1 Expand $(9-3x)^{\frac{1}{2}}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [4]

$(9-3x)^{\frac{1}{2}} = 9^{\frac{1}{2}}\left(1-\frac{x}{3}\right)^{\frac{1}{2}}$

$= 3\left(1-\frac{x}{3}\right)^{\frac{1}{2}}$

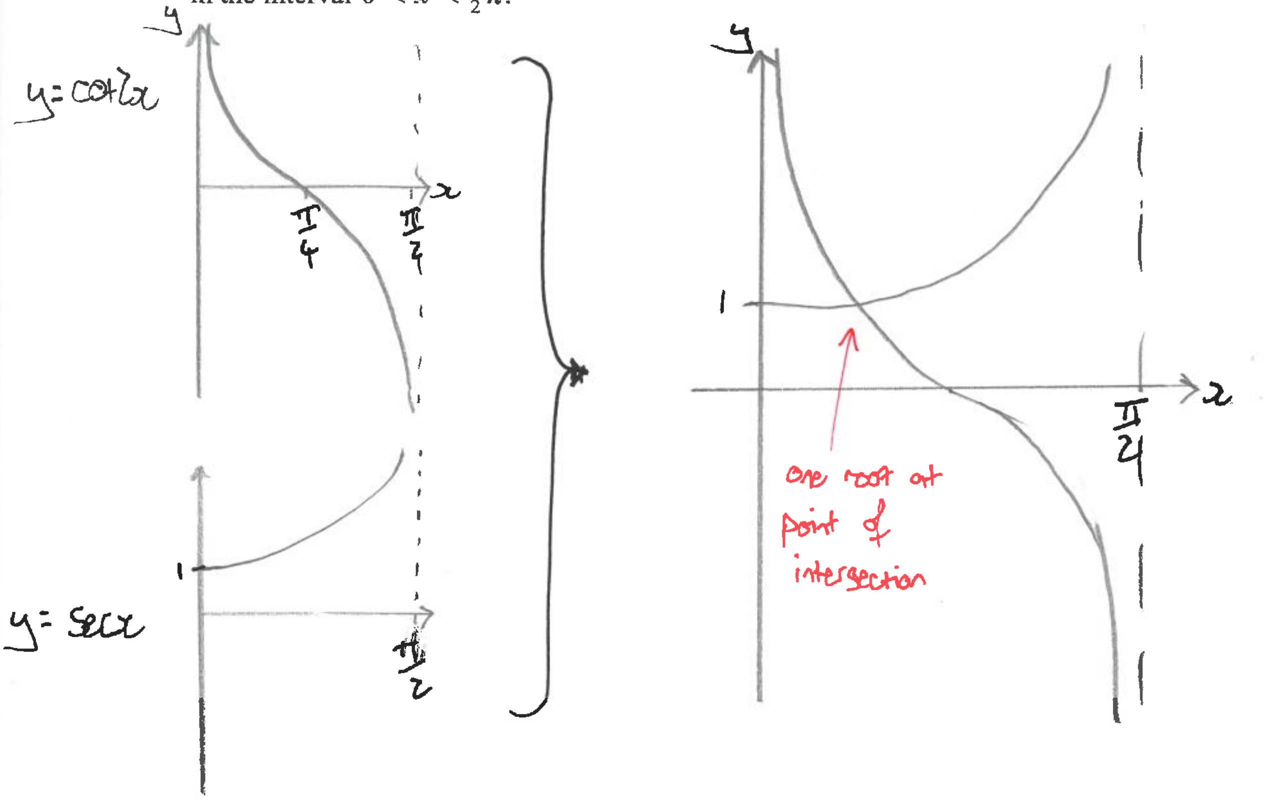
$= 3\left[1+\frac{1}{2}\left(-\frac{x}{3}\right)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(-\frac{x}{3}\right)^2+\ldots\right]$

$= 3\left[1-\frac{1}{6}x-\frac{1}{6}\times\frac{x^2}{9}+\ldots\right]$

$= 3\left[1-\frac{1}{6}x-\frac{1}{54}x^2+\ldots\right]$

$= 3-\frac{1}{2}x-\frac{1}{18}x^2$

2 **(a)** By sketching a suitable pair of graphs, show that the equation $\cot 2x = \sec x$ has exactly one root in the interval $0 < x < \frac{1}{2}\pi$. [2]



