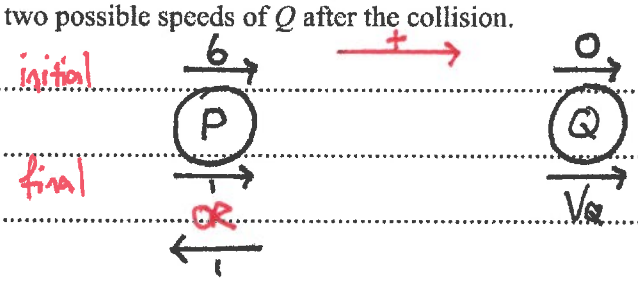
1 Two particles P and Q of masses 0.2 kg and 0.5 kg respectively are at rest on a smooth horizontal plane. Particle P is projected with a speed $6\,\mathrm{m\,s^{-1}}$ directly towards Q. After P and Q collide, P moves with a speed of $1\,\mathrm{m\,s^{-1}}$.

Find the two possible speeds of Q after the collision. [3]



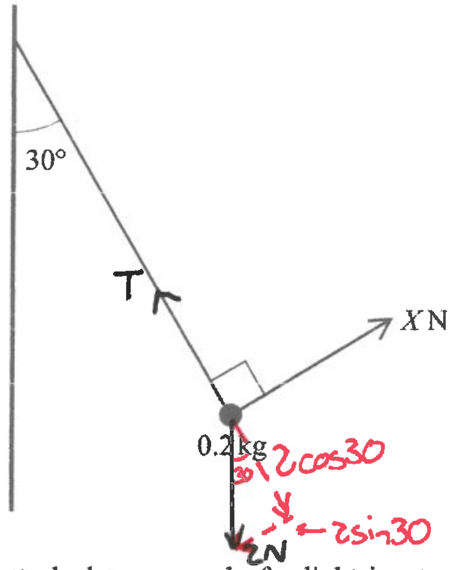
①: P moves to the right after collision:

$$m_P u_P + m_Q u_Q = m_P V_P + m_Q V_Q$$
$$0.2 \times 6 + 0 = 0.2 \times 1 + 0.5 V_Q$$
$$1.2 = 0.2 + 0.5 V_Q$$
$$1 = 0.5 V_Q$$
$$\underline{\underline{V_Q = 2\,\mathrm{ms^{-1}}}}$$

②: P moves to the left after collision:

$$m_P u_P + m_Q u_Q = m_P V_P + m_Q V_Q$$
$$0.2 \times 6 + 0 = 0.2 \times -1 + 0.5 V_Q$$
$$1.2 = -0.2 + 0.5 V_Q$$
$$1.4 = 0.5 V_Q$$
$$\underline{\underline{V_Q = 2.8\,\mathrm{ms^{-1}}}}$$

2



A particle of mass 0.2 kg is attached to one end of a light inextensible string. The other end of the string is attached to a fixed point on a vertical wall. The particle is held in equilibrium by a force of magnitude X N, perpendicular to the string, with the string taut and making an angle of 30° with the wall (see diagram).

Find the tension in the string and the value of X. [3]

Note: It's easier to resolve in the directions of T and X, rather than vertically and horizontally, but both methods work.

$R(\nwarrow)$: $T - 2\cos 30 = 0$

$T = 2\cos 30$

$T = \sqrt{3}$ N

$R(\nearrow)$: $X - 2\sin 30 = 0$

$X = 2\sin 30$

$X = 1$ N

3 A car travels along a straight road with constant acceleration $a\,\mathrm{m\,s^{-2}}$, where $a > 0$. The car passes through points A, B and C in that order. The speed of the car at A is $u\,\mathrm{m\,s^{-1}}$ in the direction AB. The distance BC is twice the distance AB. The car takes 8 seconds to travel from A to B and 10 seconds to travel from B to C.

