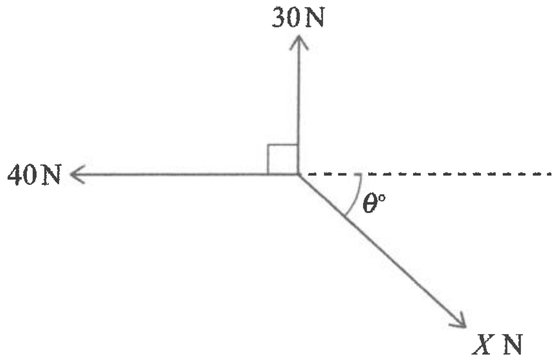
1



Three coplanar forces of magnitudes 40 N, 30 N and X N act at a point in the directions shown in the diagram.

Given that the forces are in equilibrium, find the values of θ and X. [4]

$R(\rightarrow): \quad X\cos\theta - 40 = 0$

$X\cos\theta = 40 \quad (1)$

$R(\uparrow): \quad 30 - X\sin\theta = 0$

$X\sin\theta = 30 \quad (2)$

$(2) \div (1): \quad \dfrac{X\sin\theta}{X\cos\theta} = \dfrac{30}{40}$

$\tan\theta = \dfrac{3}{4}$

$\theta = 36.9°$ STO

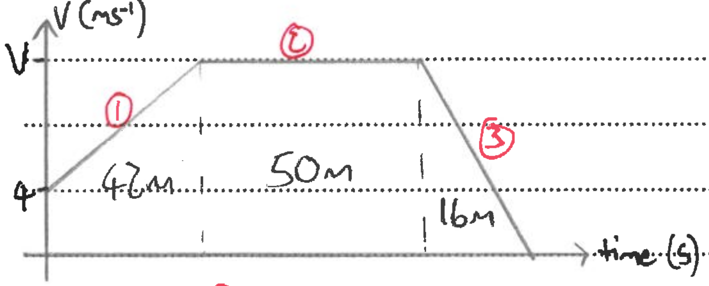
$\rightarrow (1): \quad X\cos(36.869...) = 40$

$X = \dfrac{40}{\cos(36.869..)}$

$X = 50\text{ N}$

2 A cyclist is travelling along a straight horizontal road at a speed of $4\,\text{m s}^{-1}$ when she passes a point O. She accelerates at a constant rate for a distance of 42 m, reaching a speed of $V\,\text{m s}^{-1}$. She maintains the speed of $V\,\text{m s}^{-1}$ for 50 m and then decelerates at $2\,\text{m s}^{-2}$ before coming to rest. The distance travelled while decelerating is 16 m.

(a) Find the value of V. [2]



Using section ③:

$s = 16$	$v^2 = u^2 + 2as$
$u:$	$0^2 = u^2 + 2(-2)(16)$
$v: 0$	$0 = u^2 - 64$
$a: -2$	$u^2 = 64$
$t:$	$u = 8\,\text{ms}^{-1}$

(b) Find the total time for which she is in motion from the instant that she passes O. [3]

①: $\text{Area} = \frac{1}{2}(4+8) \times t$

$42 = \frac{1}{2} \times 12 \times t$

$42 = 6t$

$t = 7\text{s}$

② $\text{Area} = 8 \times t$

$50 = 8t$

$t = 6.25\text{s}$

③ $\text{Area} = \frac{1}{2} \times 8 \times t$

$16 = 4t$

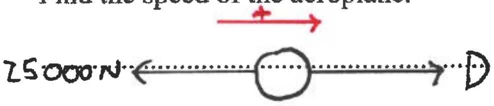
$t = 4\text{s}$

$\text{Total} = 17.25\text{s}$

3 An aeroplane is flying at a constant speed.

(a) The aeroplane is flying horizontally. The aeroplane's engines are producing a constant power of 5500 kW, and the aeroplane experiences a constant horizontal resistance force of 25 kN.

Find the speed of the aeroplane. [2]



$R(\rightarrow)\quad F = m\underbrace{a}_{=0}$ (constant speed)

$D - 25000 = 0$

$\underline{D = 25000\,N}$

$\text{Power} = D \times V$

$5\,500\,000 = 25000\,V$

$\underline{V = 220\,ms^{-1}}$

(b) The aeroplane then ascends 300 m in 50 s, while maintaining the same speed. The resistance force is no longer constant, and the work done against the resistance force in ascending the 300 m is 270 000 kJ. The mass of the aeroplane is 60 000 kg.