(b) Show that if a sequence of real values given by the iterative formula

$$x_{n+1} = \frac{1}{2}\tan^{-1}(\cos x_n)$$

converges, then it converges to the root in part **(a)**. [1]

$$x = \frac{1}{2}\tan^{-1}(\cos x)$$

$$2x = \tan^{-1}(\cos x)$$

$$\tan 2x = \cos x$$

$\div \cos x$ $\quad\quad$ $\div \cos x$

$$\frac{\tan 2x}{\cos x} = 1$$

$\div \tan 2x$ $\quad\quad$ $\div \tan 2x$

$$\frac{1}{\cos x} = \frac{1}{\tan 2x}$$

$$\sec x = \cot 2x \quad \text{QED}$$

3 The square roots of $6-8\text{i}$ can be expressed in the Cartesian form $x+\text{i}y$, where x and y are real and exact.

By first forming a quartic equation in x or y, find the square roots of $6-8\text{i}$ in exact Cartesian form. [5]

$\sqrt{6-8i} = x + iy$ square both sides

$6-8i = (x+iy)^2$

$6-8i = x^2 + 2xyi - y^2$

Real component: Imaginary:

$6 = x^2 - y^2$ (1) $\quad -8 = 2xy$

$y = \frac{-4}{x}$ (2)

(2) → (1):

$6 = x^2 - \left(\frac{-4}{x}\right)^2$

$6 = x^2 - \frac{16}{x^2}$

$\times x^2 \quad \times x^2 \quad \times x^2$

$6x^2 = x^4 - 16$

$x^4 - 6x^2 - 16 = 0$

Let $Y = x^2$:

$Y^2 - 6Y - 16 = 0$

$(Y-8)(Y+2) = 0$

$Y = 8 \qquad Y = -2$

$x^2 = 8 \qquad x^2 = -2$ × No real sol^ns

$x = \pm\sqrt{8}$

$= \pm 2\sqrt{2}$

→ (2): $x = 2\sqrt{2}$:

$y = \frac{-4}{2\sqrt{2}}$

$= \frac{-2}{\sqrt{2}} = -\sqrt{2}$

→ $x = -2\sqrt{2}$:

$y = \frac{-4}{-2\sqrt{2}}$

$= \frac{2}{\sqrt{2}}$

$= \sqrt{2}$

→ $2\sqrt{2} - \sqrt{2}i$ or $-2\sqrt{2} + \sqrt{2}i$

4 Solve the equation $5^x = 5^{x+2} - 10$. Give your answer correct to 3 decimal places. [3]

$5^x = 5^x \times 5^2 - 10$

$5^x = 5^x \times 25 - 10$

Let $Y = 5^x$:

$Y = 25Y - 10$

$24Y = 10$

$Y = \frac{5}{12}$

$5^x = \frac{5}{12}$

$\log_5 5^x = \log_5\left(\frac{5}{12}\right)$

$x = \log_5\left(\frac{5}{12}\right)$

$= -0.544$

5 (a) The complex number u is given by

$$u = \frac{(\cos\frac{1}{7}\pi + i\sin\frac{1}{7}\pi)^4}{\cos\frac{1}{7}\pi - i\sin\frac{1}{7}\pi}.$$

Find the exact value of arg u. [2]

TOP: When multiplying complex numbers in mod-arg form, we add the arguments: $\frac{\pi}{7} + \frac{\pi}{7} + \frac{\pi}{7} + \frac{\pi}{7} = \frac{4\pi}{7}$

$\rightarrow \cos\left(\frac{4\pi}{7}\right) + i\sin\left(\frac{4\pi}{7}\right)$

Bottom: Cosine is an even function, so $\cos\left(\frac{\pi}{7}\right) = \cos\left(\frac{-\pi}{7}\right)$

Sine is an odd function, so $\sin\left(\frac{-\pi}{7}\right) = -\sin\left(\frac{\pi}{7}\right)$

So: $\cos\left(\frac{\pi}{7}\right) - i\sin\left(\frac{\pi}{7}\right) = \cos\left(\frac{-\pi}{7}\right) + i\sin\left(\frac{-\pi}{7}\right)$

$\rightarrow \dfrac{\cos\left(\frac{4\pi}{7}\right) + i\sin\left(\frac{4\pi}{7}\right)}{\cos\left(\frac{-\pi}{7}\right) + i\sin\left(\frac{-\pi}{7}\right)}$ dividing, so subtract the arguments:

$\rightarrow \cos\left(\frac{5\pi}{7}\right) + i\sin\left(\frac{5\pi}{7}\right)$ $\rightarrow$ arg $= \underline{\frac{5\pi}{7}}$

(b) The complex numbers u and u^* are plotted on an Argand diagram.