(a) Find u in terms of a. [4]

AB: $s = x$	$s = ut + \frac{1}{2}at^2$
$u = u$	$x = u \times 8 + \frac{1}{2} \times a \times 8^2$
$v =$	$= 8u + \frac{1}{2} \times a \times 64$
$a = a$	$x = 8u + 32a$ ①
$t = 8$	

Distance BC $= 2x$ so distance AC $= x + 2x = 3x$:

AC: $s = 3x$	$s = ut + \frac{1}{2}at^2$
$u = u$	$3x = u \times 18 + \frac{1}{2} \times a \times 18^2$
$v =$	$= 18u + \frac{1}{2} \times a \times 324$
$a = a$	$3x = 18u + 162a \quad \div 3$
$t = 18$	$x = 6u + 54a$ ②

① = ②:

$$8u + 32a = 6u + 54a$$
$$2u = 22a$$
$$\underline{\underline{u = 11a}}$$

(b) Find the speed of the car at C in terms of a. [2]

AC: $s =$	$v = u + at$
$u = 11a$	$= 11a + a \times 18$
$v =$	$v = \underline{\underline{29a\ \mathrm{ms^{-1}}}}$
$a = a$	
$t = 18$	

4 A particle travels in a straight line. The velocity of the particle at time t s after leaving a point O is $v\,\mathrm{m\,s^{-1}}$, where

$$v = kt^2 - 4t + 3.$$

The distance travelled by the particle in the first 2 s of its motion is 6 m. You may assume that $v > 0$ in the first 2 s of its motion.

↳ i.e. no stationary points, so it doesn't turn around.

(a) Find the value of k. [4]

$$S = \int_0^2 (kt^2 - 4t + 3)\,dt$$

$$= \left[\tfrac{1}{3}kt^3 - 2t^2 + 3t\right]_0^2$$

$$= \left[\tfrac{1}{3}k \times 2^3 - 2(2)^2 + 3(2)\right] - [0]$$

$$= \tfrac{1}{3}k \times 8 - 8 + 6$$

$$= \frac{8k}{3} - 2$$

$S = 6$:

$$\frac{8k}{3} - 2 = 6$$

$$\frac{8k}{3} = 8$$

$$8k = 24$$

$$k = 3$$

(b) Find the value of the minimum velocity of the particle. You do **not** need to show that this velocity is a minimum. [3]

$$v = 3t^2 - 4t + 3$$

$$\frac{dv}{dt} = 6t - 4$$

SP, so $\frac{dv}{dt} = 0$:

$$6t - 4 = 0$$

$$6t = 4$$

$$t = \tfrac{2}{3}$$

Sub. into v:

$$v = 3\left(\tfrac{2}{3}\right)^2 - 4\left(\tfrac{2}{3}\right) + 3$$

$$= 3 \times \tfrac{4}{9} - \tfrac{8}{3} + 3$$

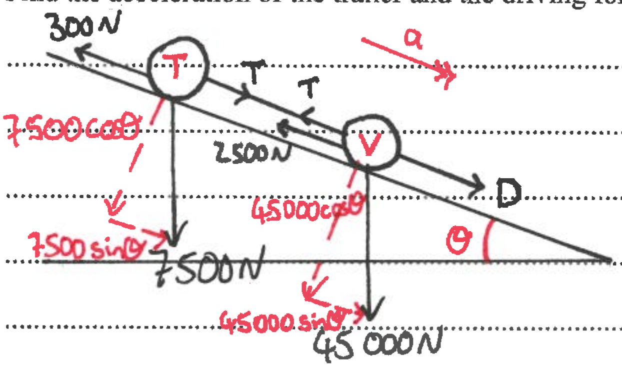
$$= \tfrac{4}{3} - \tfrac{8}{3} + 3$$

$$= \tfrac{5}{3}\ \mathrm{ms^{-1}}$$

5 A van of mass 4500 kg is towing a trailer of mass 750 kg down a straight hill inclined at an angle of θ to the horizontal where $\sin\theta = 0.05$. The van and the trailer are connected by a light rigid tow-bar which is parallel to the road. There are constant resistance forces of 2500 N on the van and 300 N on the trailer.

(a) It is given that the tension in the tow-bar is 450 N.

