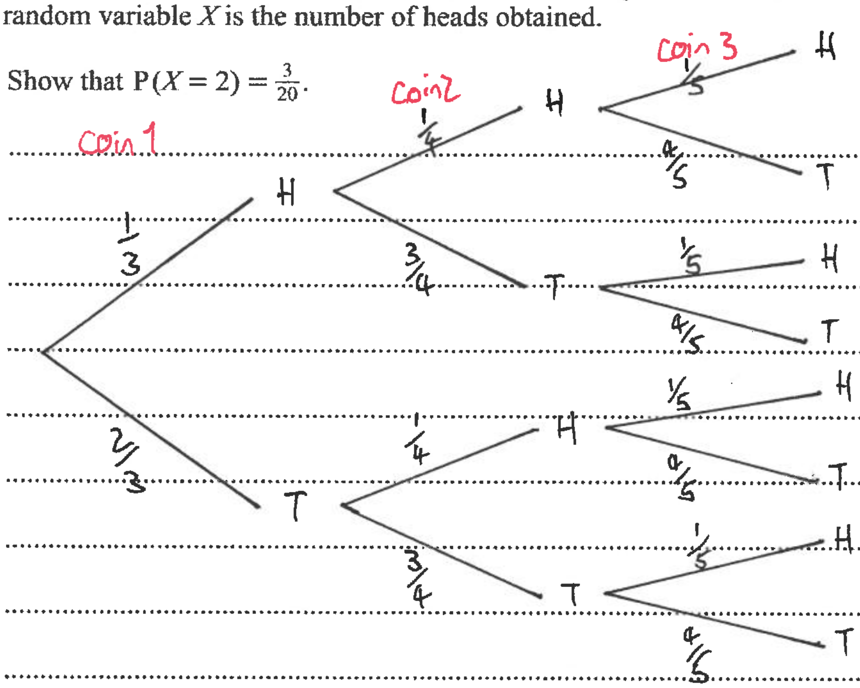
1 Jacob throws three coins at the same time.
The first coin is biased so that the probability of obtaining a head when it is thrown is $\frac{1}{3}$.
The second coin is biased so that the probability of obtaining a head when it is thrown is $\frac{1}{4}$.
The third coin is biased so that the probability of obtaining a head when it is thrown is $\frac{1}{5}$.
The random variable X is the number of heads obtained.

(a) Show that $P(X = 2) = \frac{3}{20}$. [1]



✓ $\frac{1}{3} \times \frac{1}{4} \times \frac{4}{5} = \frac{4}{60}$

✓ $\frac{1}{3} \times \frac{3}{4} \times \frac{1}{5} = \frac{3}{60}$

✓ $\frac{2}{3} \times \frac{1}{4} \times \frac{1}{5} = \frac{2}{60}$

$\frac{4}{60} + \frac{3}{60} + \frac{2}{60} = \frac{9}{60}$

$= \frac{3}{20}$

(b) Draw up the probability distribution table for X. [3]

$P(X=0)$: $\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} = \frac{24}{60}$

$P(X=1)$: $\frac{1}{3} \times \frac{3}{4} \times \frac{4}{5} + \frac{2}{3} \times \frac{1}{4} \times \frac{4}{5} + \frac{2}{3} \times \frac{3}{4} \times \frac{1}{5} = \frac{26}{60}$

$P(X=2)$: $\frac{9}{60}$ (from part a)

$P(X=3)$: $\frac{1}{3} \times \frac{1}{4} \times \frac{1}{5} = \frac{1}{60}$

x	0	1	2	3
$P(X=x)$	$24/60$	$26/60$	$9/60$	$1/60$

(c) Given that $E(X) = \frac{47}{60}$, find $Var(X)$. [2]

$Var(X) = 0^2 \times \frac{24}{60} + 1^2 \times \frac{26}{60} + 2^2 \times \frac{9}{60} + 3^2 \times \frac{1}{60} - (E(x))^2$

$= 0 + \frac{26}{60} + \frac{36}{60} + \frac{9}{60} - \left(\frac{47}{60}\right)^2$

$= \frac{71}{60} - \frac{2209}{3600}$

$= \frac{2051}{3600}$

2 Last year, an online store sold a large number of computers. 55% of the computers were made by company F, 30% were made by company G and 15% were made by company H.

A random sample of 3 customers who each bought a computer from this store is chosen.

(a) Find the probability that the 3 customers bought computers all made by different companies. [1]

Customer

1	2	3
F	G	H
F	H	G
G	F	H
G	H	F
H	F	G
H	G	F

$0.55 \times 0.3 \times 0.15 = 0.02475$

$0.55 \times 0.15 \times 0.3 = 0.02475$

all the same!