Describe the single geometrical transformation that maps u onto u^* and state the exact value of arg u^*. [2]

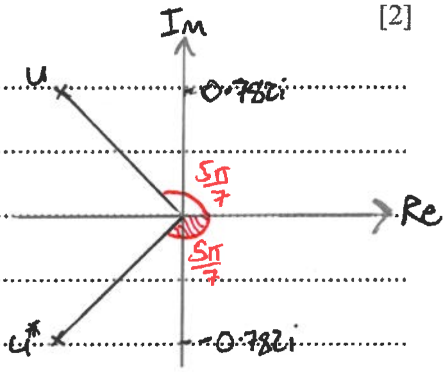
optional

$u = \cos\left(\frac{5\pi}{7}\right) + i\sin\left(\frac{5\pi}{7}\right)$

$= -0.623 + 0.782i$

$u^* = \cos\left(\frac{5\pi}{7}\right) - i\sin\left(\frac{5\pi}{7}\right)$

$= -0.623 - 0.782i$

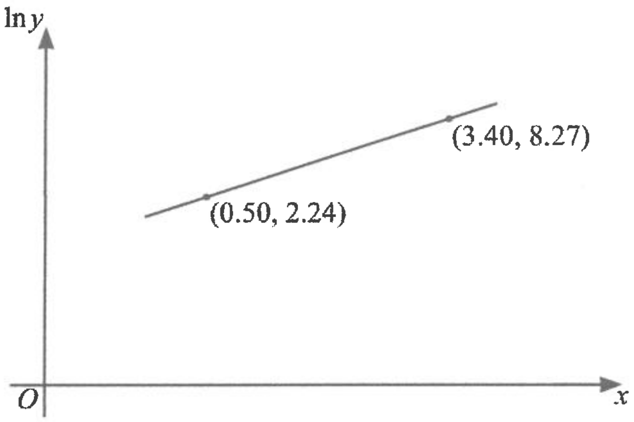


$\rightarrow$ u is mapped onto u^* by a reflection in the Real axis.

arg $u^* = \underline{\frac{-5\pi}{7}}$

DO NOT WRITE IN THIS MARGIN

6



The variables x and y satisfy the equation $ay = b^x$, where a and b are constants. The graph of $\ln y$ against x is a straight line passing through the points (0.50, 2.24) and (3.40, 8.27), as shown in the diagram.

Find the values of a and b. Give each value correct to 1 significant figure. [4]

$ay = b^x$

$\ln(ay) = \ln b^x$

$\ln a + \ln y = x \ln b$

$\ln y = x \ln b - \ln a$

Sub. (0.5, 2.24):

$2.24 = 0.5 \ln b - \ln a$ ①

Sub. (3.4, 8.27):

$8.27 = 3.4 \ln b - \ln a$ ②

② − ①:

$6.03 = 2.9 \ln b$

$\ln b = \frac{6.03}{2.9}$

$b = e^{\frac{6.03}{2.9}}$

$b = 7.998\ldots$ STO

$= 8$ (1sf)

→ ①:

$2.24 = 0.5\ln(7.998..) - \ln a$

$\ln a = 0.5\ln(7.998..) - 2.24$

$\ln a = -1.200\ldots$

$a = e^{-1.200}$

$= 0.301$

$= 0.3$ (1sf)

7 (a) Show that the equation $\tan^3 x + 2\tan 2x - \tan x = 0$ may be expressed as

$$\tan^4 x - 2\tan^2 x - 3 = 0$$

for $\tan x \neq 0$. [3]

