1 A summary of 20 values of x gives

$$\Sigma(x-30) = 439, \qquad \Sigma(x-30)^2 = 12\,405.$$

A summary of another 25 values of x gives

$$\Sigma(x-30) = 470, \qquad \Sigma(x-30)^2 = 11346.$$

(a) Find the mean of all 45 values of x. [2]

First 20 values: $\Sigma(x-30) = 439$

$\Sigma x = 439 + 20 \times 30$

$= 1039$

Second 25 values: $\Sigma(x-30) = 470$

$\Sigma x = 470 + 25 \times 30$

$= 1220$

Sum of all 45 values: $1039 + 1220 = 2259$

$\bar{x} = \dfrac{2259}{45} = \underline{50.2}$

(b) Find the standard deviation of all 45 values of x. [2]

Standard deviation of $x-30$ = Standard deviation of x,

so use coded values:

$$\sigma = \sqrt{\frac{\Sigma(x-30)^2}{n} - \left(\frac{\Sigma(x-30)}{n}\right)^2}$$

$$\sigma = \sqrt{\frac{12405 + 11346}{45} - \left(\frac{439 + 470}{45}\right)^2}$$

$$= \sqrt{\frac{23\,751}{45} - \left(\frac{909}{45}\right)^2}$$

$= \underline{10.9}$

2 The lengths of the tails of adult raccoons of a certain species are normally distributed with mean 28 cm and standard deviation 3.3 cm.

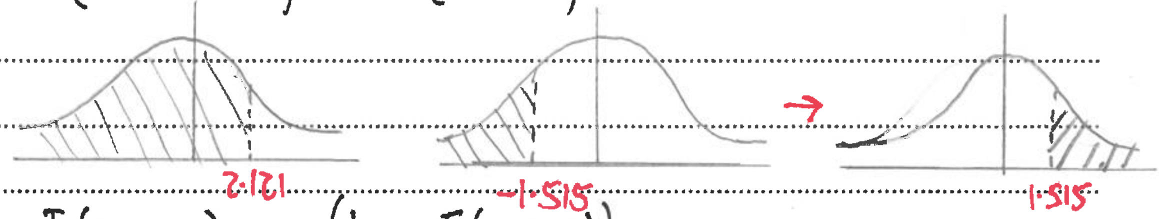
(a) Find the probability that a randomly chosen adult raccoon of this species has a tail length between 23 cm and 35 cm. [4]

$P(23 < L < 35)$

$= P\left(\frac{23-28}{3.3} < Z < \frac{35-28}{3.3}\right)$

$= P(-1.515 < Z < 2.121)$

$= P(Z < 2.121) - P(-1.515)$



$= \Phi(2.121) - (1 - \Phi(1.515))$

$= 0.9830 - (1 - 0.9351)$

$= 0.9830 - 0.0649$

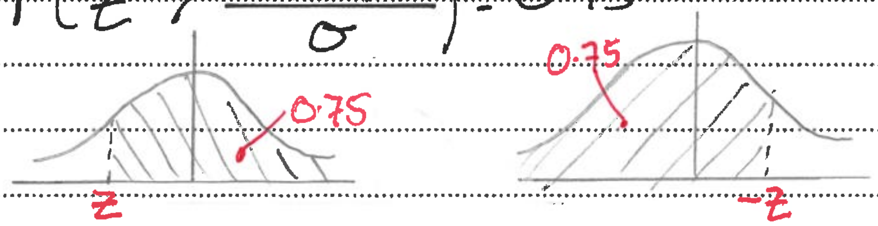
$= \underline{0.9181}$

The masses of adult raccoons of this species are normally distributed with mean 8.5 kg and standard deviation σ kg. 75% of adult raccoons of this species have mass greater than 7.6 kg.

(b) Find the value of σ. [3]

