1 A cyclist and bicycle have a total mass of 72 kg. The cyclist rides along a horizontal road against a total resistance force of 28 N.

Find the total work done by the cyclist to increase his speed from $8\,\mathrm{m\,s^{-1}}$ to $16\,\mathrm{m\,s^{-1}}$ while travelling a distance of 100 metres. [3]

$Work_{IN} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + Work_{out}$

$(work = F \times d$

$W + \frac{1}{2} \times 72 \times 8^2 + 0 = \frac{1}{2} \times 72 \times 16^2 + 0 + 28 \times 100$

$W + 2304 = 9216 + 2800$

$W + 2304 = 12016$

$W = 9712\,J$

2 A particle P moves in a straight line. At time t s after leaving a point O on the line, P has velocity $v \text{ m s}^{-1}$, where $v = 44t - 6t^2 - 36$.

(a) Find the set of values of t for which the acceleration of the particle is positive. [2]

$a = \frac{dv}{dt} = 44 - 12t$

a positive when $\frac{dv}{dt} > 0$:

$44 - 12t > 0$

$44 > 12t$

$\frac{11}{3} > t$

$t < \frac{11}{3}$

(b) Find the two values of t at which P returns to O. [3]

P returns to O when $s = 0$:

$s = \int (44t - 6t^2 - 36)\,dt$

$s = 22t^2 - 2t^3 - 36t + C$

$s = 0$ when $t = 0$:

$0 = 0 + 0 + 0 + C$

$s = 22t^2 - 2t^3 - 36t$

$= -2t^3 + 22t^2 - 36t$

$= -2t(t^2 - 11t + 18)$ ← factorise

$= -2t(t-2)(t-9)$

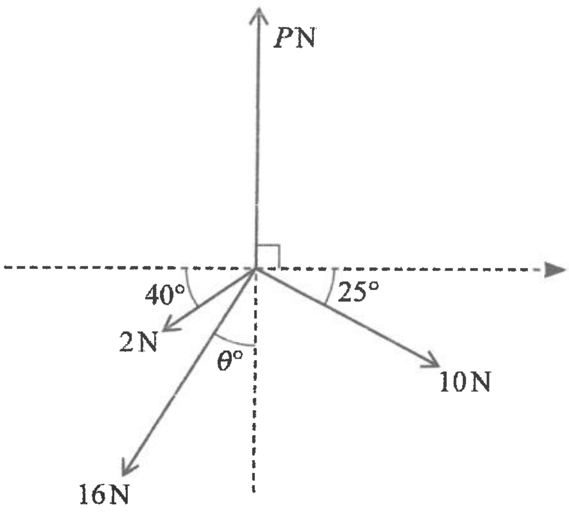
$s = 0$:

$-2t(t-2)(t-9) = 0$

$-2t = 0$ or $t - 2 = 0$ or $t - 9 = 0$

$t = 0\text{s}$ $\quad t = 2\text{s}$ $\quad t = 9\text{s}$

3



Four coplanar forces of magnitude P N, 10 N, 16 N and 2 N act at a point in the directions shown in the diagram. It is given that the forces are in equilibrium.

Find the values of θ and P. [6]

$R(\rightarrow)$: $10\cos 25 - 16\sin\theta - 2\cos 40 = 0$

$16\sin\theta = 10\cos 25 - 2\cos 40$

$\sin\theta = \dfrac{10\cos 25 - 2\cos 40}{16}$

$\theta = 28.1°$ (STORE full answer)

$R(\uparrow)$: $P - 10\sin 25 - 2\sin 40 - 16\cos(28.07\ldots) = 0$

$P = 10\sin 25 + 2\sin 40 + 16\cos(28.07\ldots)$

$= 19.6\text{N}$

4 A car has mass 1400 kg. When the speed of the car is $v\,\text{m}\,\text{s}^{-1}$ the magnitude of the resistance to motion is kv^2 N where k is a constant.

(a) The car moves at a constant speed of $24\,\text{m}\,\text{s}^{-1}$ up a hill inclined at an angle of α to the horizontal where $\sin\alpha = 0.12$. At this speed the magnitude of the resistance to motion is 480 N.