Find the average power of the aeroplane's engines. [4]

$\text{Work}_{IN} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + \text{Work}_{OUT}$

$\text{Work}_{IN} + \frac{1}{2} \times 60000 \times 220^2 + 0 = \frac{1}{2} \times 60000 \times 220^2 + 60000 \times 10 \times 300 + 270\,000\,000$

$\text{Work}_{IN} + \cancel{1\,452\,000\,000} = \cancel{1\,452\,000\,000} + 180\,000\,000 + 270\,000\,000$

$\text{Work}_{IN} = 450\,000\,000\,J$

$\text{Power} = \dfrac{\text{Work done}}{\text{time}}$

$= \dfrac{450\,000\,000}{50}$

$= \underline{9\,000\,000\,W}$

During the subsequent motion, B does **not** reach the pulley. When A reaches the ground, it comes to rest.

(b) Given that the greatest height of B above the ground is 1.2 m, find the value of x. [3]

Find an expression for velocity of B when A hits floor:

$+\uparrow$

$s = x$	$v^2 = u^2 + 2as$
$u = 0$	$v^2 = 0^2 + 2 \times 5 \times x$
$v =$	$v^2 = 10x$
$a = 5$	$v = \sqrt{10x}$
$t =$	

Now find an expression for how much higher B goes once string goes slack (ie. $a = -10\,\text{ms}^{-2}$)

$s =$	$v^2 = u^2 + 2as$
$u = \sqrt{10x}$	$0^2 = (\sqrt{10x})^2 + 2 \times -10 \times s$
$v = 0$	$0 = 10x - 20s$
$a = -10$	$20s = 10x$
$t =$	$s = \frac{1}{2}x$

B starts x m off the ground: x

It moves up x m before A hits ground: x

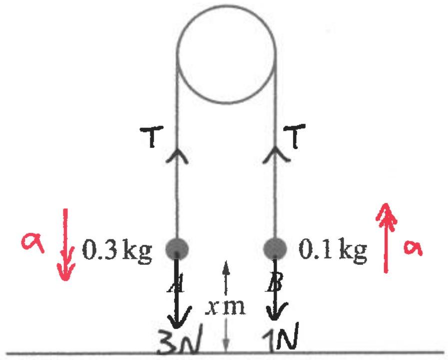
Then it moves $\frac{1}{2}x$ m higher when the string is slack: $\frac{1}{2}x$

$$x + x + \frac{1}{2}x = 1.2\text{m}$$

$$2.5x = 1.2$$

$$x = \underline{0.48\text{m}}$$

4



Two particles A and B have masses 0.3 kg and 0.1 kg respectively. The particles are attached to the ends of a light inextensible string. The string passes over a fixed smooth pulley, and the particles hang vertically below the pulley. Both particles are initially at a height of x m above horizontal ground (see diagram). The system is released from rest.

(a) Find the tension in the string and the acceleration of the particles. [4]

A: R(↓): $3 - T = ma$

$3 - T = 0.3a$ ①

B: R(↑): $T - 1 = ma$

$T - 1 = 0.1a$ ②

① + ②: $3 - 1 = 0.4a$

$2 = 0.4a$

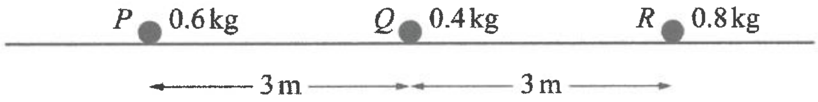
$a = 5 \text{ ms}^{-2}$

→ ②: $T - 1 = 0.1 \times 5$

$T - 1 = 0.5$

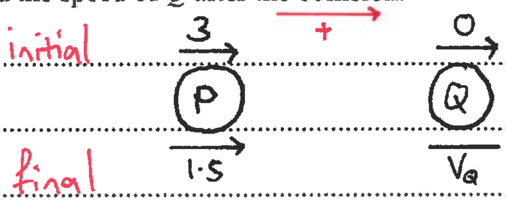
$T = 1.5 \text{N}$

5



Three particles P, Q and R, of masses 0.6 kg, 0.4 kg and 0.8 kg respectively, are at rest in a straight line on a smooth horizontal plane. The distance from P to Q is 3 m, and the distance from Q to R is also 3 m (see diagram). P is projected directly towards Q with speed $3\,\text{m}\,\text{s}^{-1}$. After P and Q collide, P continues to move in the same direction with speed $1.5\,\text{m}\,\text{s}^{-1}$.