$P(M > 7.6) = 0.75$

$P\left(Z > \frac{7.6-8.5}{\sigma}\right) = 0.75$



Critical value

$\underline{0.75} = \Phi(0.674)$

$\rightarrow z = -0.674$

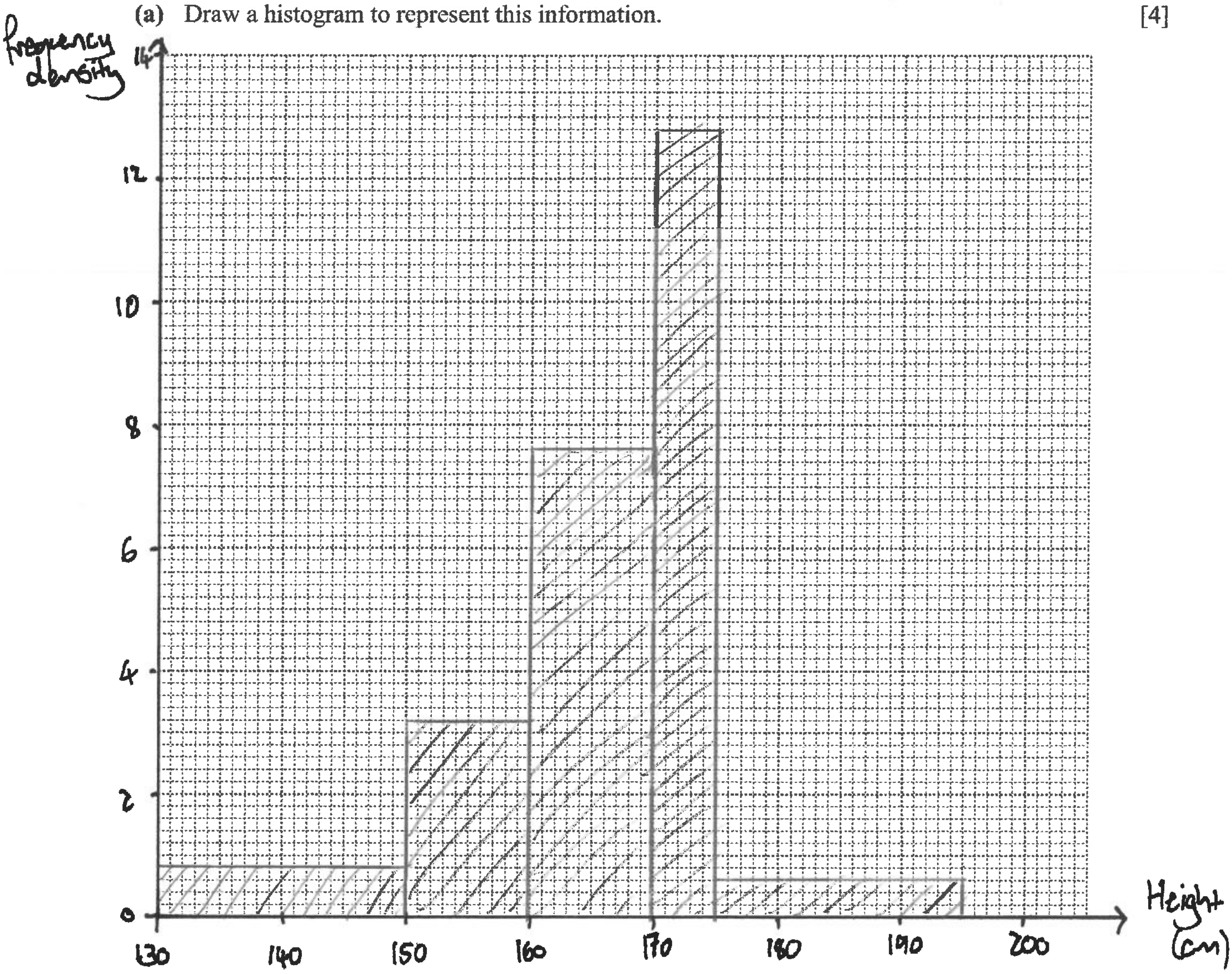
$\rightarrow \frac{7.6-8.5}{\sigma} = -0.674$

$-0.9 = -0.674\sigma \rightarrow \underline{\sigma = 1.34}$

3 The heights, in cm, of 200 adults in Barimba are summarised in the following table.

Height (h cm)	$130 \leqslant h < 150$	$150 \leqslant h < 160$	$160 \leqslant h < 170$	$170 \leqslant h < 175$	$175 \leqslant h < 195$
Frequency	16	32	76	64	12
Class width	20	10	10	5	20
f.d.	0·8	3·2	7·6	12·8	0·6

(a) Draw a histogram to represent this information. [4]



(b) The interquartile range is R cm. Show that R is **not** greater than 15. [2]