$\rightarrow 0.02475 \times 6 = \underline{0.1485}$

A random sample of 12 customers who each bought a computer from this store is chosen.

(b) Find the probability that fewer than 10 of these customers bought a computer made by company F. [3]

$F \sim B(12, 0.55)$

$P(B<10) = 1 - [P(10) + P(11) + P(12)]$

$= 1 - [{}^{12}C_{10} \times 0.55^{10} \times 0.45^{2} + {}^{12}C_{11} \times 0.55^{11} \times 0.45^{1} + {}^{12}C_{12} \times 0.55^{12} \times 0.45^{0}]$

$= 1 - (0.03385.. + 0.007522.. + 0.0007662..)$

$= \underline{0.958}$

A random sample of 140 customers who each bought a computer from this store is chosen.

(c) Use a suitable approximation to find the probability that more than 24 of these customers bought a computer made by company *H*. [5]

$X \sim B(140, 0.15)$

$\mu = 140 \times 0.15$ $\quad\quad$ $\sigma^2 = 21 \times 0.85$

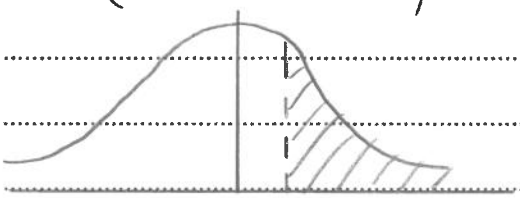
$= 21$ $\quad\quad$ $= 17.85$

$X \sim N(21, 17.85)$

$P(X > 24) \rightarrow P(X > 24.5)$ (continuity correction)