$\tan^3 x + 2\tan 2x - \tan x = 0$

$\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$

$\tan^3 x + 2\left(\dfrac{2\tan x}{1-\tan^2 x}\right) - \tan x = 0$

$\tan^3 x + \dfrac{4\tan x}{1-\tan^2 x} - \tan x = 0$ $\quad \times(1-\tan^2 x)$

$(1-\tan^2 x)(\tan^3 x) + 4\tan x - (1-\tan^2 x)(\tan x) = 0$

$\tan^3 x - \tan^5 x + 4\tan x - \tan x + \tan^3 x = 0$

$-\tan^5 x + 2\tan^3 x + 3\tan x = 0$ $\quad \times -1$

$\tan^5 x - 2\tan^3 x - 3\tan x = 0$

$\tan x(\tan^4 x - 2\tan^2 x - 3) = 0$

$\tan x = 0$ or $\tan^4 x - 2\tan^2 x - 3 = 0$ ✓

$\times$ $\tan x \neq 0$

$\rightarrow$ $\tan^4 x - 2\tan^2 x - 3 = 0$ QED

$0 < 2\theta < 2\pi$

(b) Hence solve the equation $\tan^3 2\theta + 2\tan 4\theta - \tan 2\theta = 0$ for $0 < \theta < \pi$. Give your answers in exact form. [3]

from (a): $\tan^4 2\theta - 2\tan^2 2\theta - 3 = 0$

Let $Y = \tan^2 2\theta$:

$Y^2 - 2Y - 3 = 0$

$(Y-3)(Y+1) = 0$

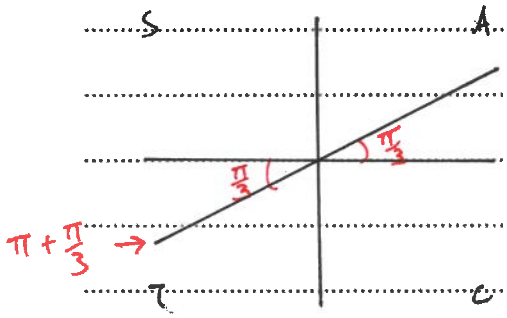
$Y = 3$ or $Y = -1$

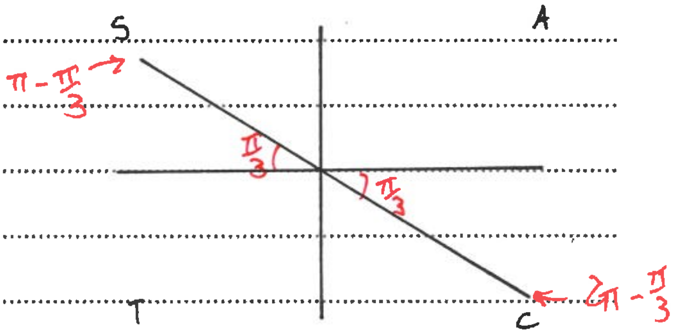
$\tan^2 2\theta = 3$ $\qquad$ $\tan^2 2\theta = -1$

$\tan 2\theta = \pm\sqrt{3}$ $\qquad$ × No sol^{n}s

$\tan 2\theta = \sqrt{3}$ $\qquad$ $\tan 2\theta = -\sqrt{3}$

$2\theta = \frac{\pi}{3}$ $\qquad$ $2\theta = \frac{-\pi}{3}$





$2\theta = \frac{\pi}{3}, \frac{4\pi}{3}$ $\qquad$ $2\theta = \frac{2\pi}{3}, \frac{5\pi}{3}$

$\theta = \frac{\pi}{6}, \frac{2\pi}{3}$ $\qquad$ $\theta = \frac{\pi}{3}, \frac{5\pi}{6}$

$\theta = \frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}$

8 The parametric equations of a curve are

$$x = \tan^2 2t, \qquad y = \cos 2t,$$

for $0 < t < \frac{1}{4}\pi$.

(a) Show that $\frac{dy}{dx} = -\frac{1}{2}\cos^3 2t$. [4]

$x = \tan^2 2t$

chain rule: $x = u^2 \qquad u = \tan 2t$

$\frac{dx}{du} = 2u \qquad \frac{du}{dt} = 2\sec^2 2t$

$\frac{dx}{dt} = \frac{dx}{du} \times \frac{du}{dt}$

$= 2u \times 2\sec^2 2t$

$= 2\tan 2t \times 2\sec^2 2t$

$= 4\tan 2t \sec^2 2t$

$y = \cos 2t$

$\frac{dy}{dt} = -2\sin 2t$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$= -2\sin 2t \times \frac{1}{4\tan 2t \sec^2 2t}$

$= \frac{-\sin 2t}{2\tan 2t \sec^2 2t}$

$= \frac{-\sin 2t}{2\tan 2t \times \frac{1}{\cos^2 2t}}$ $\times \cos^2 2t$ $\times \cos^2 2t$

