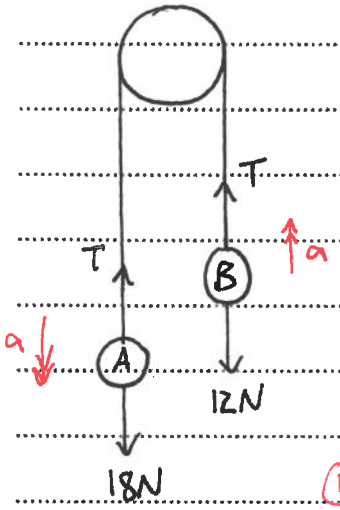
1 Two particles, of masses 1.8 kg and 1.2 kg, are connected by a light inextensible string that passes over a fixed smooth pulley. The particles hang vertically. The system is released from rest.

Find the magnitude of the acceleration of the particles and find the tension in the string. [4]



A:

R(↓): $18 - T = 1.8a$ ①

B:

R(↑): $T - 12 = 1.2a$ ②

①+②: $6 = 3a$

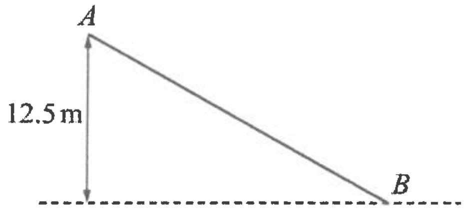
$a = 2\,\text{ms}^{-2}$

→ ②: $T - 12 = 1.2 \times 2$

$T - 12 = 2.4$

$T = 14.4\,\text{N}$

2



A particle of mass 7.5 kg, starting from rest at A, slides down an inclined plane AB. The point B is 12.5 metres vertically below the level of A, as shown in the diagram.

(a) Given that the plane is smooth, use an energy method to find the speed of the particle at B. [2]

$$\text{Work}_{IN} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + \text{Work}_{OUT}$$

$$0 + 0 + 0 = \tfrac{1}{2}\times 7.5\times v^2 + 7.5\times 10\times -12.5 + 0$$

$$0 = 3.75v^2 - 937.5$$

$$3.75v^2 = 937.5$$

$$v^2 = 250$$

$$v = 15.8\,\text{ms}^{-1}$$

(b) It is given instead that the plane is rough and the particle reaches B with a speed of $8\,\text{m s}^{-1}$. The plane is 25 m long and the constant frictional force has magnitude F N.

Find the value of F. [3]

$$\text{Work}_{IN} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + \text{Work}_{OUT}$$

$$0 + 0 + 0 = \tfrac{1}{2}\times 7.5\times 8^2 + 7.5\times 10\times -12.5 + F\times 25$$

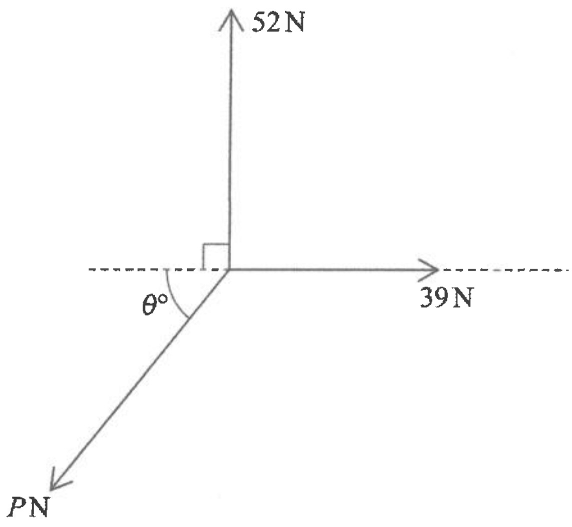
work = F×d

$$0 = 240 - 937.5 + 25F$$

$$25F = 697.5$$

$$F = 27.9\,\text{N}$$

3



Coplanar forces of magnitudes 52 N, 39 N and P N act at a point in the directions shown in the diagram. The system is in equilibrium.