$P\left(Z > \dfrac{24.5 - 21}{\sqrt{17.85}}\right)$

$= P(Z > 0.828)$



0.828

$= 1 - \Phi(0.828)$

$= 1 - 0.7961$

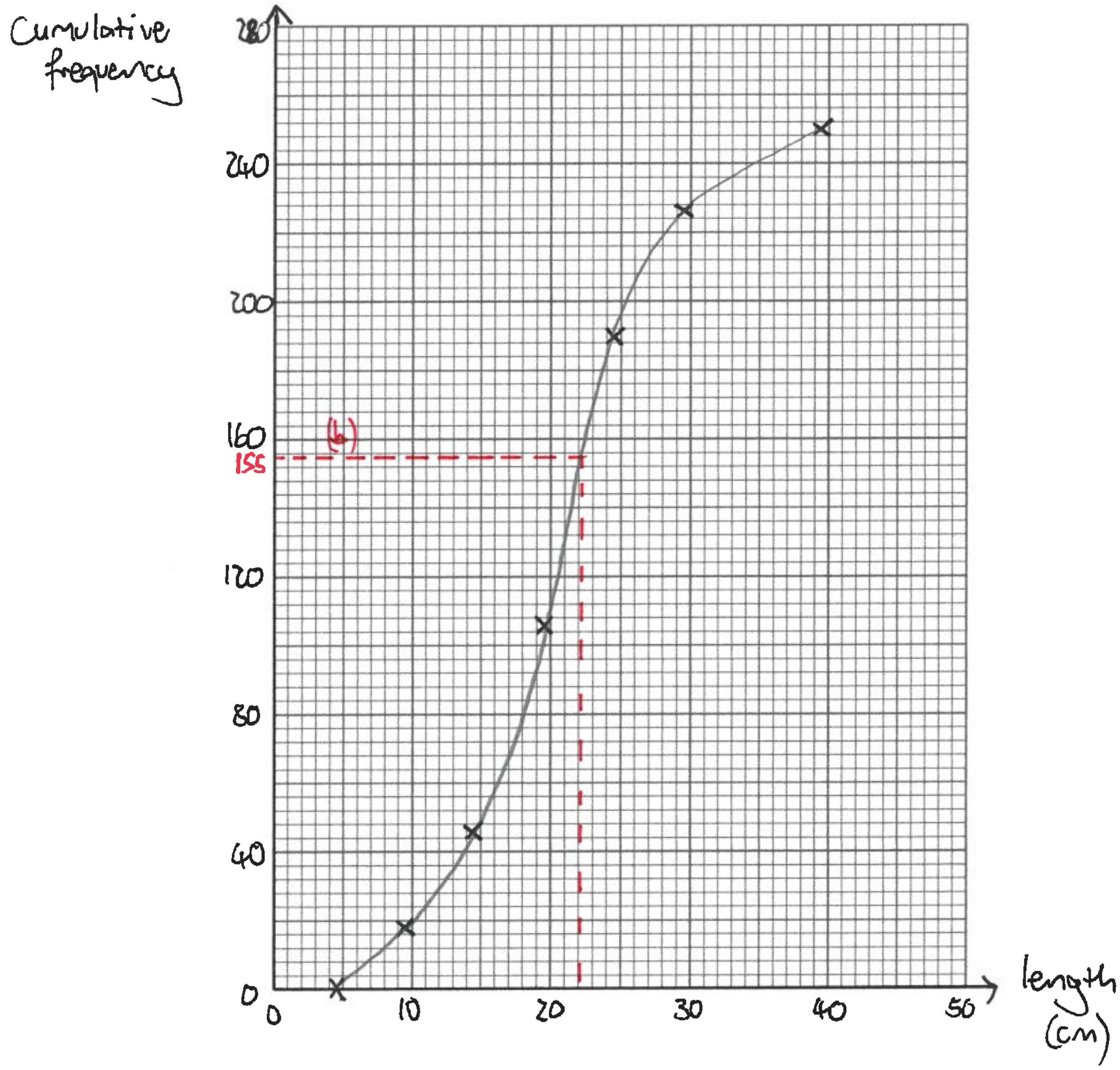
$= 0.2039$

3 The lengths of 250 leaves of a certain type of plant are measured, correct to the nearest centimetre. The results are summarised in the table below.

4.5 9.5 14.5 19.5 24.5 29.5 39.5

Length (cm)	5 – 9	10 – 14	15 – 19	20 – 24	25 – 29	30 – 39
Frequency	18	28	60	72	48	24
c.f.	18	46	106	178	226	250

(a) On the grid below, draw a cumulative frequency graph to illustrate this information. [4]



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(b) 38% of these leaves are of length k cm or more.

Use your graph to find an estimate for k. [2]

38% are more than k, so 62% are less than k:

$0.62 \times 250 = 155$

$k \approx 22\text{cm}$

(c) Calculate an estimate for the mean length of these 250 leaves. [3]

Mid-point (x)	Frequency (f)	$f \times x$
7	18	126
12	28	336
17	60	1020
22	72	1584
27	48	1296
34.5	24	828
	$\Sigma f = 250$	$\Sigma fx = 5190$

$$\bar{x} = \frac{\Sigma fx}{\Sigma f}$$

$$= \frac{5190}{250}$$

$$= 20.76$$

4 Eddie has 16 toy cars, of which 8 are white, 5 are black and 3 are silver. He places all the cars in a bag and selects three of them at random, without replacement.

(a) Find the probability that all three cars are the same colour. [3]

pick 3 whites from the 8

$P(WWW) = \frac{^{8}C_{3}}{^{16}C_{3}}$ pick any 3 cars from all 16

$= \frac{56}{560}$

$P(BBB) = \frac{^{5}C_{3}}{^{16}C_{3}}$

$= \frac{10}{560}$

$P(SSS) = \frac{^{3}C_{3}}{^{16}C_{3}}$

$= \frac{1}{560}$

P(all the same colour) $= \frac{56}{560} + \frac{10}{560} + \frac{1}{560}$

$= \frac{67}{560}$

(b) Find the probability that, when the 3 cars are selected, at least one car is white and at least one car is black. [4]

1 black from 5

W W B = ${}^{8}C_{2} \times {}^{5}C_{1} = 140$

2 whites from 8

W B B = ${}^{8}C_{1} \times {}^{5}C_{2} = 80$

W B S = ${}^{8}C_{1} \times {}^{5}C_{1} \times {}^{3}C_{1} = 120$

Probability $= \dfrac{140 + 80 + 120}{{}^{16}C_{3}}$

$= \dfrac{340}{560}$

$= \dfrac{17}{28}$

5 The mass of peaches sold per day in a supermarket is normally distributed with mean 65.8 kg and standard deviation 9.6 kg.

(a) Find the probability that the mass of peaches sold on any given day is between 56 kg and 75 kg. [3]