$= \frac{-\sin 2t \cos^2 2t}{2\tan 2t}$

$= \frac{-\sin 2t \cos^2 2t}{2 \times \frac{\sin 2t}{\cos 2t}}$ $\times \cos 2t$ $\times \cos 2t$

$= \frac{-\cancel{\sin 2t}\cos^3 2t}{2\cancel{\sin 2t}}$

$= \frac{-\cos^3 2t}{2}$

$= -\frac{1}{2}\cos^3 2t$ QED

(b) Hence find the equation of the normal to the curve at the point where $t = \frac{1}{8}\pi$. Give your answer in the form $y = mx + c$. [4]

Find x and y-coords at $t = \frac{\pi}{8}$:

$x = \tan^2\left(\frac{2\pi}{8}\right)$

$= 1$

$y = \cos\left(\frac{2\pi}{8}\right)$

$= \frac{\sqrt{2}}{2}$

$\rightarrow$ Coordinates of point are: $\left(1, \frac{\sqrt{2}}{2}\right)$

Find gradient at $t = \frac{\pi}{8}$:

$M = -\frac{1}{2}\cos^3\left(\frac{2\pi}{8}\right)$

$m = \frac{-\sqrt{2}}{8}$

Gradient of Normal:

$M_{perp} = \frac{8}{\sqrt{2}} \begin{matrix} \times\sqrt{2} \\ \times\sqrt{2} \end{matrix}$

$= \frac{8\sqrt{2}}{2} = 4\sqrt{2}$

Equation of Normal:

$y - y_1 = m(x - x_1)$

$y - \frac{\sqrt{2}}{2} = 4\sqrt{2}(x - 1)$

$y - \frac{\sqrt{2}}{2} = 4\sqrt{2}x - 4\sqrt{2}$

$y = 4\sqrt{2}x - 4\sqrt{2} + \frac{\sqrt{2}}{2}$

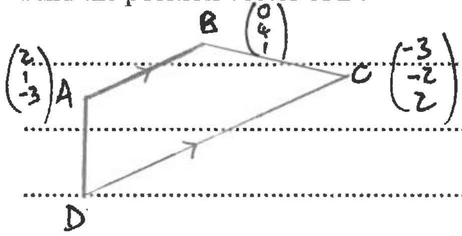
$y = 4\sqrt{2}x - \frac{7\sqrt{2}}{2}$

9 With respect to the origin O, the points A, B and C have position vectors given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 0 \\ 4 \\ 1 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} -3 \\ -2 \\ 2 \end{pmatrix}.$$

(a) The point D is such that $ABCD$ is a trapezium with $\overrightarrow{DC} = 3\overrightarrow{AB}$.

Find the position vector of D. [2]



$$\overrightarrow{AB} = \underline{b} - \underline{a} = \begin{pmatrix} 0 \\ 4 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \\ 4 \end{pmatrix}$$

$$\overrightarrow{DC} = 3\begin{pmatrix} -2 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} -6 \\ 9 \\ 12 \end{pmatrix}$$

$$\overrightarrow{OD} = \overrightarrow{OC} + \overrightarrow{CD}$$
$$= \begin{pmatrix} -3 \\ -2 \\ 2 \end{pmatrix} + \begin{pmatrix} 6 \\ -9 \\ -12 \end{pmatrix} = \begin{pmatrix} 3 \\ -11 \\ -10 \end{pmatrix}$$

(b) The diagonals of the trapezium intersect at the point P.

Find the position vector of P. [5]

Eqⁿ of line through A and C:

$$\overrightarrow{AC} = \underline{c} - \underline{a}$$
$$= \begin{pmatrix} -3 \\ -2 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$$
$$= \begin{pmatrix} -5 \\ -3 \\ 5 \end{pmatrix}$$

$$\underline{r} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} -5 \\ -3 \\ 5 \end{pmatrix}$$

point on line (A) direction vector $\overrightarrow{AC}$

Eq^n of line through B and D:

$\vec{BD} = \underline{d} - \underline{b}$

$= \begin{pmatrix} 3 \\ -11 \\ -10 \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 3 \\ -15 \\ -11 \end{pmatrix}$

$\underline{r} = \begin{pmatrix} 0 \\ 4 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -15 \\ -11 \end{pmatrix}$

Solve simultaneously: $\begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} -5 \\ -3 \\ 5 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -15 \\ -11 \end{pmatrix}$

$\underline{i}$: $2 - 5\lambda = 3\mu$ $\times 5$ $\quad$ $\underline{j}$: $1 - 3\lambda = 4 - 15\mu$ (2)

