1 The polynomial $4x^3 + ax^2 + 5x + b$, where a and b are constants, is denoted by $p(x)$. It is given that $(2x+1)$ is a factor of $p(x)$. When $p(x)$ is divided by $(x-4)$ the remainder is equal to 3 times the remainder when $p(x)$ is divided by $(x-2)$. 3R

R

Find the values of a and b. [5]

$(2x+1)$ is a factor, so $p\left(-\frac{1}{2}\right) = 0$:

$4\left(\frac{-1}{2}\right)^3 + a\left(\frac{-1}{2}\right)^2 + 5\left(\frac{-1}{2}\right) + b = 0$

$4 \times \frac{-1}{8} + a \times \frac{1}{4} - \frac{5}{2} + b = 0$

$\frac{-1}{2} + \frac{a}{4} - \frac{5}{2} + b = 0$

$\frac{a}{4} + b = 3$

$\times 4 \quad \times 4 \quad \times 4$

$a + 4b = 12$ (1)

When $p(x)$ is divided by $(x-4)$.... so $p(4) = 3R$: ← remainder

$4(4)^3 + a(4)^2 + 5(4) + b = 3R$

$256 + 16a + 20 + b = 3R$

$16a + b = 3R - 276$ (2)

When $p(x)$ is divided by $(x-2)$.... so $p(2) = R$:

$4(2)^3 + a(2)^2 + 5(2) + b = R$

$32 + 4a + 10 + b = R$

$4a + b = R - 42 \quad \times 3$

$12a + 3b = 3R - 126$ (3)

(2) − (3): $4a - 2b = -150 \quad \times 2$

$8a - 4b = -300$ (4)

(1) + (4): $9a = -288$

$\underline{\underline{a = -32}}$

→ (1): $a + 4b = 12$

$-32 + 4b = 12$

$4b = 44$

$\underline{\underline{b = 11}}$

2 Find the exact value of $\int_1^3 x^2 \ln 3x \, dx$. Give your answer in the form $a \ln b + c$, where a and c are rational and b is an integer. [5]

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E

$u = \ln 3x \qquad v = \frac{1}{3}x^3$

$\frac{du}{dx} = \frac{3}{3x} = \frac{1}{x} \qquad \frac{dv}{dx} = x^2$

$= \left[\frac{1}{3}x^3 \ln|3x| - \int \frac{1}{3}x^3 \times \frac{1}{x} \, dx\right]_1^3$

$= \left[\frac{1}{3}x^3 \ln|3x| - \int \frac{1}{3}x^2 \, dx\right]_1^3$

$= \left[\frac{1}{3}x^3 \ln|3x| - \frac{1}{9}x^3\right]_1^3$

$= \left[\frac{1}{3} \times 3^3 \times \ln 9 - \frac{1}{9} \times 3^3\right] - \left[\frac{1}{3} \times 1^3 \times \ln 3 - \frac{1}{9} \times 1^3\right]$

$= [9\ln 9 - 3] - \left[\frac{1}{3}\ln 3 - \frac{1}{9}\right]$

$= 9\ln 9 - 3 - \frac{1}{3}\ln 3 + \frac{1}{9}$

$= 9\ln 9 - \frac{1}{3}\ln 3 - \frac{26}{9}$

$= 9\ln 3^2 - \frac{1}{3}\ln 3 - \frac{26}{9}$

$= 18\ln 3 - \frac{1}{3}\ln 3 - \frac{26}{9}$

$= \frac{53}{3}\ln 3 - \frac{26}{9}$

3 The equation of a curve is $\ln(x+y) = 3x^2y$.

Find the gradient of the curve at the point $(1, 0)$. [4]

differentiate $\ln(x+y)$: chain rule:

$Y = \ln t \qquad t = x + y$

$\frac{dY}{dt} = \frac{1}{t} \qquad \frac{dt}{dx} = 1 + \frac{dy}{dx}$

$\frac{dY}{dx} = \frac{1}{t} \times \left(1 + \frac{dy}{dx}\right)$

$= \frac{1}{x+y}\left(1 + \frac{dy}{dx}\right)$

differentiate $3\overset{u}{x^2}\overset{v}{y}$:

$u = 3x^2 \qquad v = y$

$\frac{du}{dx} = 6x \qquad \frac{dv}{dx} = \frac{dy}{dx}$

$\rightarrow 3x^2\frac{dy}{dx} + 6xy$

Whole thing: $\frac{1}{x+y}\left(1 + \frac{dy}{dx}\right) = 3x^2\frac{dy}{dx} + 6xy$

Sub. $(1,0)$:

$\frac{1}{1+0}\left(1 + \frac{dy}{dx}\right) = 3 \times 1^2 \times \frac{dy}{dx} + 6 \times 1 \times 0$

$1\left(1 + \frac{dy}{dx}\right) = 3\frac{dy}{dx} + 0$

$1 + \frac{dy}{dx} = 3\frac{dy}{dx}$

$1 = 2\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1}{2}$

4 (a) Show that $\sec^4\theta - \tan^4\theta \equiv 1 + 2\tan^2\theta$. [3]

LHS: $\sec^4\theta - \tan^4\theta$

$= (\sec^2\theta + \tan^2\theta)(\sec^2\theta - \tan^2\theta)$ (difference of two squares)

$1 + \tan^2\theta = \sec^2\theta$

$= (\sec^2\theta + \tan^2\theta)(1 + \tan^2\theta - \tan^2\theta)$

$= (\sec^2\theta + \tan^2\theta) \times 1$

$1 + \tan^2\theta = \sec^2\theta$

$= 1 + \tan^2\theta + \tan^2\theta$

$= \underline{1 + 2\tan^2\theta}$ QED

(b) Hence or otherwise solve the equation $\sec^4 2\alpha - \tan^4 2\alpha = 2\tan^2 2\alpha \sec^2 2\alpha$ for $0° < \alpha < 180°$. [5]

from (a): $\sec^4 2\alpha - \tan^4 2\alpha \equiv 1 + 2\tan^2 2\alpha$

$\rightarrow \quad 1 + 2\tan^2 2\alpha = 2\tan^2 2\alpha \sec^2 2\alpha$

$\sec^2 2\alpha = 1 + \tan^2 2\alpha$

$1 + 2\tan^2 2\alpha = 2\tan^2 2\alpha(1 + \tan^2 2\alpha)$

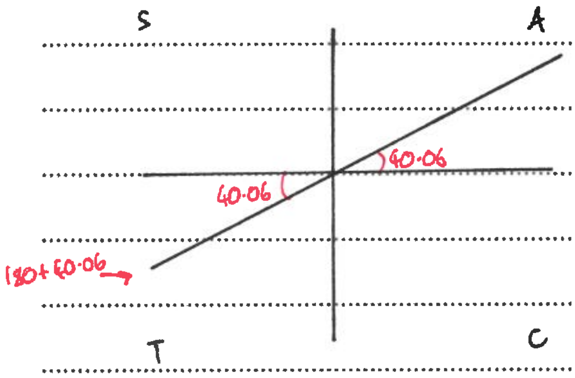
$1 + 2\tan^2 2\alpha = 2\tan^2 2\alpha + 2\tan^4 2\alpha$

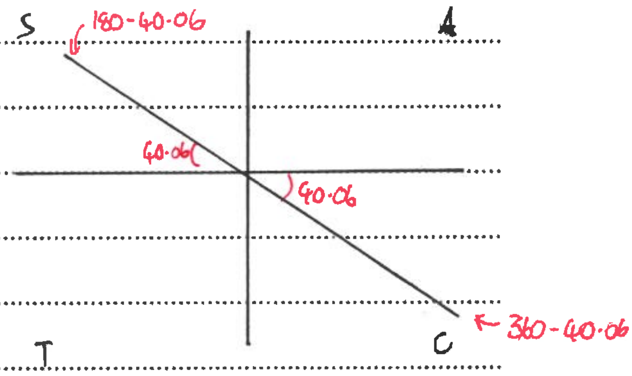
$1 = 2\tan^4 2\alpha$

$\tan^4 2\alpha = \frac{1}{2}$

$\tan 2\alpha = \sqrt[4]{\frac{1}{2}}$ or $\tan 2\alpha = -\sqrt[4]{\frac{1}{2}}$

