1 Rajesh applies once every year for a ticket to a music festival. The probability that he is successful in any particular year is 0.3, independently of other years.

(a) Find the probability that Rajesh is successful for the first time on his 7th attempt. [1]

$X \sim \text{Geo}(0.3)$

$P(X=7) = 0.3 \times 0.7^6$

$= \underline{0.0353}$

(b) Find the probability that Rajesh is successful for the first time before his 6th attempt. [2]

$P(X<6) = P(X \leq 5)$

$= 1 - q^5$ ← probability of 5 failures

$= 1 - 0.7^5$

$= \underline{0.832}$

(c) Find the probability that Rajesh is successful for the second time on his 10th attempt. [2]

Probability of 1 success in first nine attempts:

$X \sim B(9, 0.3)$

$P(1) = {}^9C_1 \times 0.3^1 \times 0.7^8$

$= 0.1556\ldots$ STO

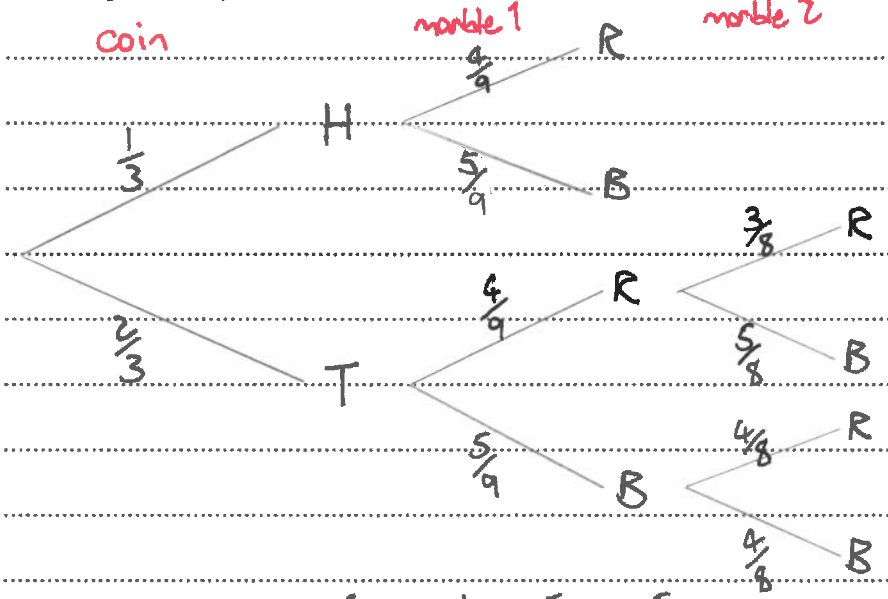
Probability of this × probability of success on 10th attempt:

$0.1556 \times 0.3 = \underline{0.0467}$

2 Seva has a coin which is biased so that when it is thrown the probability of obtaining a head is $\frac{1}{3}$. He also has a bag containing 4 red marbles and 5 blue marbles.

Seva throws the coin. If he obtains a head, he selects one marble from the bag at random. If he obtains a tail, he selects two marbles from the bag at random and without replacement.

(a) Find the probability that Seva selects at least one red marble. [3]



$\frac{1}{3} \times \frac{4}{9} = \frac{4}{27}$ ✓

$\frac{1}{3} \times \frac{5}{9} = \frac{5}{27}$

$\frac{2}{3} \times \frac{4}{9} \times \frac{3}{8} = \frac{1}{9}$ ✓

$\frac{2}{3} \times \frac{4}{9} \times \frac{5}{8} = \frac{5}{27}$ ✓

$\frac{2}{3} \times \frac{5}{9} \times \frac{4}{8} = \frac{5}{27}$ ✓

$\frac{2}{3} \times \frac{5}{9} \times \frac{4}{8} = \frac{5}{27}$

$P(\text{at least one red}) = \frac{4}{27} + \frac{1}{9} + \frac{5}{27} + \frac{5}{27}$

$= \frac{17}{27}$

(b) Find the probability that Seva obtains a head given that he selects no red marbles. [2]