(i) Find the value of k. [1]

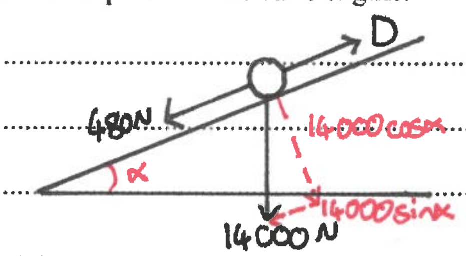
$kv^2 = 480$

$k \times 24^2 = 480$

$576k = 480$

$k = 5/6$

(ii) Find the power of the car's engine. [3]



$a = 0$ (constant speed)

$R(\nearrow)$: $F = ma$

$D - 14000\sin\alpha - 480 = 0$

$D - 14000(0.12) - 480 = 0$

$D - 1680 - 480 = 0$

$D = 2160$ N

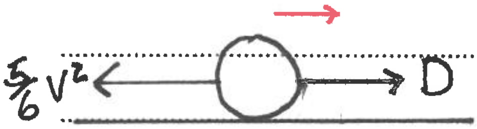
Power $= D \times V$

$= 2160 \times 24$

$= 51\,840$ W

(b) The car now moves at a constant speed on a straight level road.

Given that its engine is working at 54 kW, find this speed. [3]



Power $= Dv$

$54000 = Dv$

$D = \dfrac{54000}{v}$

$R(\rightarrow)$: $F = ma$, $a = 0$

$D - \frac{5}{6}v^2 = 0$

$\dfrac{54000}{v} - \dfrac{5}{6}v^2 = 0$

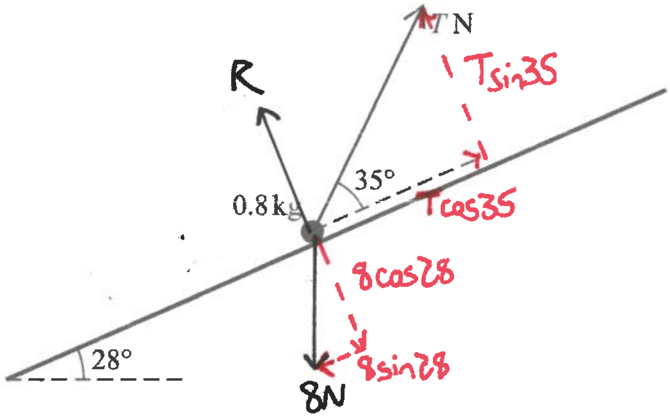
$\dfrac{5v^2}{6} = \dfrac{54000}{v}$

$5v^3 = 324000$

$v^3 = 64800$

$v = 40.2\,\text{ms}^{-1}$

5



A particle of mass 0.8 kg lies on a rough plane which is inclined at an angle of 28° to the horizontal. The particle is kept in equilibrium by a force of magnitude T N. This force acts at an angle of 35° above a line of greatest slope of the plane (see diagram). The coefficient of friction between the particle and the plane is 0.2 .

Find the least and greatest possible values of T. [8]

For greatest value of T, particle is about to move up the plane and friction is acting down the plane:

$R(\nwarrow)$: $R + T\sin 35 - 8\cos 28 = 0$

$R = \underline{8\cos 28 - T\sin 35}$

$R(\nearrow)$: $T\cos 35 - 8\sin 28 - F = 0$

$T\cos 35 - 8\sin 28 - \mu R = 0$

$T\cos 35 - 8\sin 28 - 0.2(8\cos 28 - T\sin 35) = 0$

$T\cos 35 - 8\sin 28 - 1.6\cos 28 + 0.2T\sin 35 = 0$

$T\cos 35 + 0.2T\sin 35 = 8\sin 28 + 1.6\cos 28$

$T(\cos 35 + 0.2\sin 35) = 8\sin 28 + 1.6\cos 28$

$$T = \frac{8\sin 28 + 1.6\cos 28}{\cos 35 + 0.2\sin 35}$$

$T = \underline{\underline{5.53\text{ N}}}$

continued...

For least value, particle is about to move down the plane and friction is acting up the plane.