$2\alpha = 40.06°$ $\qquad$ $2\alpha = -40.06°$



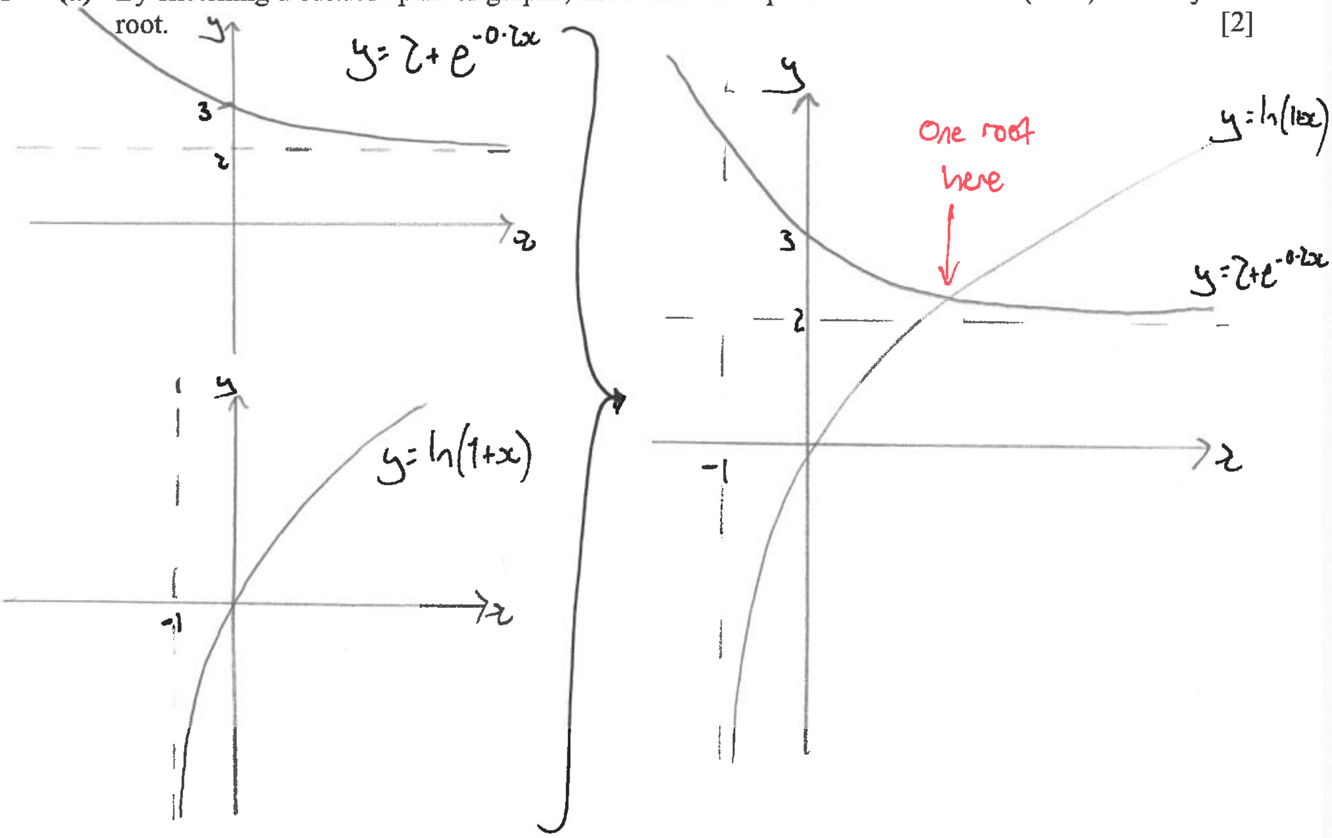


$2\alpha = 40.06°, 220.06°$ $\qquad$ $2\alpha = 139.94°, 319.94°$

$2\alpha = 40.06°, 139.94°, 220.06°, 319.94°$

$\alpha = 20.0°, 70.0°, 110.0°, 160.0°$

5 (a) By sketching a suitable pair of graphs, show that the equation $2 + e^{-0.2x} = \ln(1 + x)$ has only one root. [2]



(b) Show by calculation that this root lies between 7 and 9. [2]

Set =0: $2 + e^{-0.2x} - \ln(1+x) = 0$

Sub. $x = 7$: $2 + e^{-0.2(7)} - \ln(1+7) = 0.167$

Sub. $x = 9$: $2 + e^{-0.2(9)} - \ln(1+9) = -0.137$

Sign change (and continuous function) between $x = 7$ and $x = 9$ shows there's a root between 7 and 9