$P(H \cap \text{No red}) = P(H | \text{No red}) \times P(\text{No red})$

$P(H | \text{No red}) = \dfrac{P(H \cap \text{No red})}{P(\text{No red})}$

$P(H \cap \text{No red}) = \frac{5}{27}$

$P(\text{No red}) = \frac{5}{27} + \frac{5}{27} = \frac{10}{27}$

$P(H | \text{No red}) = \dfrac{5/27}{10/27} = \frac{1}{2}$

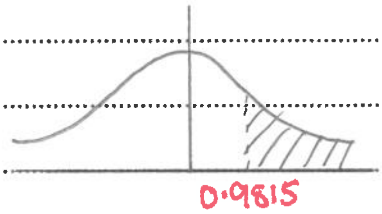
3 The weights of oranges can be modelled by a normal distribution with mean 131 grams and standard deviation 54 grams. Oranges are classified as small, medium or large. A large orange weighs at least 184 grams and 20% of oranges are classified as small.

(a) Find the percentage of oranges that are classified as large. [3]

$P(W > 184)$

$= P\left(Z > \dfrac{184 - 131}{54}\right)$

$= P(Z > 0.9815)$



$= 1 - \Phi(0.9815)$

$= 1 - 0.8369$ "add 4" in table because in the middle of 0.981 and 0.982

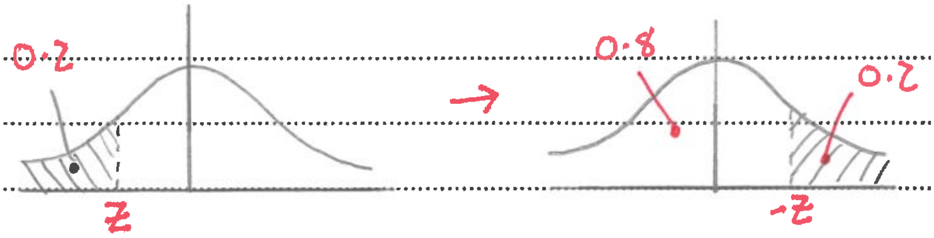
$= 0.1631$

Percentage: 16.3% (3sf)

(b) Find the greatest possible weight of a small orange. [3]

$P(W < S) = 0.2$

$P\left(Z < \frac{S-131}{54}\right) = 0.2$



$0.8 = \Phi(0.842)$

$-z = 0.842$

$z = -0.842$

$\frac{S-131}{54} = -0.842$

$S - 131 = -45.468$

$S = 85.532$

$= \underline{85.5\text{g}}$ (3sf)

4 The back-to-back stem-and-leaf diagram shows the annual salaries of 19 employees at each of two companies, Petral and Ravon.

			Petral						Ravon		
			3	0	0	30	2	6			
9	9	8	2	2	1	31	1	5			
		5	5	4	0	32	0	0	2		
			7	5	3	33	0	4	8	9	
				1	0	34	1	1	3	4	6
						35	3				
					8	36	7	9			

Key: 2 | 31 | 5 means \$31 200 for a Petral employee and \$31 500 for a Ravon employee.

(a) Find the median and the interquartile range of the salaries of the Petral employees. [3]