Find the values of P and θ. [4]

$R(\uparrow)$: $52 - P\sin\theta = 0$

$P\sin\theta = 52$ (1)

$R(\rightarrow)$: $39 - P\cos\theta = 0$

$P\cos\theta = 39$ (2)

(1) ÷ (2) : $\frac{P\sin\theta}{P\cos\theta} = \frac{52}{39}$

$\tan\theta = \frac{4}{3}$

$\theta = 53.1°$ STO

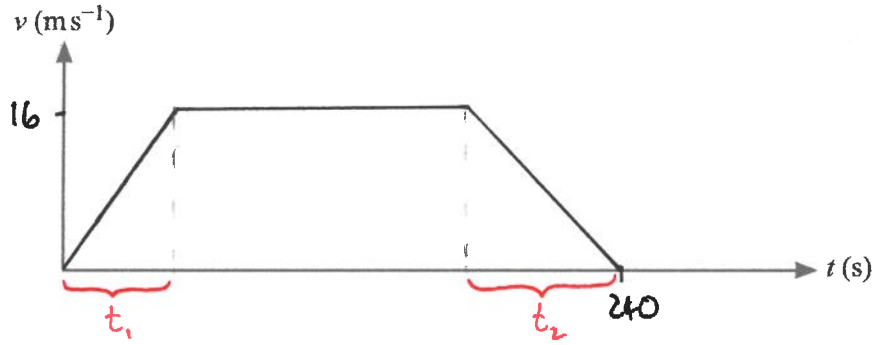
→ (1) : $P\sin(53.13..) = 52$

$P = \frac{52}{\sin(53.13..)}$

$= 65N$

4 A bus travels between two stops, A and B. The bus starts from rest at A and accelerates at a constant rate of $a\,\mathrm{m\,s^{-2}}$ until it reaches a speed of $16\,\mathrm{m\,s^{-1}}$. It then travels at this constant speed before decelerating at a constant rate of $0.75a\,\mathrm{m\,s^{-2}}$, coming to rest at B. The total time for the journey is 240 s.

(a) Sketch the velocity-time graph for the bus's journey from A to B. [1]



(b) Find an expression, in terms of a, for the length of time that the bus is travelling with constant speed. [2]

t_1: $V = u + at$ $\qquad$ t_2: $V = u + at$

$16 = 0 + at_1$ $\qquad$ $0 = 16 + (-0.75a)t_2$

$t_1 = \frac{16}{a}$ $\qquad$ $0.75at_2 = 16 \rightarrow t_2 = \frac{64}{3a}$

Time at constant speed $= 240 - \frac{16^{\times 3}}{a_{\times 3}} - \frac{64}{3a}$

$= 240 - \frac{112}{3a}$

(c) Given that the distance from A to B is 3000 m, find the value of a. [3]

Area $= \frac{1}{2}(a+b) \times h$

$= \frac{1}{2}\left(240 + 240 - \frac{112}{3a}\right) \times 16$

$= 8\left(480 - \frac{112}{3a}\right)$

$3000 = 3840 - \frac{896}{3a}$

$\frac{896}{3a} = 840$

$896 = 2520a$ $\rightarrow$ $a = \frac{16}{45}\ \mathrm{ms^{-2}}$

5 A particle, A, is projected vertically upwards from a point O with a speed of $80\,\text{m s}^{-1}$. One second later a second particle, B, with the same mass as A, is projected vertically upwards from O with a speed of $100\,\text{m s}^{-1}$. At time T s after the first particle is projected, the two particles collide and coalesce to form a particle C.

(a) Show that $T = 3.5$. [4]

A: $\uparrow +$ $s = s$, $u = 80$, $v =$, $a = -10$, $t = t_A$

$s = ut + \frac{1}{2}at^2$

$= 80t_A + \frac{1}{2}(-10)t_A^2$

$= 80t_A - 5t_A^2$ (1)

B: $\uparrow +$ $s = s$, $u = 100$, $v =$, $a = -10$, $t = t_A - 1$

B has been in the air for 1s less than A.

$s = ut + \frac{1}{2}at^2$

$= 100(t_A - 1) + \frac{1}{2}(-10)(t_A - 1)^2$

$= 100t_A - 100 - 5(t_A^2 - 2t_A + 1)$

$= 100t_A - 100 - 5t_A^2 + 10t_A - 5$

$= -5t_A^2 + 110t_A - 105$ (2)

(1) = (2):

$80t_A - \cancel{5t_A^2} = -\cancel{5t_A^2} + 110t_A - 105$

$30t_A = 105$

$t_A = 3.5\text{s}$

(b) Find the height above O at which the particles collide. [1]

Sub. $t = 3.5$ into (1):

$s = 80 \times 3.5 - 5 \times 3.5^2$

$= 280 - 61.25$

$= 218.75\text{m}$

(c) Find the time from A being projected until C returns to O. [5]

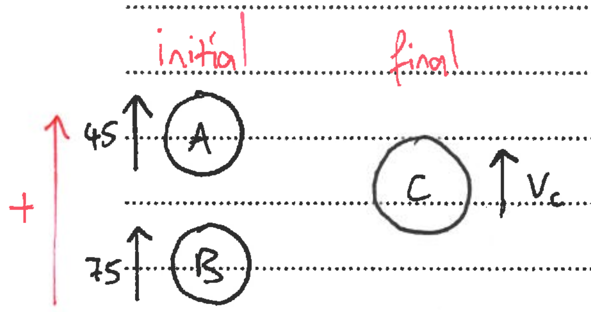
Find velocities of A and B when they collide at $t = 3.5$s:

A: $\uparrow +$

$s = 218.75$	$v = u + at$
$u = 80$	$= 80 + (-10) \times 3.5$
$v =$	$= 45 \text{ ms}^{-1}$ (upwards)
$a = -10$	
$t = 3.5$	

B: $\uparrow +$

$s = 218.75$	$v = u + at$
$u = 100$	$= 100 + (-10) \times 2.5$
$v =$	$= 75 \text{ ms}^{-1}$ (upwards)
$a = -10$	
$t = 2.5$	



$m_A u_A + m_B u_B = m_c V_c$

$45m + 75m = 2m V_c$

$120m = 2m V_c$

$V_c = 60 \text{ ms}^{-1}$

C: $\uparrow +$

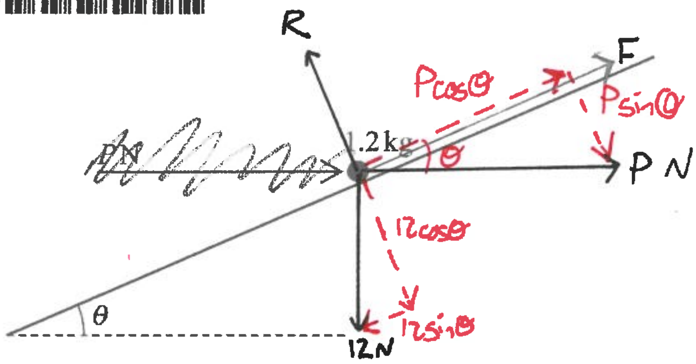
$s = -218.75$	$s = ut + \frac{1}{2}at^2$
$u = 60$	$-218.75 = 60t + \frac{1}{2}(-10)t^2$
$v =$	$-218.75 = 60t - 5t^2$
$a = -10$	$5t^2 - 60t - 218.75 = 0$
$t =$	$t^2 - 12t - 43.75 = 0$

$$t = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(1)(-43.75)}}{2(1)}$$

$t = 14.9$s ✓ or $t = -2.93$ ✗

Time since A projected $= 14.9 + 3.5 = 18.4$ s

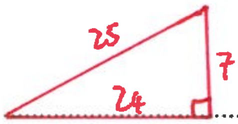
6



A particle of mass 1.2 kg is placed on a rough plane which is inclined at an angle θ to the horizontal, where $\sin\theta = \frac{7}{25}$. The particle is kept in equilibrium by a horizontal force of magnitude P N acting in a vertical plane containing a line of greatest slope (see diagram). The coefficient of friction between the particle and the plane is 0.15 .

Find the least possible value of P. [6]

$\sin\theta = \frac{7}{25}$ →



$\cos\theta = \frac{24}{25}$ $\tan\theta = \frac{7}{24}$

P is at its least when the particle is on the point of sliding down the plane, so friction is acting up the plane.

$R(\nwarrow)$: $R - 12\cos\theta - P\sin\theta = 0$

$R - 12\times\frac{24}{25} - P\times\frac{7}{25} = 0$

$R - 11.52 - \frac{7}{25}P = 0$ $\times 25$

$25R - 288 - 7P = 0$

$25R - 7P = 288$ ①

$R(\swarrow)$: $12\sin\theta - P\cos\theta - F = 0$

$12\times\frac{7}{25} - P\times\frac{24}{25} - \mu R = 0$

$3.36 - \frac{24}{25}P - 0.15R = 0$ $\times 25$

$84 - 24P - 3.75R = 0$

$3.75R + 24P = 84$ ②

Cont...

① × 24 : $600R - 168P = 6912$ +
② × 7 : $26.25R + 168P = 588$
add : $626.25R = 7500$
$R = 11.97...$

→ ②: $3.75(11.97...) + 24P = 84$
$44.91... + 24P = 84$
$24P = 39.089...$
$P = 1.63\ N$

DO NOT WRITE IN THIS MARGIN

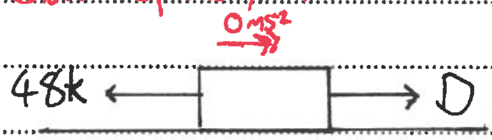
7 A car has mass 1200 kg. When the car is travelling at a speed of $v\,\mathrm{m\,s^{-1}}$, there is a resistive force of magnitude kv N. The maximum power of the car's engine is 92.16 kW.

(a) The car travels along a straight level road.