(c) Use the iterative formula

$$x_{n+1} = \exp\left(2 + e^{-0.2x_n}\right) - 1$$

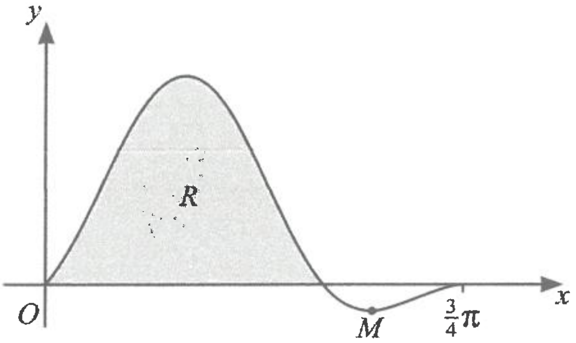
to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places.

[$\exp(x)$ is an alternative notation for e^x.] [3]

$x_1 = 7.0000$
$x_2 = 8.4555$
$x_3 = 7.8846$
$x_4 = 8.0849$
$x_5 = 8.0115$
$x_6 = 8.0380$
$x_7 = 8.0283$
$x_8 = 8.0318$
$x_9 = 8.0306$

x_7, x_8 and x_9 all round to 8.03 to 2dp so $x = 8.03$

6



u v

The diagram shows the curve $y = \sin 2x(1 + \sin 2x)$, for $0 \leqslant x \leqslant \frac{3}{4}\pi$, and its minimum point M. The shaded region bounded by the curve that lies above the x-axis and the x-axis itself is denoted by R.

(a) Given that the x-coordinate of M lies in the interval $\frac{1}{2}\pi < x < \frac{3}{4}\pi$, find the exact coordinates of M. [4]

$u = \sin 2x \qquad V = 1 + \sin 2x \qquad \frac{\pi}{2} < x < \frac{3\pi}{4}$

$\frac{du}{dx} = 2\cos 2x \qquad \frac{dv}{dx} = 2\cos 2x \qquad \pi < 2x < \frac{3\pi}{2}$

$\frac{dy}{dx} = 2\sin 2x \cos 2x + 2\cos 2x(1 + \sin 2x)$

$= 2\sin 2x \cos 2x + 2\cos 2x + 2\sin 2x \cos 2x$

$= 4\sin 2x \cos 2x + 2\cos 2x$

SP, so $\frac{dy}{dx} = 0$:

$4\sin 2x \cos 2x + 2\cos 2x = 0$

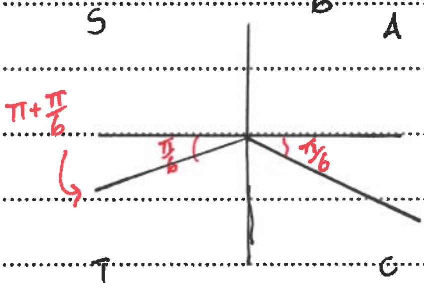
$2\cos 2x(2\sin 2x + 1) = 0$

$2\cos 2x = 0$ or $2\sin 2x + 1 = 0$

$\cos 2x = 0 \qquad 2\sin 2x = -1$

$2x = \frac{\pi}{2} \qquad \sin 2x = \frac{-1}{2}$

$\frac{\pi}{2}$ and others from CAST diagram outside range. $\qquad 2x = \frac{-\pi}{6}$



Sub. $x = \frac{7\pi}{12}$:

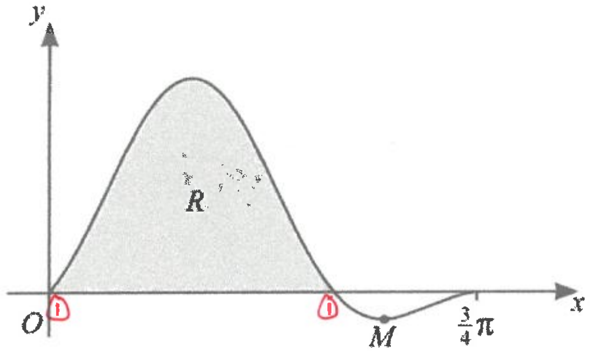
$y = \sin\left(2 \times \frac{7\pi}{12}\right)\left(1 + \sin\left(2 \times \frac{7\pi}{12}\right)\right)$

$= \frac{-1}{4}$

→ $\left(\frac{7\pi}{12}, \frac{-1}{4}\right)$

$2x = \frac{-\pi}{6}, \frac{7\pi}{6}$

outside range → $x = \frac{-\pi}{12}, \frac{7\pi}{12}$



The diagram shows the curve $y = \sin 2x(1 + \sin 2x)$, for $0 \leqslant x \leqslant \frac{3}{4}\pi$, and its minimum point M. The shaded region bounded by the curve that lies above the x-axis and the x-axis itself is denoted by R.