$Q_1: \frac{19+1}{4} = 5^{th}$ $\quad Q_2: \frac{19+1}{2} = 10^{th}$

$Q_3: \frac{3(19+1)}{4} = 15^{th}$

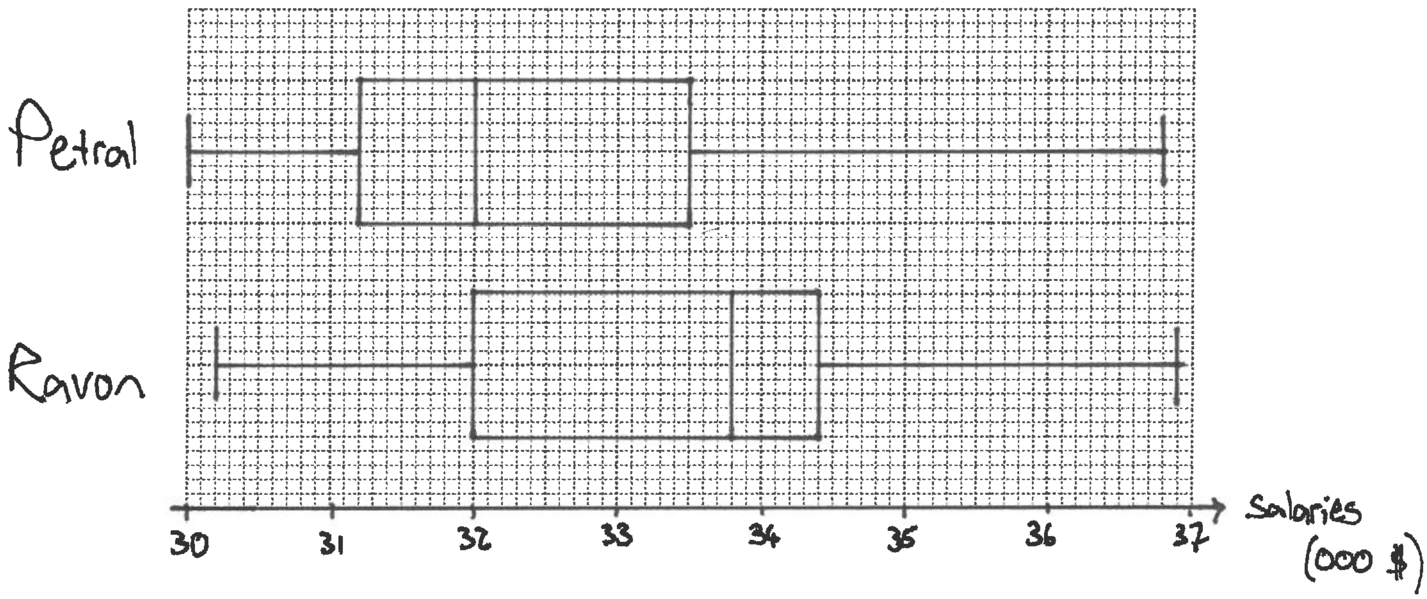
Median $= \$32\,000$

IQR $= 33\,500 - 31\,200$

$= \$2300$

The median salary of the Ravon employees is \$33 800, the lower quartile is \$32 000 and the upper quartile is \$34 400.

(b) Represent the data shown in the back-to-back stem-and-leaf diagram by a pair of box-and-whisker plots in a single diagram. [3]



(c) Comment on whether the mean or the median would be a better representation of the data for the employees at Petral. [1]

The median would be better because it isn't affected by the outlier (\$36 800).

5 Jasmine has one \$5 coin, two \$2 coins and two \$1 coins. She selects two of these coins at random. The random variable X is the total value, in dollars, of these two coins.

(a) Show that $P(X = 7) = 0.2$. [1]

\$7 = one \$5 and one \$2:

$${}^{1}C_{1} \times {}^{2}C_{1} = 2$$

one \$5 from one, one \$2 from two

$$P(X=7) = \frac{2}{{}^{5}C_{2}} = 0.2 \quad \text{QED}$$

any two coins from five

(b) Draw up the probability distribution table for X. [3]

\$2: $\frac{{}^{2}C_{2}}{{}^{5}C_{2}} = 0.1$ \$3: $\frac{{}^{2}C_{1} \times {}^{2}C_{1}}{{}^{5}C_{2}} = 0.4$

\$4: $\frac{{}^{2}C_{2}}{{}^{5}C_{2}} = 0.1$ \$5: X

\$6: $\frac{{}^{1}C_{1} \times {}^{2}C_{1}}{{}^{5}C_{2}} = 0.2$ \$7: 0.2 (part (a))

x	2	3	4	6	7
$P(X=x)$	0.1	0.4	0.1	0.2	0.2

(c) Find the value of $\mathrm{Var}(X)$. [3]

$E(x)$:

$E(x) = 2\times0.1 + 3\times0.4 + 4\times0.1 + 6\times0.2 + 7\times0.2$

$= 0.2 + 1.2 + 0.4 + 1.2 + 1.4$

$= \underline{4.4}$

$\mathrm{Var}(x)$:

$\mathrm{Var}(x) = 2^2\times0.1 + 3^2\times0.4 + 4^2\times0.1 + 6^2\times0.2 + 7^2\times0.2 - (E(x))^2$

$= 0.4 + 3.6 + 1.6 + 7.2 + 9.8 - (4.4)^2$

$= 22.6 - 19.36$

$= \underline{3.24}$

6 The residents of Mahjing were asked to classify their local bus service:

- 25% of residents classified their service as good.
- 60% of residents classified their service as satisfactory.
- 15% of residents classified their service as poor.

