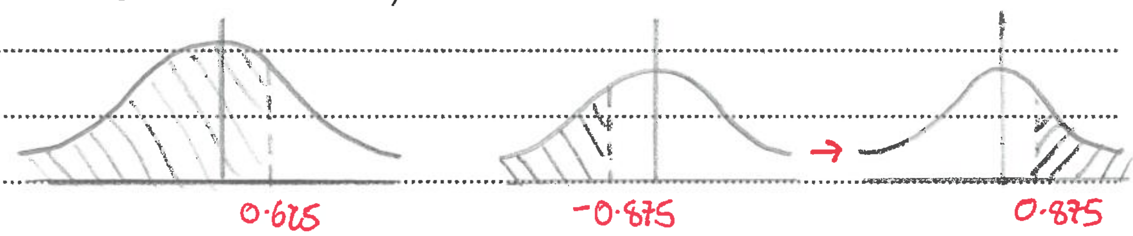
(a) Find the probability that a randomly chosen green apple weighs between 83 grams and 95 grams. [4]

$P(83 < W < 95)$

$P\left(\frac{83-90}{8} < Z < \frac{95-90}{8}\right)$

$P(-0.875 < Z < 0.625)$

$= P(Z < 0.625) - P(Z < -0.875)$



$= \Phi(0.625) - (1 - \Phi(0.875))$

$= 0.7340 - (1 - 0.8092)$

$= 0.7340 - 0.1908$

$= \underline{0.5432}$

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(b) The shop also sells red apples. 60% of the red apples sold by the shop weigh more than 80 grams. 160 red apples are chosen at random from the shop.

Use a suitable approximation to find the probability that fewer than 105 of the chosen red apples weigh more than 80 grams. [5]

$R \sim B(160, 0.6)$ ← P(weigh more than 80g)

$\mu = 160 \times 0.6$ $\qquad \sigma^2 = 96 \times 0.4$

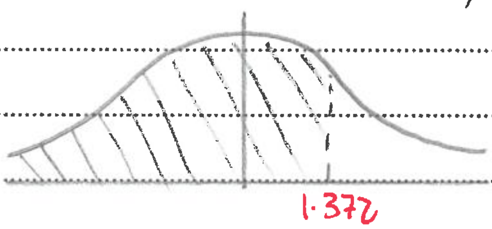
$= 96$ $\qquad = 38.4$

$R \sim N(96, 38.4)$

$P(R < 105) \rightarrow P(R < 104.5)$ (continuity correction)

$P\left(Z < \frac{104.5 - 96}{\sqrt{38.4}}\right)$

$= P(Z < 1.372)$



$= \Phi(1.372)$

$= 0.915$

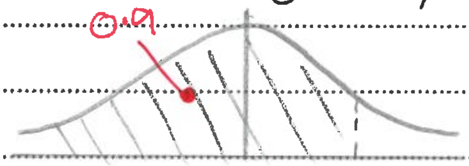
6 The heights of the female students at Breven college are normally distributed:

- **90% of the female students have heights less than 182.7 cm.**
- 40% of the female students have heights less than 162.5 cm.

(a) Find the mean and the standard deviation of the heights of the female students at Breven college. [5]

$P(H < 182.7) = 0.9$

$P\left(Z < \dfrac{182.7 - \mu}{\sigma}\right) = 0.9$



$0.9 = \Phi(1.282)$

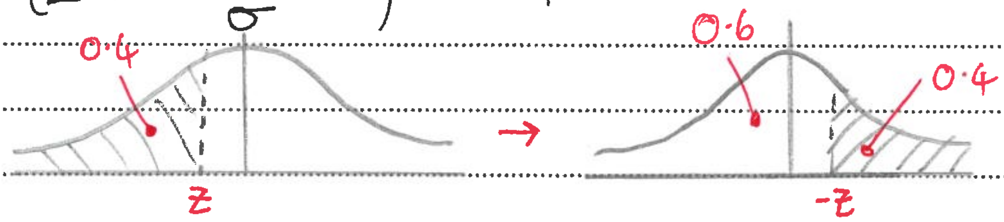
↑ critical value

$\rightarrow \dfrac{182.7 - \mu}{\sigma} = 1.282$

$182.7 - \mu = 1.282\sigma$ ①

$P(H < 162.5) = 0.4$

$P\left(Z < \dfrac{162.5 - \mu}{\sigma}\right) = 0.4$



$0.6 = \Phi(0.253)$ so $z = -0.253$

$\rightarrow \dfrac{162.5 - \mu}{\sigma} = -0.253$

$162.5 - \mu = -0.253\sigma$ ②

① − ②: $20.2 = 1.535\sigma$

$\sigma = 13.159\ldots$ STO

$\sigma = 13.2$

$\rightarrow$ ①: $182.7 - \mu = 1.282(13.159)$

$182.7 - \mu = 16.87$

$-\mu = -165.8$

$\mu = 165.8$

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Ten female students are chosen at random from those at Breven college.

(b) Find the probability that fewer than 8 of these 10 students have heights more than 162.5 cm. [3]

$P(H > 162.5) = 0.6$ (from the start of the question)

$X \sim B(10, 0.6)$

$P(X < 8) = 1 - (P(8) + P(9) + P(10))$

$= 1 - ({}^{10}C_8 \times 0.6^8 \times 0.4^2 + {}^{10}C_9 \times 0.6^9 \times 0.4^1 + {}^{10}C_{10} \times 0.6^{10} \times 0.4^0)$

$= \underline{\underline{0.833}}$

7 (a) How many different arrangements are there of the 9 letters in the word INTELLECT in which the two Ts are together? [2]

INTTEELLC

TT

one object

$$\frac{8!}{2! \times 2!} = \underline{10\,080}$$

2 Es 2 Ls

(b) How many different arrangements are there of the 9 letters in the word INTELLECT in which there is a T at each end and the two Es are not next to each other? [3]

T at each end:

T _ _ _ _ _ _ _ T

fixed fixed

$$\text{two Es} \rightarrow \frac{7!}{2! \times 2!} \leftarrow \text{two Ls} = 1260$$

T at each end and Es together:

T EE _ _ _ _ _ _ T

fixed one object fixed

$$\text{two Ls} \rightarrow \frac{6!}{2!} = 360$$

T at ends and Es separate $= 1260 - 360$

$= \underline{900}$

Four letters are selected at random from the 9 letters in the word INTELLECT.

(c) Find the percentage of the possible selections which contain at least one E and exactly one T. [4]

We are picking letters at random, not looking for different selections, so the Es, Ts and Ls are treated as distinguishable.

E T _ _ $\quad {}^2C_1 \times {}^2C_1 \times {}^5C_2 = 40$

1 E from 2 $\quad$ 1 T from 2 $\quad$ 2 letters from INLLC

E E T _ $\quad {}^2C_2 \times {}^2C_1 \times {}^5C_1 = 10$

both Es $\quad$ 1 T $\quad$ 1 from INLLC

$40 + 10 = \underline{50}$ selections

percentage: $\dfrac{50}{{}^9C_4} \times 100$

$= \underline{39.7\%}$