(i) The car has a greatest possible constant speed of $48\,\mathrm{m\,s^{-1}}$.

Show that $k = 40$. [1]

At constant speed, $a = 0$:



$D - 48k = 0$

$D = 48k$

Power $= Dv$

$92\,160 = 48k \times 48$

$92\,160 = 2304k$

$k = 40$ QED

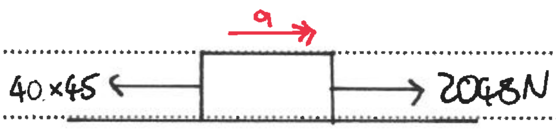
(ii) At an instant when its speed is $45\,\mathrm{m\,s^{-1}}$, find the greatest possible acceleration of the car. [3]

Greatest possible acceleration is when engine is at max. power:

Power $= Dv$

$92\,160 = D \times 45$

$D = 2048$ N



$R(\rightarrow)$: $F = ma$

$2048 - 1800 = 1200a$

$248 = 1200a$

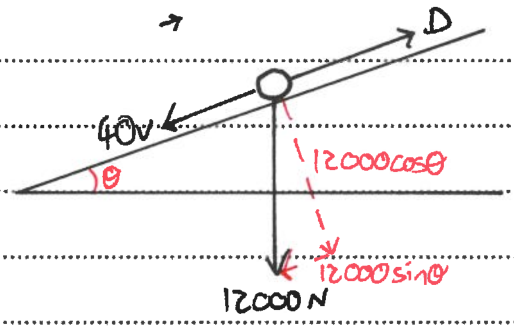
$a = \frac{31}{150}\,\mathrm{ms^{-2}}$

$\sin\theta = 0.15$

(b) The car now travels at a constant speed up a hill inclined at an angle of $\sin^{-1}0.15$ to the horizontal.

Find the greatest possible speed of the car going up the hill. [4]

At highest speed, engine is at max. and can't accelerate any more, so $a = 0$.



$R(\nearrow)$: $D - 12000\sin\theta - 40v = ma$ $\quad =0$

$D - 12000 \times 0.15 - 40v = 0$

$D - 40v = 1800$ ①

$\text{Power} = D \times v$

$92160 = Dv$

$D = \dfrac{92160}{v}$ ②

Sub. ② → ①: $\dfrac{92160}{v} - 40v = 1800$ $\quad \times v$

$92160 - 40v^2 = 1800v$

$40v^2 + 1800v - 92160 = 0$ $\quad \div 40$

$v^2 + 45v - 2304 = 0$

$v = \dfrac{-45 \pm \sqrt{45^2 - 4(1)(-2304)}}{2(1)}$

$v = \underline{30.5\text{ ms}^{-1}}$ or $v = -75.5\text{ ms}^{-1}$ ✗

uphill downhill

DO NOT WRITE IN THIS MARGIN

8 A particle P moves in a straight line, passing through a point O with velocity 4.2 m s^{-1}. At time t s after P passes O, the acceleration, $a\text{ m s}^{-2}$, of P is given by $a = 0.6t - 2.7$.

Find the distance P travels between the times at which it is at instantaneous rest. [7]

Find velocity: $V = \int (0.6t - 2.7)\,dt$

$V = 0.3t^2 - 2.7t + C$

$V = 4.2$ at $t = 0$:

$4.2 = 0 - 0 + C$

$\rightarrow \underline{V = 0.3t^2 - 2.7t + 4.2}$

Find when P is at rest ($v = 0$):

$0.3t^2 - 2.7t + 4.2 = 0 \quad \div 0.3$

$t^2 - 9t + 14 = 0$

$(t - 7)(t - 2) = 0$

$\underline{t = 7}$ or $\underline{t = 2}$

Find displacement: $S = \int (0.3t^2 - 2.7t + 4.2)\,dt$

$= 0.1t^3 - 1.35t^2 + 4.2t + C$

$S = 0$ at $t = 0$:

$0 = 0 - 0 + 0 + C$

$\rightarrow \underline{S = 0.1t^3 - 1.35t^2 + 4.2t}$

Find displacement at $t = 2$ and $t = 7$:

$\underline{t = 2}$:

$S = 0.1(2)^3 - 1.35(2)^2 + 4.2(2)$

$= 0.8 - 5.4 + 8.4$

$= \underline{3.8\text{ m}}$

cont..

$t=7$:

$$s = 0.1(7)^3 - 1.35(7)^2 + 4.2(7)$$
$$= 34.3 - 66.15 + 29.4$$
$$= -2.45$$

Distance travelled between $t=2$ and $t=7$:

$$d = 3.8 - -2.45$$
$$= 6.25\text{ m}$$