(a) A random sample of 110 residents of Mahjing is chosen.

Use a suitable approximation to find the probability that fewer than 22 residents classified their bus service as good. [5]

$G \sim B(110, 0.25)$

$\mu = 110 \times 0.25$ $\qquad$ $\sigma^2 = 27.5 \times 0.75$

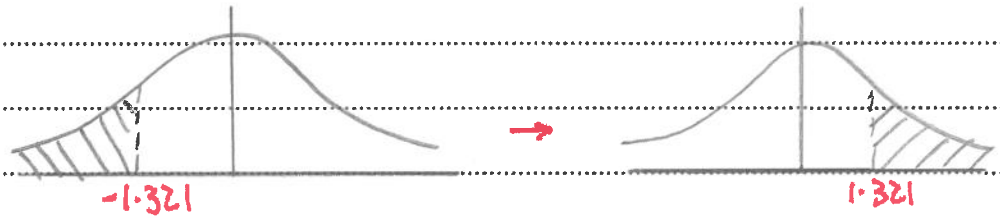
$= 27.5$ $\qquad$ $= 20.625$

$G \sim N(27.5, 20.625)$

$P(G < 22) \rightarrow P(G < 21.5)$ (continuity correction)

$P\left(Z < \dfrac{21.5 - 27.5}{\sqrt{20.625}}\right)$

$= P(Z < -1.321)$



$= 1 - \Phi(1.321)$

$= 1 - 0.9068$

$= 0.0932$

(b) For a random sample of 10 residents of Mahjing, find the probability that fewer than 8 classified their bus service as good or satisfactory. [3]

$X \sim B(10, 0.85)$ probability of good or satisfactory

$P(X<8) = 1 - (P(8) + P(9) + P(10))$

$= 1 - ({}^{10}C_8 \times 0.85^8 \times 0.15^2 + {}^{10}C_9 \times 0.85^9 \times 0.15^1 + {}^{10}C_{10} \times 0.85^{10} \times 0.15^0)$

$= \underline{\underline{0.180}}$

(c) Three residents of Mahjing are selected at random.

Find the probability that one resident classified the bus service as good, one as satisfactory and one as poor. [2]

P(Good, Satisfactory, poor in that order):

$= 0.25 \times 0.6 \times 0.15$

$= 0.0225$

Could be in any order, so multiply by 3! :

$0.0225 \times 3! = \underline{\underline{0.135}}$

7 (a) How many different arrangements are there of the 10 letters in the word REGENERATE? [1]

RREEEEGNAT

$$\frac{10!}{2! \times 4!} = \underline{75\,600}$$

2 Rs 4 Es

(b) How many different arrangements are there of the 10 letters in the word REGENERATE in which the 4 Es are together and the 2 Rs have exactly 3 letters in between them? [4]

fixed fixed

R ___ ___ ___ R

Not enough space for the 4 Es, so pick three letters from GNAT and permutate them:

$^4P_3$

Now treat the group with the Rs as one object and permutate with the four Es and one remaining letter:

R - - - R EEEE 1 remaining letter

$3!$

$\rightarrow {}^4P_3 \times 3! = \underline{144}$

(c) Find the probability that a randomly chosen arrangement of the 10 letters in the word REGENERATE is one in which the consonants (G, N, R, R, T) and vowels (A, E, E, E, E) alternate, so that no two consonants are next to each other and no two vowels are next to each other. [5]

Picking letters at random, NOT looking for different arrangements, so the Rs and Es are treated as distinguishable:

Arrange the consonants first:

$$5!$$

Arrange the vowels into either the first five spaces or the last five spaces:

either: $^5P_5$ ← 5 spaces (5 letters)

or

or: $^5P_5$

$$\rightarrow 5! \times {}^5P_5 \times 2 = 28800$$

Total possible arrangements: $10! = 3628800$

$$\text{probability} = \frac{28800}{3628800} = \frac{1}{126}$$