Find the acceleration of the trailer and the driving force of the van's engine. [4]



Van:

$R(\searrow)$: $F = ma$

$D + 45000\sin\theta - 2500 - T = 4500a$ (T = 450)

$D + 45000(0.05) - 2500 - 450 = 4500a$

$D + 2250 - 2500 - 450 = 4500a$

$D - 700 = 4500a$ (1)

Trailer:

$R(\searrow)$: $F = ma$

$T + 7500\sin\theta - 300 = 750a$

$450 + 7500(0.05) - 300 = 750a$

$450 + 375 - 300 = 750a$

$525 = 750a$

$a = 0.7\ \text{ms}^{-2}$

Sub into (1):

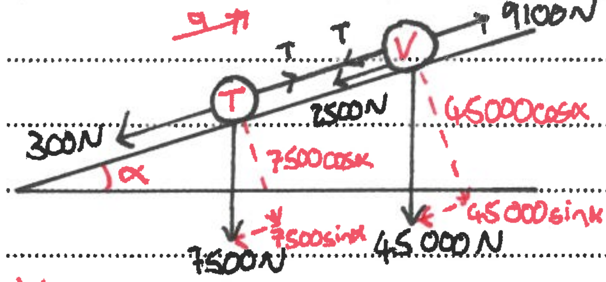
$D - 700 = 4500 \times 0.7$

$D - 700 = 3150$

$D = 3850\text{N}$

On another occasion, the van and trailer ascend a straight hill inclined at an angle of α to the horizontal where $\sin\alpha = 0.09$. The driving force of the van's engine is now 9100 N, and the speed of the van at the bottom of the hill is $20\,\mathrm{m\,s^{-1}}$. The resistances to motion are unchanged.

(b) **(i)** Find the acceleration of the van and the tension in the tow-bar. [5]



Van:

$R(\nearrow)$: $9100 - 45000\sin\alpha - 2500 - T = 4500a$

$9100 - 45000(0.09) - 2500 - T = 4500a$

$2550 - T = 4500a$ ①

Trailer:

$R(\nearrow)$: $T - 7500\sin\alpha - 300 = 750a$

$T - 7500(0.09) - 300 = 750a$

$T - 975 = 750a$ ②

①+②: $2550 - 975 = 5250a$

$1575 = 5250a$

$a = 0.3\,\mathrm{ms^{-2}}$

②: $T - 975 = 750(0.3)$

$T - 975 = 225$

$T = 1200\,\mathrm{N}$

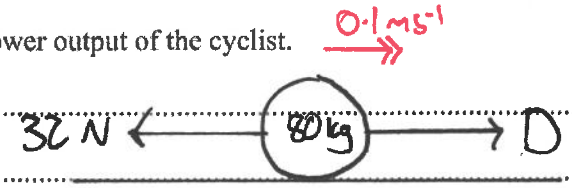
(ii) Find the speed of the van when it has travelled a distance of 375 m up the hill. [2]

+↗

$s = 375$	$v^2 = u^2 + 2as$
$u = 20$	$= 20^2 + 2 \times 0.3 \times 375$
$v =$	$= 400 + 225$
$a = 0.3$	$= 625$
$t =$	$v = 25\,\mathrm{ms^{-1}}$

6 A cyclist is travelling along a straight horizontal road. The total mass of the cyclist and her bicycle is 80 kg. There is a constant resistance force of magnitude 32 N to the cyclist's motion. At an instant when she is travelling at $7\,\mathrm{m\,s^{-1}}$, her acceleration is $0.1\,\mathrm{m\,s^{-2}}$.

(a) Find the power output of the cyclist. [3]



R(→): $F = ma$

$D - 32 = 80 \times 0.1$

$D - 32 = 8$

$D = 40\,\mathrm{N}$

Power $= D \times v$

$= 40 \times 7$

$= 280\,\mathrm{W}$

(b) Find the steady speed that the cyclist can maintain if her power output and the resistance force are both unchanged. [2]

At steady speed, $a = 0$:

R(→): $F = m\underbrace{a}_{=0}$

$D - 32 = 0$

$D = 32\,\mathrm{N}$

Power $= D \times v$

$280 = 32v$

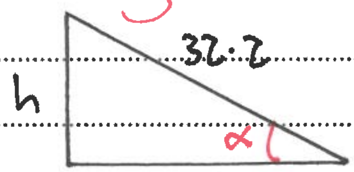
$v = 8.75\,\mathrm{m\,s^{-1}}$

$\sin\alpha = \frac{1}{20}$

The cyclist later descends a straight hill of length 32.2 m, inclined at an angle of $\sin^{-1}\left(\frac{1}{20}\right)$ to the horizontal. Her power output is now 120 W, and the resistance force now has variable magnitude such that the work done against this force in descending the hill is 1128 J. The time taken to descend the hill is 4 s.