$R(\nearrow)$: $T\cos 35 + F - 8\sin 28 = 0$

$T\cos 35 + \mu R - 8\sin 28 = 0$

$T\cos 35 + 0.2(8\cos 28 - T\sin 35) - 8\sin 28 = 0$

$T\cos 35 + 1.6\cos 28 - 0.2T\sin 35 - 8\sin 28 = 0$

$T\cos 35 - 0.2T\sin 35 = 8\sin 28 - 1.6\cos 28$

$T(\cos 35 - 0.2\sin 35) = 8\sin 28 - 1.6\cos 28$

$$T = \frac{8\sin 28 - 1.6\cos 28}{\cos 35 - 0.2\sin 35}$$

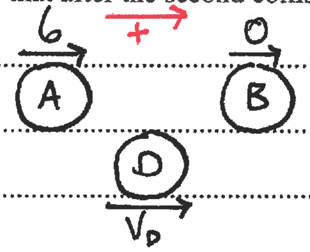
$T = \underline{3.33\text{N}}$

6 Three particles A, B and C of masses 5 kg, 1 kg and 2 kg respectively lie at rest in that order on a straight smooth horizontal track XYZ. Initially A is at X, B is at Y and C is at Z. Particle A is projected towards B with a speed of $6\,\text{m s}^{-1}$ and at the same instant C is projected towards B with a speed of $v\,\text{m s}^{-1}$. In the subsequent motion, A collides and coalesces with B to form particle D. Particle D then collides and coalesces with C to form particle E and E moves towards Z.

(a) Show that after the second collision the speed of E is $\frac{15-v}{4}\,\text{m s}^{-1}$. [3]

initial

final



$m_A u_A + m_B u_B = m_D V_D$

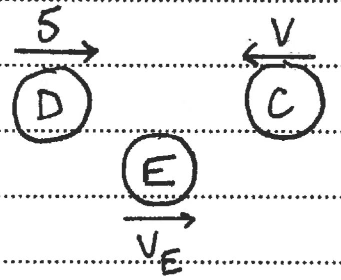
$5 \times 6 + 0 = 6V_D$

$30 = 6V_D$

$\underline{V_D = 5\,\text{ms}^{-1}}$

initial

final



$m_D u_D + m_C u_C = m_E V_E$

$6 \times 5 + 2 \times -V = 8V_E$

$30 - 2V = 8V_E$

$V_E = \dfrac{30 - 2V}{8}$

$= \dfrac{15 - V}{4}\,\text{ms}^{-1}$ QED

(b) The total loss of kinetic energy of the system due to the two collisions is 63 J.

Use the result from **(a)** to show that $v = 3$. [3]

Initial KE (A and C moving):

$KE_{init} = \frac{1}{2} \times 5 \times 6^2 + \frac{1}{2} \times 2 \times V^2$

$= 90 + V^2$

Final KE (only E moving):

$KE_{fin} = \frac{1}{2} \times 8 \times \left(\frac{15-V}{4}\right)^2$

$= 4 \times \dfrac{225 - 30V + V^2}{16}$

$= \frac{1}{4}(225 - 30V + V^2)$

Loss of KE = 63J :

$KE_{init} - KE_{fin} = 63$

$$90 + v^2 - \tfrac{1}{4}(225 - 30v + v^2) = 63 \quad \times 4$$

$$360 + 4v^2 - 225 + 30v - v^2 = 252$$

$$3v^2 + 30v - 117 = 0$$

$$v = \frac{-30 \pm \sqrt{30^2 - 4(3)(-117)}}{2(3)}$$

$v = 3$ or $v = -13$

V has to be positive, or C would be moving away from B:

$\underline{V = 3ms^{-1}}$

(c) It is given that the distance XY is 36 m and the distance YZ is 98 m.

(i) Find the time between the two collisions. [4]

Collision A + B: $\text{time} = \frac{\text{distance}}{\text{speed}} = \frac{36}{6} = \underline{6s}$

In these 6 seconds, how far has C travelled from Z?

$\text{distance} = 6 \times 3 = \underline{18m}$

Collision D + C: So there is $98 - 18 = \underline{80m}$ between D + C

$\text{distance}_D = \text{speed}_D \times \text{time} \qquad \text{distance}_C = \text{speed}_C \times \text{time}$

$d_D = 5t \qquad d_C = 3t$

$\rightarrow 80 = 5t + 3t$

$80 = 8t$

$\underline{t = 10s}$ between collisions

(ii) Find the time between the instant that A is projected from X and the instant that E reaches Z. [1]

How far from Z do D+C collide? → sub. into d_C (and add 18)