200 adults, so $Q_1 = 50^{th}$ person
and $Q_3 = 150^{th}$ person

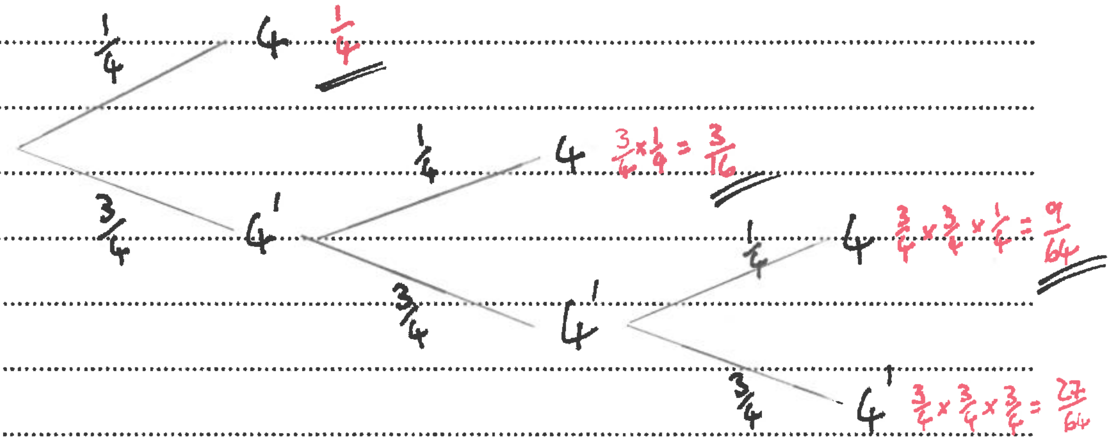
so Q_1 lies in $160 \leq h < 170$
and Q_3 lies in $170 \leq h < 175$

Max. value for IQR would use the lowest possible value of Q_1 and the highest possible value of Q_3:

$Q_3(\text{max}) - Q_1(\text{min})$
$= 175 - 160$
Max IQR $= 15$
→ so R cannot be greater than 15.

4 A game for two players is played using a fair 4-sided dice with sides numbered 1, 2, 3 and 4. One turn consists of throwing the dice repeatedly up to a maximum of three times. When a 4 is obtained, no further throws are made during that turn. A player who obtains a 4 in their turn scores 1 point.

(a) Show that the probability that a player obtains a 4 in one turn is $\frac{37}{64}$. [2]



$\frac{1}{4} + \frac{3}{16} + \frac{9}{64} = \frac{37}{64}$ QED

Xeno and Yao play this game.

(b) Find the probability that neither Xeno nor Yao score any points in their first two turns. [1]

$P(\text{no points}) = \frac{27}{64}$

$P(\text{four} \times \text{no points}) = \left(\frac{27}{64}\right)^4$

$= 0.0317$

(c) Xeno and Yao each have three turns.

Find the probability that Xeno scores 2 more points than Yao. [3]

$W \sim B(3, \frac{37}{64})$

Xeno 2, Yao 0:

$P(X=2) \times P(Y=0)$:

${}^3C_2 \times \left(\frac{37}{64}\right)^2 \times \left(\frac{27}{64}\right) \times {}^3C_0 \times \left(\frac{37}{64}\right)^0 \times \left(\frac{27}{64}\right)^3$

$= 0.03176\ldots$ STO

Xeno 3, Yao 1:

$P(X=3) \times P(Y=1)$:

${}^3C_3 \times \left(\frac{37}{64}\right)^3 \times \left(\frac{27}{64}\right)^0 \times {}^3C_1 \times \left(\frac{37}{64}\right)^1 \times \left(\frac{27}{64}\right)^2$

$= 0.05964\ldots$ STO

$\rightarrow 0.03176\ldots + 0.05964\ldots = \underline{\underline{0.0914}}$

5 In a certain area in the Arctic the probability that it snows on any given day is 0.7, independent of all other days.

(a) Find the probability that in a week (7 days) it snows on at least five days. [3]

$S \sim B(7, 0.7)$

$P(S \geq 5) = P(5) + P(6) + P(7)$

$= {}^7C_5 \times 0.7^5 \times 0.3^2 + {}^7C_6 \times 0.7^6 \times 0.3^1 + {}^7C_7 \times 0.7^7 \times 0.3^0$

$= \underline{0.647}$ STO

A week in which it snows on at least five days out of seven is called a 'white' week.

(b) Find the probability that in three randomly chosen weeks at least one is a white week. [2]

$W \sim B(3, 0.647)$

$P(W \geq 1) = 1 - P(0)$

$= 1 - {}^3C_0 \times 0.647^0 \times 0.353^3$

$= 1 - 0.0440$

$= \underline{0.956}$

In a different area in the Arctic, the probability that a week is a white week is 0.8 .

(c) Use a suitable approximation to find the probability that in 60 randomly chosen weeks fewer than 47 are white weeks. [5]

$W \sim B(60, 0.8)$

$\mu = 60 \times 0.8$ $\qquad\qquad$ $\sigma^2 = 48 \times 0.2$

$= 48$ $\qquad\qquad\qquad\quad$ $= 9.6$

$W \sim N(48, 9.6)$

$P(W < 47) \rightarrow P(W < 46.5)$ (continuity correction)

$P\left(Z < \dfrac{46.5 - 48}{\sqrt{9.6}}\right)$

$= P(Z < -0.484)$

-0.484 $\rightarrow$ 0.484

$= 1 - \Phi(0.484)$

$= 1 - 0.6858$

$= \underline{0.3142}$

6 Harry has three coins:

- One coin is biased so that the probability of obtaining a head when it is thrown is $\frac{1}{3}$. A
- The second coin is biased so that the probability of obtaining a head when it is thrown is $\frac{1}{4}$. B
- The third coin is biased so that the probability of obtaining a head when it is thrown is $\frac{1}{5}$. C

Harry throws the three coins. The random variable X is the number of heads that he obtains.