$\hookrightarrow 0 \leqslant 2x \leqslant \frac{3\pi}{2}$

(b) Find the exact area of the region R. [4]

① Find points of intersection with x-axis:

$\sin 2x(1 + \sin 2x) = 0$

$\sin 2x = 0$ or $1 + \sin 2x = 0$

$2x = 0$ or π $\qquad$ $\sin 2x = -1$

$x = 0$ or $\frac{\pi}{2}$ ✓

$2x = \frac{-\pi}{2}$ or $\frac{3\pi}{2}$

$x = \frac{-\pi}{4}$ or $\frac{3\pi}{4}$

Not in range $\qquad$ Not the point we want

$$\int_0^{\frac{\pi}{2}} (\sin 2x + \sin^2 2x)\,dx$$

$\cos 2A = 1 - 2\sin^2 A$

$\cos 4x = 1 - 2\sin^2 2x$

$\sin^2 2x = \frac{1}{2} - \frac{1}{2}\cos 4x$

$$\int_0^{\pi/2} \left(\sin 2x + \frac{1}{2} - \frac{1}{2}\cos 4x\right)dx$$

$$= \left[-\frac{1}{2}\cos 2x + \frac{1}{2}x - \frac{1}{8}\sin 4x\right]_0^{\pi/2}$$

$$= \left[\frac{-1}{2}\cos(\pi) + \frac{1}{2}\left(\frac{\pi}{2}\right) - \frac{1}{8}\sin(2\pi)\right] - \left[\frac{-1}{2}\cos(0) + 0 - \frac{1}{8}\sin(0)\right]$$

$$= \left[\frac{-1}{2}(-1) + \frac{\pi}{4} - 0\right] - \left[\frac{-1}{2}(1) + 0 - 0\right]$$

$$= \left[\frac{1}{2} + \frac{\pi}{4}\right] - \left[\frac{-1}{2}\right]$$

$$= 1 + \frac{\pi}{4}$$

7 Let $f(x) = \dfrac{5x^2 + 8x + 5}{(1+2x)(2+x^2)}$.

(a) Express $f(x)$ in partial fractions. [5]

$$\frac{5x^2 + 8x + 5}{(1+2x)(2+x^2)} \equiv \frac{A}{1+2x} + \frac{Bx + C}{2+x^2}$$

$$5x^2 + 8x + 5 \equiv A(2+x^2) + (Bx+C)(1+2x)$$

$x = \frac{-1}{2}$:

$$5\left(\tfrac{-1}{2}\right)^2 + 8\left(\tfrac{-1}{2}\right) + 5 = A\left(2 + \left(\tfrac{-1}{2}\right)^2\right) + \left(\tfrac{-1}{2}B + C\right) \times 0$$

$$\frac{5}{4} - 4 + 5 = \frac{9}{4}A + 0$$

$$\frac{9}{4} = \frac{9}{4}A$$

$$A = 1$$

$x = 0$:

$$0 + 0 + 5 = A(2) + C(1)$$

$$5 = 2A + C$$

$\leftarrow A=1$

$$5 = 2 + C$$

$$C = 3$$

$x = 1$:

$$5 + 8 + 5 = A(2 + 1^2) + (B + C)(1+2)$$

$$18 = 3A + (B+C)(3)$$

$$18 = 3A + 3B + 3C$$

$\leftarrow A=1$ $\leftarrow C=3$

$$18 = 3 + 3B + 9$$

$$6 = 3B$$

$$B = 2$$

$$f(x) = \frac{1}{1+2x} + \frac{2x+3}{(2+x^2)}$$

(b) Hence find the coefficient of x^3 in the expansion of f(x). [4]