(c) Given that the speed of the cyclist at the top of the hill is $7.5\,\text{m s}^{-1}$, find her speed at the bottom of the hill. [6]

Change in height:



$\sin\alpha = \frac{h}{32.2}$

$\frac{1}{20} = \frac{h}{32.2}$

$h = 1.61\text{m}$ (downwards)

$\text{Power} = \frac{\text{Work done}}{\text{time}}$

$120 = \frac{\text{Work done}}{4}$

Work done $= 480\text{J}$

$\text{Work}_{\text{in}} + KE_{\text{init}} + PE_{\text{init}} = KE_{\text{fin}} + PE_{\text{fin}} + \text{Work}_{\text{out}}$

$480 + \frac{1}{2}\times 80\times 7.5^2 + 0 = \frac{1}{2}\times 80\times v^2 + 80\times 10\times -1.61 + 1128$

$480 + 2250 = 40v^2 - 1288 + 1128$

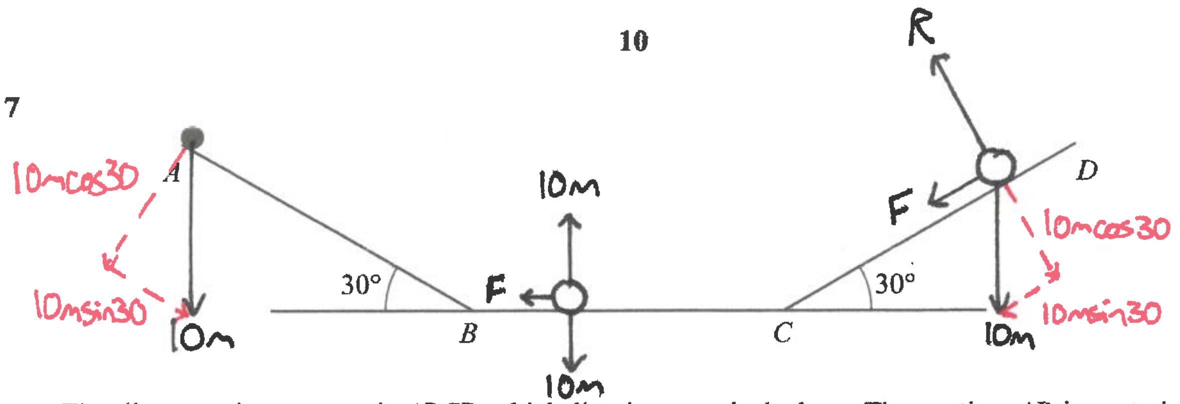
$2730 = 40v^2 - 160$

$40v^2 = 2890$

$v^2 = 72.25$

$v = 8.5\text{ms}^{-1}$

7



The diagram shows a track $ABCD$ which lies in a vertical plane. The section AB is a straight line inclined at an angle of 30° to the horizontal and is smooth. The section BC is a horizontal straight line and is rough. The section CD is a straight line inclined at an angle of 30° to the horizontal and is rough. The lengths AB, BC and CD are each 2 m.

A particle is released from rest at A. The coefficient of friction between the particle and both BC and CD is μ. There is no change in the speed of the particle when it passes through either of the points B or C.

(a) It is given that $\mu = 0.1$.