(a) Draw up the probability distribution table for X. [4]

Coin

	A	B	C		
1H	H	T	T	$\frac{1}{3} \times \frac{3}{4} \times \frac{4}{5} = \frac{1}{5}$	+ = $\frac{13}{30}$
	T	H	T	$\frac{2}{3} \times \frac{1}{4} \times \frac{4}{5} = \frac{2}{15}$	
	T	T	H	$\frac{2}{3} \times \frac{3}{4} \times \frac{1}{5} = \frac{1}{10}$	
2H	H	H	T	$\frac{1}{3} \times \frac{1}{4} \times \frac{4}{5} = \frac{1}{15}$	+ = $\frac{3}{20}$
	H	T	H	$\frac{1}{3} \times \frac{3}{4} \times \frac{1}{5} = \frac{1}{20}$	
	T	H	H	$\frac{2}{3} \times \frac{1}{4} \times \frac{1}{5} = \frac{1}{30}$	
3H	H	H	H	$\frac{1}{3} \times \frac{1}{4} \times \frac{1}{5} = \frac{1}{60}$	$\frac{1}{60}$
0H	T	T	T	$\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} = \frac{2}{5}$	$\frac{2}{5}$

x	0	1	2	3
$P(X = x)$	$\frac{2}{5}$	$\frac{13}{30}$	$\frac{3}{20}$	$\frac{1}{60}$

Harry has two other coins, each of which is biased so that the probability of obtaining a head when it is thrown is p. He throws all five coins at the same time. The random variable Y is the number of heads that he obtains.

(b) Given that $P(Y = 0) = 6P(Y = 5)$, find the value of p. [3]

Other two coins:

$P(H) = p$

$P(T) = 1 - p$

$P(Y=5) = \frac{1}{60} \times p^2$

first 3 coins ↗ ↖ other 2

$P(Y=0) = \frac{2}{5} \times (1-p)^2$

→ $\frac{2}{5}(1-p)^2 = 6 \times \frac{1}{60}p^2$

$\frac{2}{5}(1-2p+p^2) = \frac{1}{10}p^2$

$\frac{2(1-2p+p^2)}{5} = \frac{p^2}{10}$

$20(1-2p+p^2) = 5p^2$

$20 - 40p + 20p^2 = 5p^2$

$15p^2 - 40p + 20 = 0$

$3p^2 - 8p + 4 = 0$

$(3p-2)(p-2) = 0$

$p = \frac{2}{3}$ or $p = 2$ X probability can't be > 1.

7 The eight digits 1, 2, 2, 3, 4, 4, 4, 5 are arranged in a line.

(a) How many different arrangements are there of these 8 digits? [1]

$$\frac{8!}{2! \times 3!} = \underline{3360}$$

two 2s three 4s

(b) Find the number of different arrangements of the 8 digits in which there is a 2 at the beginning, a 2 at the end and the three 4s are not all together. [4]

With 2s at ends and no other restrictions:

fixed fixed

2 __ __ __ __ __ __ 2

$$\frac{6!}{3!} = 120$$

three 4s

With 2s at ends and 4s together:

fixed fixed

2 __ __ __ __ 2

one object → 444

$$4! = 24$$

With 2s at ends and 4s not together:

$$120 - 24 = \underline{96}$$

Three digits are selected at random from the eight digits 1, 2, 2, 3, 4, 4, 4, 5.

(c) Find the probability that the three digits are all different. [5]

Picking digits at random, NOT looking for different selections, so the 2s and 4s are distinguishable.

one 4, one 2: 4 2 ___ $\quad {}^3C_1 \times {}^2C_1 \times {}^3C_1 = 18$

pick one 4 from 3. pick one 2 from 2 pick one from 1,3,5

one 4: 4 ___ ___ $\quad {}^3C_1 \times {}^3C_2 = 9$

pick two from 1,3,5

one 2: 2 ___ ___ $\quad {}^2C_1 \times {}^3C_2 = 6$

no 4s/2s: ___ ___ ___ $\quad {}^3C_3 = 1$

$18 + 9 + 6 + 1 = \underline{34}$

Without any restrictions: ${}^8C_3 = 56$

pick any three digits from the eight.

$$\text{Probability} = \frac{34}{56} = \frac{17}{28}$$