$f(x) = (1+2x)^{-1}$ A $+ (2x+3)(2+x^2)^{-1}$ B

A: $(1+2x)^{-1} = 1 + (-1)(2x) + \frac{(-1)(-2)}{2!}(2x)^2 + \frac{(-1)(-2)(-3)}{3!}(2x)^3 + \ldots$

x^3 term: $\frac{(-1)(-2)(-3)}{3!}(2x)^3 = -1 \times 8x^3$

$= -8x^3$

B: $(2x+3)(2+x^2)^{-1} = (2x+3) \times 2^{-1}\left(1+\frac{x^2}{2}\right)^{-1}$

$= \frac{1}{2}(2x+3)\left(1+\frac{x^2}{2}\right)^{-1}$

$= \frac{1}{2}(2x+3)\left[1 + (-1)\left(\frac{x^2}{2}\right) + \frac{(-1)(-2)}{2!}\left(\frac{x^2}{2}\right)^2 + \ldots\right]$

$= \left(x+\frac{3}{2}\right)\left[1 - \frac{x^2}{2} + \frac{x^4}{4} + \ldots\right]$

x^3 term: $x \times \frac{-x^2}{2} = \frac{-1}{2}x^3$

x^3 term for whole thing:

$-8x^3 - \frac{1}{2}x^3 = \frac{-17}{2}x^3$

Coeff only: $= \frac{-17}{2}$

8 (a) Given that $z = 1+yi$ and that y is a real number, express $\frac{1}{z}$ in the form $a+bi$, where a and b are functions of y. [2]

$$\frac{1}{z} = \frac{1}{1+yi} \times \frac{(1-yi)}{(1-yi)}$$

$$= \frac{1-yi}{1-\cancel{yi}+\cancel{yi}+y^2}$$

$$= \frac{1-yi}{1+y^2}$$

$$= \frac{1}{1+y^2} + \frac{-y}{1+y^2}i$$

(b) Show that $\left(a-\frac{1}{2}\right)^2 + b^2 = \frac{1}{4}$, where a and b are the functions of y found in part (a). [3]

LHS: $\left(\frac{1}{1+y^2} - \frac{1}{2}\right)^2 + \left(\frac{-y}{1+y^2}\right)^2$

$$\left(\frac{1}{1+y^2} - \frac{1}{2}\right)\left(\frac{1}{1+y^2} - \frac{1}{2}\right) + \left(\frac{-y}{1+y^2}\right)^2$$

$$\frac{1}{(1+y^2)^2} - \frac{1}{2} \times \frac{1}{1+y^2} - \frac{1}{2} \times \frac{1}{1+y^2} + \frac{1}{4} + \frac{y^2}{(1+y^2)^2}$$

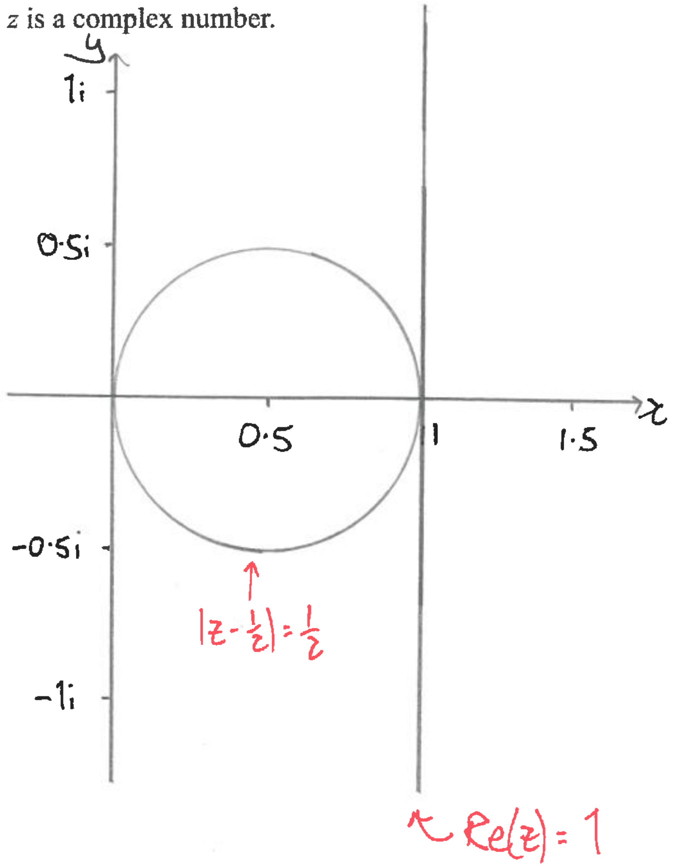
$$\frac{1}{(1+y^2)^2} - \frac{1 \times (1+y^2)}{1+y^2 \times (1+y^2)} + \frac{y^2}{(1+y^2)^2} + \frac{1}{4}$$