$10 - 25\lambda = 15\mu$ (1)

(1) + (2): $11 - 28\lambda = 4$

$28\lambda = 7$

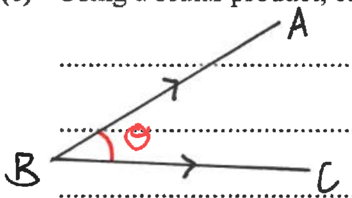
$\lambda = \frac{1}{4}$

sub into eq^n of line:

$\underline{r} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} + \frac{1}{4} \begin{pmatrix} -5 \\ -3 \\ 5 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} + \begin{pmatrix} -5/4 \\ -3/4 \\ 5/4 \end{pmatrix} = \begin{pmatrix} 3/4 \\ 1/4 \\ -7/4 \end{pmatrix}$

(c) Using a scalar product, calculate angle ABC. [4]



$\vec{BA} = \begin{pmatrix} 2 \\ -3 \\ -4 \end{pmatrix}$ $\quad$ $\vec{BC} = \underline{c} - \underline{b}$

$= \begin{pmatrix} -3 \\ -2 \\ 2 \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \\ 1 \end{pmatrix} = \begin{pmatrix} -3 \\ -6 \\ 1 \end{pmatrix}$

$\vec{BA} \cdot \vec{BC} = |\vec{BA}||\vec{BC}| \cos\theta$

$\begin{pmatrix} 2 \\ -3 \\ -4 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ -6 \\ 1 \end{pmatrix} = \sqrt{2^2 + (-3)^2 + (-4)^2} \times \sqrt{(-3)^2 + (-6)^2 + 1^2} \cos\theta$

$-6 + 18 - 4 = \sqrt{29}\sqrt{46} \cos\theta$

$8 = \sqrt{29} \times \sqrt{46} \cos\theta$

$\cos\theta = \dfrac{8}{\sqrt{29} \times \sqrt{46}}$

$\theta = 77.3^\circ$

10 A balloon in the shape of a sphere has volume V and radius r. Air is pumped into the balloon at a constant rate of 40π starting when time $t = 0$ and $r = 0$. At the same time, air begins to flow out of the balloon at a rate of $0.8\pi r$. The balloon remains a sphere at all times.

(a) Show that r and t satisfy the differential equation

$$\frac{dr}{dt} = \frac{50 - r}{5r^2}.$$ [3]

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = 4\pi r^2$

* $\frac{dV}{dt} = 40\pi - 0.8\pi r$

$\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt}$

$= \frac{1}{4\pi r^2} \times (40\pi - 0.8\pi r)$

$= \frac{40\cancel{\pi} - 0.8\cancel{\pi} r}{4\cancel{\pi} r^2}$

$= \frac{40 - 0.8r}{4r^2}$ ÷4 ÷4

$= \frac{10 - 0.2r}{r^2}$ ×5 ×5

$= \frac{50 - r}{5r^2}$ QED

(b) Find the quotient and remainder when $5r^2$ is divided by $50 - r$. ← $-r + 50$ [3]

$$\begin{array}{r|l} & -5r - 250 \\ \hline -r + 50 & 5r^2 + 0r + 0 \\ & 5r^2 - 250r \\ \hline & \quad 250r + 0 \\ & \quad 250r - 12500 \\ \hline & \qquad 12500 \end{array}$$

→ $\frac{5r^2}{50 - r} = \underbrace{-5r - 250}_{\text{quotient}} + \frac{12500}{50 - r}$ remainder

(c) Solve the differential equation in part (a), obtaining an expression for t in terms of r. [6]

$$\frac{dr}{dt} = \frac{50-r}{5r^2}$$

$$\frac{5r^2}{50-r}\,dr = dt$$

$$\int\left(-5r - 250 + \frac{12500}{50-r}\right)dr = \int 1\,dt$$

$$-\frac{5}{2}r^2 - 250r - 12500\ln|50-r| = t + C$$

Sub. $t=0$, $r=0$:

$$0 - 0 - 12500\ln 50 = 0 + C$$

$$C = -12500\ln 50$$

$$-\frac{5}{2}r^2 - 250r - 12500\ln|50-r| = t - 12500\ln 50$$

$$t = -\frac{5}{2}r^2 - 250r - 12500\ln(50-r) + 12500\ln 50$$

(d) Find the value of t when the radius of the balloon is 12. [1]

$$t = -\frac{5}{2}(12)^2 - 250(12) - 12500\ln 38 + 12500\ln 50$$

$$= 70.5$$

11 Let $f(x) = \dfrac{2e^{2x}}{e^{2x} - 3e^x + 2}$. u v

(a) Find $f'(x)$ and hence find the exact coordinates of the stationary point of the curve with equation $y = f(x)$. [5]