Find the distance which the particle has moved up the section CD when its speed is $1\,\mathrm{m\,s^{-1}}$. [5]

AB: (smooth)

$R(\searrow)$: $F = ma$ | $s = 2$ | $v^2 = u^2 + 2as$
$10m\sin 30 = ma$ | $u = 0$ | $= 0^2 + 2\times5\times2$
$5m = ma$ | $v =$ | $v^2 = 20$
$a = 5$ | $a = 5$ | $v = \sqrt{20}$
$t =$

BC: (rough)

$R(\rightarrow)$: $F = ma$ | $s = 2$ | $v^2 = u^2 + 2as$
$-\mu R = ma$ | $u = \sqrt{20}$ | $= \sqrt{20}^2 + 2(-1)\times2$
$-0.1\times10m = ma$ | $v =$ | $= 20 - 4$
$-1m = ma$ | $a = -1$ | $v^2 = 16$
$a = -1$ | $t =$ | $v = 4$

CD: (rough)

$R(\nearrow)$: $F = ma$ | $s =$ | $v^2 = u^2 + 2as$
$-10m\sin 30 - \mu R = ma$ | $u = 4$ | $1^2 = 4^2 + 2\times-5.866\times s$
$-5m - 0.1\times10m\cos 30 = ma$ | $v = 1$ | $1 = 16 - 11.73s$
$-5m - \frac{\sqrt{3}}{2}m = ma$ | $a = -5.866$ | $11.73s = 15$
$-5 - \frac{\sqrt{3}}{2} = a$ | $t =$ | $s = 1.28\text{m}$
$a = -5.866$ 5sf

(b) It is given instead that with a different value of μ the particle travels 1 m up the track from C before it comes instantaneously to rest.

Find the value of μ and the speed of the particle at the instant that it passes C for the second time. [4]

No change to AB section.

BC: (rough)

$R(\rightarrow)$: $F = ma$

$-\mu R = ma$

$-\mu \times 10m = ma$

$a = -10\mu$

$s = 2$
$u = \sqrt{20}$
$v =$
$a = -10\mu$
$t =$

$v^2 = u^2 + 2as$
$= \sqrt{20}^2 + 2 \times -10\mu \times 2$
$= 20 - 40\mu$
$v = \sqrt{20 - 40\mu}$

1m up CD: (rough)

$R(\nearrow)$: $F = ma$

$-10m\sin 30 - \mu R = ma$

$-5m - \mu \times 10m\cos 30 = ma$

$-5m - 5\sqrt{3}\mu m = ma$

$a = -5 - 5\sqrt{3}\mu$

$+\nearrow$

$s = 1$
$u = \sqrt{20 - 40\mu}$
$v = 0$
$a = -5 - 5\sqrt{3}\mu$
$t =$

$v^2 = u^2 + 2as$

$0^2 = \sqrt{20 - 40\mu}^2 + 2(-5 - 5\sqrt{3}\mu) \times 1$

$0 = 20 - 40\mu - 10 - 10\sqrt{3}\mu$

$0 = 10 - 40\mu - 10\sqrt{3}\mu$

$40\mu + 10\sqrt{3}\mu = 10$

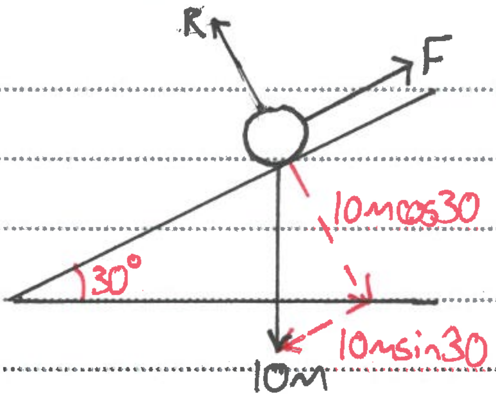
$\mu(40 + 10\sqrt{3}) = 10$

$\mu = \dfrac{10}{40 + 10\sqrt{3}}$

$\mu = 0.174$ STO

CONTINUED

On way back down:



R(↙): $F = ma$

$10m\sin30 - \mu R = ma$

$5m - 0.174 \times 10m\cos30 = ma$

$5m - 1.511m = ma$

$3.489m = ma$

$a = 3.489...$ STO

$s = 1$	$v^2 = u^2 + 2as$
$u = 0$	$v^2 = 0^2 + 2 \times 3.489 \times 1$
$v =$	$v^2 = 6.9783$
$a = 3.489$	$v = 2.64\ ms^{-1}$
$t =$	