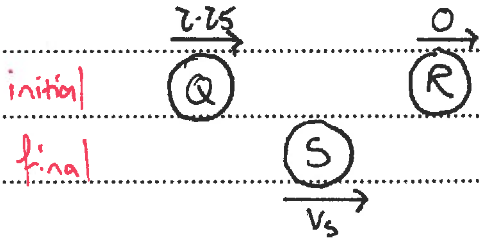
(a) Find the speed of Q after the collision. [2]



$$M_P U_P + M_Q U_Q = M_P V_P + M_Q V_Q$$
$$0.6 \times 3 + 0 = 0.6 \times 1.5 + 0.4 V_Q$$
$$1.8 = 0.9 + 0.4 V_Q$$
$$0.9 = 0.4 V_Q$$
$$V_Q = \underline{\underline{2.25\,\text{ms}^{-1}}}$$

In the subsequent collision between Q and R, these particles coalesce.

(b) Find the speed of the combined particle after this collision. [1]



$$M_Q U_Q + M_R U_R = M_S V_S$$
$$0.4 \times 2.25 + 0 = 1.2 \times V_S$$
$$0.9 = 1.2 V_S$$
$$V_S = \underline{\underline{0.75\,\text{ms}^{-1}}}$$

(c) Find the time that it takes from when P is initially projected until the instant at which P collides with the combined particle. [4]

Time until P and Q collide:

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

$$= \frac{3}{3}$$

$$= 1\text{s}$$

Time for Q to reach R:

$$\text{time} = \frac{3}{2.25}$$

$$= \tfrac{4}{3}\text{s}$$

Where is P when Q and R collide?

$$\text{distance} = 1.5 \times \tfrac{4}{3}$$

$= 2\text{m}$ ← to the right of where Q started, so 1m behind R.

So P has 1m more to cover than the combined particle(s).

For them to collide, they need to be in the same place at the same time:

P:

$$\text{distance} = \text{speed} \times \text{time}$$

$$x + 1 = 1.5t \quad (1)$$

Q:

$$\text{distance} = \text{speed} \times \text{time}$$

$$x = 0.75t \quad (2)$$

sub. (2) into (1):

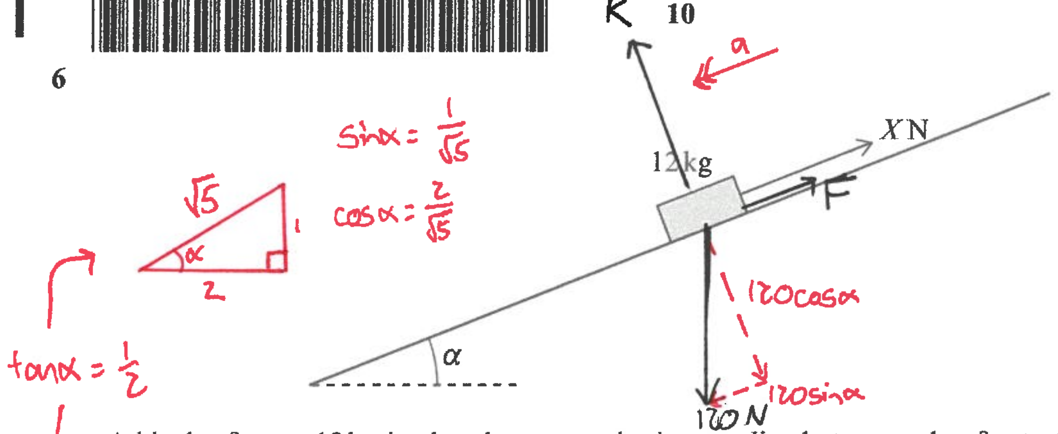
$$0.75t + 1 = 1.5t$$

$$1 = 0.75t$$

$$t = \tfrac{4}{3}\text{s}$$

$$\text{Total Time} = 1 + \tfrac{4}{3} + \tfrac{4}{3} = \tfrac{11}{3}\text{ s}$$

10



A block of mass 12 kg is placed on a rough plane inclined at an angle of α to the horizontal, where $\alpha = \tan^{-1}0.5$. A force of X N is applied to the block, directly up the plane (see diagram). The coefficient of friction between the block and the plane is μ.

(a) It is given that $\mu = 0.15$ and $X = 20$.

Find the time that it takes for the block to move 2 m down the plane from rest. [6]

Since it's moving down the plane, friction must be acting up the plane.