$$\frac{1}{(1+y^2)^2} - \frac{1+y^2}{(1+y^2)^2} + \frac{y^2}{(1+y^2)^2} + \frac{1}{4}$$

$$\frac{1 - 1 - y^2 + y^2}{(1+y^2)^2} + \frac{1}{4}$$

$$= 0 + \frac{1}{4} = \frac{1}{4}$$ QED

(c) On a single Argand diagram, sketch the loci given by the equations $\mathrm{Re}(z) = 1$ and $\left|z - \frac{1}{2}\right| = \frac{1}{2}$, where z is a complex number. [3]



Cartesian equation (useful for part (d)):

$$\left|x+iy-\tfrac{1}{2}\right| = \tfrac{1}{2}$$
$$\left|(x-\tfrac{1}{2})+iy\right| = \tfrac{1}{2}$$
$$(x-\tfrac{1}{2})^2 + y^2 = \tfrac{1}{4} \leftarrow r^2$$

(d) The complex number z is such that $\mathrm{Re}(z) = 1$. Use your answer to part **(b)** to give a geometrical description of the locus of $\frac{1}{z}$. [1]

This seems really confusing until you realise you have to use (a) and (b).

If $\mathrm{Re}(z) = 1$, complex number is $z = 1 + yi$ (as in part (a))

So $\frac{1}{z} = \frac{1}{1+y^2} + \frac{-y}{1+y^2}i$ (from part (a))

$= a + bi$

From (b), the locus of $\frac{1}{z}$: $(a-\frac{1}{2})^2 + b^2 = \frac{1}{4}$

$\rightarrow$ This is a circle, centre $(\frac{1}{2}, 0)$, radius $= \frac{1}{2}$.

For what it's worth, to properly understand this requires "Transformations on the Complex Plane" which is not on the syllabus.

9 The position vector of point A relative to the origin O is $\overrightarrow{OA} = 8\mathbf{i} - 5\mathbf{j} + 6\mathbf{k}$. The line l passes through A and is parallel to the vector $2\mathbf{i} + \mathbf{j} + 4\mathbf{k}$.

(a) State a vector equation for l. [2]

$$\underline{r} = \begin{pmatrix} 8 \\ -5 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix}$$

point on line

direction vector of line

(b) The position vector of point B relative to the origin O is $\overrightarrow{OB} = -t\mathbf{i} + 4t\mathbf{j} + 3t\mathbf{k}$, where t is a constant. The line l also passes through B.

Find the value of t. [3]

$$\begin{pmatrix} -t \\ 4t \\ 3t \end{pmatrix} = \begin{pmatrix} 8 \\ -5 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix}$$

$\underline{i}$: $-t = 8 + 2\lambda$ ①

$\underline{j}$: $4t = -5 + \lambda$ $\times 2$

$8t = -10 + 2\lambda$ ②

② - ①: $9t = -18$

$\underline{\underline{t = -2}}$

Optional: Check by finding λ:

→ ①: $-(-2) = 8 + 2\lambda$

$2 = 8 + 2\lambda$

$-6 = 2\lambda$

$\lambda = -3$

and check in $\underline{k}$:

$3(-2) = 6 + 4(-3)$

$-6 = 6 + -12$

$-6 = -6$ ✓

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(c) The line m has vector equation $\mathbf{r} = 5\mathbf{i} - \mathbf{j} + 2\mathbf{k} + \mu(a\mathbf{i} - \mathbf{j} + 3\mathbf{k})$. The acute angle between the directions of l and m is θ, where $\cos\theta = \frac{1}{\sqrt{6}}$.