$u = 2e^{2x}$ $\quad v = e^{2x} - 3e^x + 2$

$\dfrac{du}{dx} = 4e^{2x}$ $\quad \dfrac{dv}{dx} = 2e^{2x} - 3e^x$

$$\frac{dy}{dx} = \frac{(e^{2x} - 3e^x + 2) \times 4e^{2x} - 2e^{2x}(2e^{2x} - 3e^x)}{(e^{2x} - 3e^x + 2)^2}$$

$$= \frac{4e^{4x} - 12e^{3x} + 8e^{2x} - 4e^{4x} + 6e^{3x}}{(e^{2x} - 3e^x + 2)^2}$$

$$= \frac{-6e^{3x} + 8e^{2x}}{(e^{2x} - 3e^x + 2)^2}$$

SP, so $\dfrac{dy}{dx} = 0$:

$$\frac{-6e^{3x} + 8e^{2x}}{(e^{2x} - 3e^x + 2)^2} = 0 \quad \times (e^{2x} - 3e^x + 2)^2$$

$$-6e^{3x} + 8e^{2x} = 0$$

$$2e^{2x}(-3e^x + 4) = 0$$

$2e^{2x} = 0$ or $-3e^x + 4 = 0$

$\times$ No sol^ns

$3e^x = 4$

$e^x = 4/3$

$x = \ln 4/3$

Sub. into $f(x)$:

$$y = \frac{2e^{2\ln 4/3}}{e^{2\ln 4/3} - 3e^{\ln 4/3} + 2}$$

$= -16$ $\rightarrow$ $\left(\ln\frac{4}{3}, -16\right)$

11 Let $f(x) = \dfrac{2e^{2x}}{e^{2x} - 3e^{x} + 2}$.

(b) Use the substitution $u = e^{x}$ and partial fractions to find the exact value of $\int_{\ln 3}^{\ln 5} f(x)\,dx$.

Give your answer in the form $\ln a$, where a is a rational number in its simplest form. [9]

$$\int_{\ln 3}^{\ln 5} \frac{2e^{2x}}{e^{2x} - 3e^{x} + 2}\,dx$$

$$u = e^{x}$$
$$\frac{du}{dx} = e^{x}$$
$$du = e^{x}\,dx$$
$$\boxed{dx = \frac{du}{e^{x}}}$$

$$\int_{x=\ln 3}^{x=\ln 5} \frac{2e^{2x}}{e^{2x} - 3e^{x} + 2} \times \frac{du}{e^{x}}$$

$u = e^{\ln 5} = 5$ $\qquad u = e^{\ln 3} = 3$

$$= \int_{3}^{5} \frac{2e^{x}}{e^{2x} - 3e^{x} + 2}\,du$$

$u = e^{x}$

$u^2 = (e^{x})^2 = e^{2x}$

$$= \int_{3}^{5} \frac{2u}{u^2 - 3u + 2}\,du$$

Partial Fractions:

$$\frac{2u}{(u-2)(u-1)} \equiv \frac{A}{u-2} + \frac{B}{u-1}$$

$$2u \equiv A(u-1) + B(u-2)$$

$u=1$: $2 = 0 + -B$

$B = -2$

$u=2$: $4 = A$

$$\int_{3}^{5} \left(\frac{4}{u-2} + \frac{-2}{u-1}\right) du$$

$$= \int_{3}^{5} \left(4 \times \frac{1}{u-2} - 2 \times \frac{1}{u-1}\right) du$$

$$\Big[4\ln|u-2| - 2\ln|u-1|\Big]_{3}^{5}$$

$$= [4\ln 3 - 2\ln 4] - [4\ln 1 - 2\ln 2]$$
$$= 4\ln 3 - 2\ln 4 + 2\ln 2$$
$$= \ln 3^4 - \ln 4^2 + \ln 2^2$$
$$= \ln 81 - \ln 16 + \ln 4$$
$$= \ln\left(\frac{81 \times 4}{16}\right) = \ln \frac{81}{4}$$