$3t + 18 = 30 + 18 = \underline{48m}$

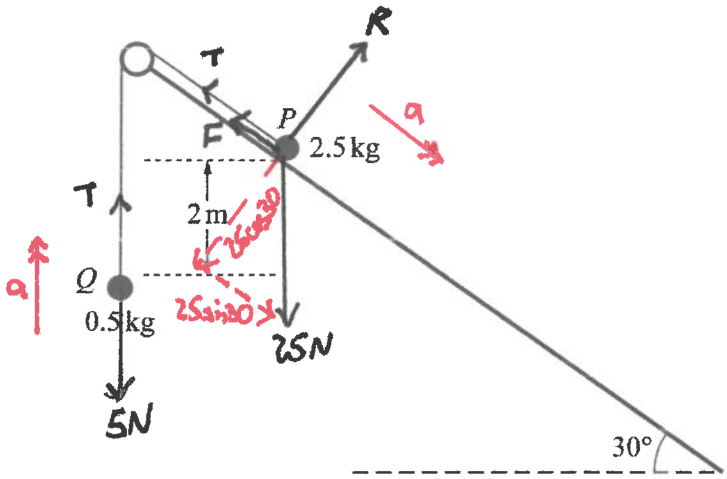
How long does it take E to travel 48m? ($V_E = 3ms^{-1}$)

$t = \frac{48}{3} = 16s$

Time from start $= 6 + 10 + 16$

$= \underline{32s}$

7



Two particles P and Q of masses 2.5 kg and 0.5 kg respectively are connected by a light inextensible string that passes over a small smooth pulley fixed at the top of a plane inclined at an angle of 30° to the horizontal. Particle P is on the plane and Q hangs below the pulley such that the level of Q is 2 m below the level of P (see diagram).

Particle P is released from rest with the string taut and slides down the plane. The plane is rough with coefficient of friction 0.2 between the plane and P.

(a) Find the acceleration of P. [5]

Q:

$R(\uparrow)$: $T - 5 = 0.5a$ ①

P:

$R(\searrow)$: $25\sin 30 - \mu R - T = 2.5a$

$12.5 - 0.2(25\cos 30) - T = 2.5a$

$12.5 - \frac{5\sqrt{3}}{2} - T = 2.5a$ ②

① + ②:

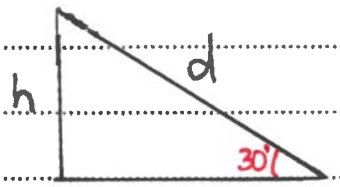
$12.5 - 5 - \frac{5\sqrt{3}}{2} = 3a$

$7.5 - \frac{5\sqrt{3}}{2} = 3a$

$a = 1.06 \text{ ms}^{-2}$ STO

(b) Use an energy method to find the speed of the particles at the instant when they are at the same vertical height. [5]

When Q moves up 'd' m, how far does P move down?



$$\sin 30 = \frac{h}{d}$$

$$h = d\sin 30$$

$$h = \tfrac{1}{2}d$$

Q and P will be at the same vertical height when the vertical distances they've travelled add up to 2m:

$$d + \tfrac{1}{2}d = 2$$

$$\tfrac{3}{2}d = 2$$

$$d = \tfrac{4}{3}\text{ m}$$

Change in height of P $= \frac{1}{2} \times \frac{4}{3} = \frac{2}{3}$ m (downwards)

Consider the Whole System:

$$\text{Work}_{IN}(P) + \text{Work}_{IN}(Q) + KE_{init}(P) + KE_{init}(Q) + PE_{init}(P) + PE_{init}(Q)$$

$$0 + T \times \tfrac{4}{3} + 0 + 0 + 0 + 0$$

$$= KE_{fin}(P) + KE_{fin}(Q) + PE_{fin}(P) + PE_{fin}(Q) + \text{Work}_{out}(P) + \text{Work}_{out}(Q)$$

$$\tfrac{1}{2}\times 2.5 \times v^2 + \tfrac{1}{2}\times 0.5 \times v^2 + 2.5 \times 10 \times -\tfrac{2}{3} + 0.5 \times 10 \times \tfrac{4}{3} + (\mu R + T)\times\tfrac{4}{3} + 0$$

$$\rightarrow \cancel{\frac{4T}{3}} = 1.25v^2 + 0.25v^2 - \frac{50}{3} + \frac{20}{3} + \frac{4}{3}\mu R + \cancel{\frac{4T}{3}}$$

$$0 = 1.5v^2 - 10 + \tfrac{4}{3} \times 0.2 \times 25\cos 30$$

$$0 = 1.5v^2 - 10 + \frac{10\sqrt{3}}{3}$$

$$1.5v^2 = 10 - \frac{10\sqrt{3}}{3}$$

$$v^2 = \frac{20}{3} - \frac{20\sqrt{3}}{9}$$

$$v = 1.68\text{ ms}^{-1}$$