Find the possible values of a. [5]

$$\underbrace{\begin{pmatrix} a \\ -1 \\ 3 \end{pmatrix}}_{\text{direction vector of } m} \bullet \underbrace{\begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix}}_{\text{direction vector of } l} = \sqrt{a^2 + (-1)^2 + 3^2} \times \sqrt{2^2 + 1^2 + 4^2}\ \underbrace{\cos\theta}_{= \frac{1}{\sqrt{6}}}$$

$$2a - 1 + 12 = \sqrt{a^2 + 1 + 9} \times \sqrt{4 + 1 + 16} \times \frac{1}{\sqrt{6}}$$

$$2a + 11 = \sqrt{a^2 + 10} \times \sqrt{21} \times \frac{1}{\sqrt{6}}$$

$$2a + 11 = \sqrt{(a^2 + 10) \times 21 \times \frac{1}{6}}$$

Square both sides

$$(2a + 11)^2 = \frac{21}{6}(a^2 + 10)$$

$$4a^2 + 44a + 121 = \frac{7}{2}(a^2 + 10)$$

$\times 2$ $\times 2$

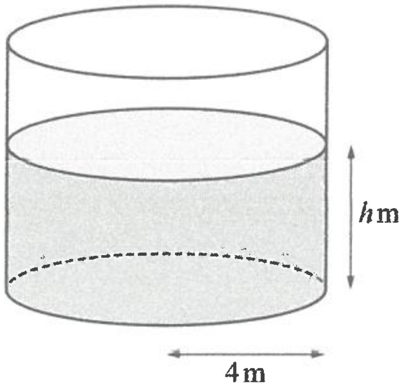
$$8a^2 + 88a + 242 = 7a^2 + 70$$

$$a^2 + 88a + 172 = 0$$

$$(a + 86)(a + 2) = 0$$

$$a = -86 \quad \text{or} \quad a = -2$$

10



A large cylindrical tank is used to store water. The base of the tank is a circle of radius 4 metres. At time t minutes, the depth of the water in the tank is h metres. There is a tap at the bottom of the tank. When the tap is open, water flows out of the tank at a rate proportional to the square root of the volume of water in the tank. *

(a) Show that $\frac{dh}{dt} = -\lambda\sqrt{h}$, where λ is a positive constant. [4]

$V = \pi \times 4^2 \times h$

$= 16\pi h$

$\frac{dV}{dh} = 16\pi$

* $\frac{dV}{dt} \propto -\sqrt{V}$ water flowing out so volume decreasing.

$\frac{dV}{dt} = -k\sqrt{V}$

$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$

$= \frac{1}{16\pi} \times -k\sqrt{V}$ $= 16\pi h$

$= \frac{1}{16\pi} \times -k\sqrt{16\pi h}$

$= \frac{-k\sqrt{16\pi}}{16\pi} \times \sqrt{h}$

$= \frac{-4k\sqrt{\pi}}{16\pi} \times \sqrt{h}$

$= \frac{-k\sqrt{\pi}}{4\pi} \times \sqrt{h}$

$= \frac{-k}{4\sqrt{\pi}} \times \sqrt{h}$

QED

$\left(\lambda = \frac{k}{4\sqrt{\pi}}\right)$

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(b) At time $t = 0$ the tap is opened. It is given that $h = 4$ when $t = 0$ and that $h = 2.25$ when $t = 20$.

Solve the differential equation to obtain an expression for t in terms of h, and hence find the time taken to empty the tank. [6]

$$\frac{dh}{dt} = -\lambda\sqrt{h}$$

$$\int \frac{1}{\sqrt{h}}\,dh = \int -\lambda\,dt$$

$$\int h^{-\frac{1}{2}}\,dh = -\int \lambda\,dt$$

$$2h^{\frac{1}{2}} = -\lambda t + C$$

Sub. $h=4$, $t=0$:

$$2(4)^{\frac{1}{2}} = 0 + C$$

$$4 = C$$

Sub. $h=2.25$, $t=20$:

$$2(2.25)^{\frac{1}{2}} = -\lambda(20) + 4$$

C

$$2(1.5) = -20\lambda + 4$$

$$3 = -20\lambda + 4$$

$$20\lambda = 1$$

$$\lambda = \frac{1}{20}$$

$$\rightarrow 2h^{\frac{1}{2}} = -\frac{1}{20}t + 4$$

$$2\sqrt{h} = \frac{-1}{20}t + 4$$

$\times 20$ $\times 20$ $\times 20$

$$40\sqrt{h} = -t + 80$$

$$t = 80 - 40\sqrt{h}$$

When tank is empty, $h=0$:

$t = 80 - 0 = $ 80 minutes to empty