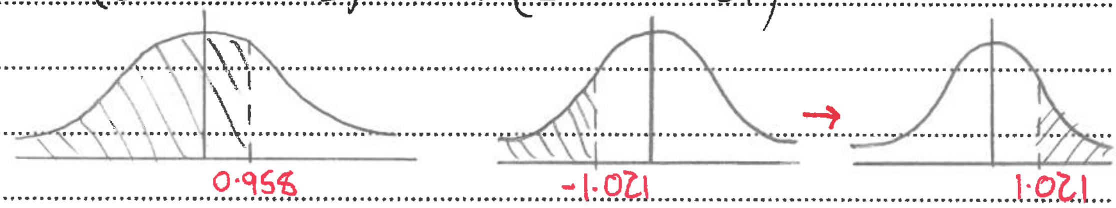
$M \sim N(65.8, 9.6^2)$

$P(56 < M < 75)$

$= P\left(\frac{56 - 65.8}{9.6} < Z < \frac{75 - 65.8}{9.6}\right)$

$= P(-1.021 < Z < 0.958)$

$= P(Z < 0.958) - P(Z < -1.021)$



$= \Phi(0.958) - (1 - \Phi(1.021))$

$= 0.8309 - (1 - 0.8463)$

$= 0.8309 - 0.1537$

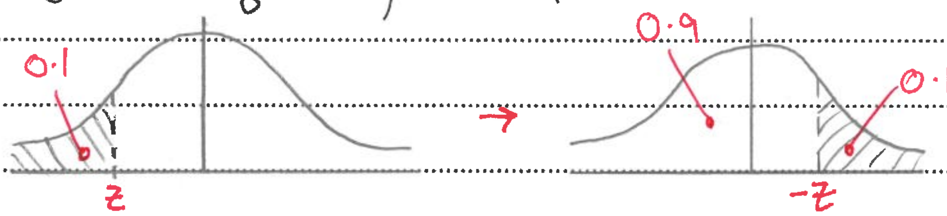
$= \underline{0.6772}$

The mass of cherries sold per day in a supermarket is normally distributed with mean 72.4 kg and standard deviation σ kg. It is known that on 10% of days less than 59.1 kg of cherries are sold.

(b) Find the value of σ. [3]

$P(C < 59.1) = 0.1$

$P\left(Z < \frac{59.1 - 72.4}{\sigma}\right) = 0.1$



critical value $\rightarrow 0.9 = \Phi(1.282)$

so $z = -1.282$

$\frac{59.1 - 72.4}{\sigma} = -1.282$

$-13.3 = -1.282\sigma$

$\sigma = 10.4$

The supermarket is open 7 days a week.

(c) Find the probability that, in a randomly chosen week, the first day on which less than 59.1 kg of cherries are sold is the fifth day of the week. [1]

$C \sim \text{Geo}(0.1)$

$P(X = 5) = 0.1 \times 0.9^4$

$= 0.06561$

(d) Find the probability that, in a randomly chosen week, the first day on which less than 59.1 kg of cherries are sold is before the fifth day of the week. [2]

$P(X < 5) = P(X \leqslant 4)$

$= 1 - 9^4$ ← probability of 4 failures

$= 1 - 0.9^4$

$= 0.3439$

6 Alissa has 10 different books from the series Squares and Circles. The books look similar except for their colour. There are 3 blue books, 2 red books, 2 yellow books, 1 orange book, 1 purple book and 1 green book.

Alissa places the books in a row on her shelf. She is only interested in the arrangement of the colours.

(a) How many different colour arrangements are there of the 10 books? [1]

$$\frac{10!}{\underset{\text{3 blues}}{3!} \times \underset{\text{2 reds}}{2!} \times \underset{\text{2 yellows}}{2!}} = 151\,200$$

(b) How many different colour arrangements are there of the 10 books in which the 3 blue books are together, but the 2 yellow books are **not** next to each other? [2]

With 3 blues together:

one object → B B B

$$\frac{8!}{2! \times 2!} = 10\,080$$

2 reds → 2! × 2! ← 2 yellows

With 3 blues together AND 2 yellows together:

B B B   Y Y

$$\frac{7!}{2!} = 2520$$

2 reds → 2!

Blues together, but not yellows: $10\,080 - 2520$

$= 7560$

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(c) How many different colour arrangements are there of the 10 books with exactly 4 books between the 2 yellow books? [3]

Pick 4 books to go between the yellows and permutate:

Y __ __ __ __ Y

fixed $\underbrace{\qquad\qquad\qquad}_{^8P_4}$ fixed

$^8P_4$ ← 4 from 8 remaining books

Treat this as one object and permutate with remaining 4 books:

one object → Y____Y __ __ __ __

$$\frac{5! \times {}^8P_4}{3! \times 2!} = \underline{16\,800}$$

3 blues → $3!$ ← 2 reds $2!$

Alissa selects 4 books from her 10 different books from the series Squares and Circles.

(d) Find the number of different selections if the 4 books include at least 1 red book, at most 1 blue book and exactly 1 yellow book. [4]

Careful: NOT different colours anymore, but different books!

This means the two yellow books are different, same w/ other colours.

Y R B __ $\quad {}^2C_1 \times {}^2C_1 \times {}^3C_1 \times {}^3C_1 = 36$

Y R B 1 from O/P/G

Y R R B $\quad {}^2C_1 \times {}^2C_2 \times {}^3C_1 = 6$

Y R __ __ $\quad {}^2C_1 \times {}^2C_1 \times {}^3C_2 = 12$

Y R R __ $\quad {}^2C_1 \times {}^2C_2 \times {}^3C_1 = 6$

$36 + 6 + 12 + 6 = \underline{60}$