$R(\nwarrow)$: $R - 120\cos\alpha = 0$

$R = 120 \times \frac{2}{\sqrt{5}}$

$= 48\sqrt{5}$ N

$R(\swarrow)$: $F = ma$

$120\sin\alpha - X - F = 12a$

$120 \times \frac{1}{\sqrt{5}} - 20 - \mu R = 12a$

$24\sqrt{5} - 20 - 0.15 \times 48\sqrt{5} = 12a$

$17.565\ldots = 12a$

$a = 1.46\ldots \text{ ms}^{-2}$ STO

$+\swarrow$

$s = 2$	$s = ut + \frac{1}{2}at^2$
$u = 0$	$2 = 0 + \frac{1}{2} \times 1.46.. \times t^2$
$v =$	$2 = 0.7319\ldots t^2$
$a = 1.46\ldots$	$t^2 = 2.7325\ldots$
$t =$	$t = 1.65$ s

(b) It is given instead that $\mu \neq 0.15$ and that when $X = 10$, the block is on the point of moving **down** the plane.

Find the value of μ and the value of X for which the block is on the point of moving **up** the plane. [4]

On the point of moving down (F acting up):

$R(\swarrow)$: $F = ma$

$120\sin\alpha - X - F = 0$ ← not moving

$24\sqrt{5} - 10 - \mu \times 120\cos\alpha = 0$

$24\sqrt{5} - 10 - 48\sqrt{5}\mu = 0$

$48\sqrt{5}\mu = -10 + 24\sqrt{5}$

$\mu = \dfrac{-10 + 24\sqrt{5}}{48\sqrt{5}}$

$\underline{\mu = 0.407}$ STO

On the point of moving up (F acting down):

$R(\nearrow)$: $F = ma$

$X - 120\sin\alpha - F = 0$

$X - 24\sqrt{5} - \mu R = 0$

$X - 24\sqrt{5} - 0.407.. \times 48\sqrt{5} = 0$

$X = 24\sqrt{5} + 0.407.. \times 48\sqrt{5}$

$= \underline{97.3\text{ N}}$

7 A particle moves in a straight line. The velocity $v\,\mathrm{m\,s^{-1}}$ of the particle t s after leaving a fixed point O is given by $v = k(20 + pt - 6t^2)$, where k and p are constants. The acceleration of the particle at $t = 1$ is $42\,\mathrm{m\,s^{-2}}$, and the displacement of the particle from O at $t = 1$ is 93 m.

(a) Show that $k = 3$ and $p = 26$. [6]

$(v = 20k + kpt - 6kt^2)$

Acceleration: $a = \frac{dv}{dt}$

$a = kp - 12kt$

$a = 42$ when $t = 1$:

$42 = kp - 12k$ ①

Displacement: $s = \int (20k + kpt - 6kt^2)\,dt$

$s = 20kt + \frac{1}{2}kpt^2 - 2kt^3 + C$

at $t = 0, s = 0$: $0 = 0 + 0 - 0 + C$

$\rightarrow s = 20kt + \frac{1}{2}kpt^2 - 2kt^3$

at $t = 1, s = 93$:

$93 = 20k + \frac{1}{2}kp - 2k$

$93 = \frac{1}{2}kp + 18k$ $\times 2$

$186 = kp + 36k$ ②

② - ①:

$144 = 48k$

$\underline{k = 3}$

$\rightarrow$ ①: $42 = 3p - 36$

$78 = 3p$

$\underline{p = 26}$

(b) Find the distance moved by the particle between the time at which its acceleration is zero and the time at which its velocity is zero. [5]

$a = 78 - 36t$ (using values of k and p from part (a))

$a = 0$:

$0 = 78 - 36t$

$36t = 78$

$t = \frac{13}{6}$ s

$V = 60 + 78t - 18t^2$

$V = 0$:

$-18t^2 + 78t + 60 = 0 \quad \div -6$

$3t^2 - 13t - 10 = 0$

$3t^2 - 15t + 2t - 10 = 0$

$3t(t-5) + 2(t-5) = 0$

$(3t+2)(t-5) = 0$

$t = -\frac{2}{3}$ ✗ or $t = 5$ ✓

$$S = \int_{\frac{13}{6}}^{5} (60 + 78t - 18t^2)\,dt$$

$$= \left[60t + 39t^2 - 6t^3\right]_{\frac{13}{6}}^{5}$$

$$= \left[60\times5 + 39\times5^2 - 6\times5^3\right] - \left[60\times\frac{13}{6} + 39\times\left(\frac{13}{6}\right)^2 - 6\times\left(\frac{13}{6}\right)^3\right]$$

$$= \left[300 + 975 - 750\right] - \left[130 + \frac{2197}{12} - \frac{2197}{36}\right]$$

$$= 525 - \frac{4537}{18}$$

$$= 273 \text{ m}$$