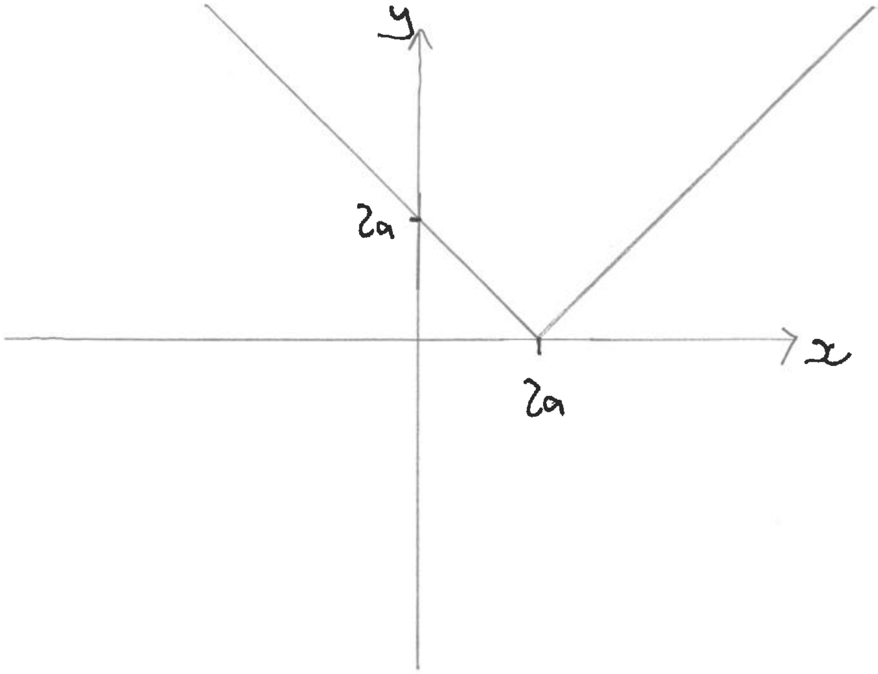
1 (a) Sketch the graph of $y = |x - 2a|$, where $a$ is a positive constant. [1]




(b) Solve the inequality $2x - 3a < |x - 2a|$. [2]

+/− method: critical values:

$+$: $2x - 3a = x - 2a$
$x - 3a = -2a$
$\underline{x = a}$

$-$: $2x - 3a = -(x - 2a)$
$2x - 3a = -x + 2a$
$3x - 3a = 2a$
$3x = 5a$
$\underline{x = \frac{5}{3}a}$

| $x < a$ | $a < x < \frac{5}{3}a$ | $x > \frac{5}{3}a$ |
|---|---|---|
| e.g. $x = 0$ | e.g. $x = \frac{4}{3}a$ | e.g. $x = 2a$ |
| $2(0) - 3a < \lvert 0 - 2a \rvert$ | $2(\frac{4}{3}a) - 3a < \lvert \frac{4}{3}a - 2a \rvert$ | $2(2a) - 3a < \lvert 2a - 2a \rvert$ |
| $-3a < 2a$ | $\frac{8}{3}a - 3a < \lvert \frac{-2}{3}a \rvert$ | $4a - 3a < 0$ |
| | $\frac{-1}{3}a < \frac{2}{3}a$ | $a < 0$ |
| ✓ | ✓ | ✗ |

so $\underline{x < \frac{5}{3}a}$

2 Express $\dfrac{6x^2-9x-16}{2x^2-5x-12}$ in partial fractions. [5]

Improper fraction → long division first.

$$2x^2-5x-12 \,\overline{\big)\, 6x^2-9x-16}\quad 3$$
$$\underline{6x^2-15x-36}$$
$$6x+20$$

$$f(x) = 3 + \frac{6x+20}{2x^2-5x-12}$$

factorise: $ac = -24 \rightarrow -8$ and $+3$

$$2x^2-8x+3x-12$$
$$2x(x-4)+3(x-4)$$
$$(2x+3)(x-4)$$

Partial Fractions:

$$\frac{6x+20}{(2x+3)(x-4)} \equiv \frac{A}{2x+3} + \frac{B}{x-4}$$

$$6x+20 \equiv A(x-4) + B(2x+3)$$

$x=4$:

$$6(4)+20 = 0 + B(2(4)+3)$$
$$44 = 11B$$
$$B = 4$$

$x=\frac{-3}{2}$:

$$6\left(\frac{-3}{2}\right)+20 = A\left(\frac{-3}{2}-4\right) + 0$$
$$11 = -\frac{11}{2}A$$
$$22 = -11A$$
$$A = -2$$

$$\rightarrow 3 - \frac{2}{2x+3} + \frac{4}{x-4}$$

3 The variables $x$ and $y$ satisfy the equation $a^{2y-1} = b^{x-y}$, where $a$ and $b$ are constants.

(a) Show that the graph of $y$ against $x$ is a straight line. [3]

$\ln a^{2y-1} = \ln b^{x-y}$

$(2y-1)\ln a = (x-y)\ln b$

$2y\ln a - \ln a = x\ln b - y\ln b$

$2y\ln a + y\ln b = x\ln b + \ln a$

$y(2\ln a + \ln b) = x\ln b + \ln a$

$y = \underbrace{\dfrac{\ln b}{2\ln a + \ln b}}_{\text{gradient}} x + \underbrace{\dfrac{\ln a}{2\ln a + \ln b}}_{\text{intercept}}$

So straight line with gradient and intercept

(b) Given that $a = b^3$, state the equation of the straight line in the form $y = px + q$, where $p$ and $q$ are rational numbers in their simplest form. [2]

Sub. $a = b^3$: $y = \dfrac{\ln b}{2\ln b^3 + \ln b}x + \dfrac{\ln b^3}{2\ln b^3 + \ln b}$

$= \dfrac{\ln b}{\ln b^6 + \ln b}x + \dfrac{\ln b^3}{\ln b^6 + \ln b}$

$= \dfrac{\ln b}{\ln b^7}x + \dfrac{\ln b^3}{\ln b^7}$

$= \dfrac{\cancel{\ln b}}{7\cancel{\ln b}}x + \dfrac{3\cancel{\ln b}}{7\cancel{\ln b}}$

$\underline{y = \dfrac{1}{7}x + \dfrac{3}{7}}$

4 The equation of a curve is $ye^{2x} + y^2e^x = 6$.

Find the gradient of the curve at the point where $y = 1$. [6]

differentiate $\underset{u}{y}\,\underset{v}{e^{2x}}$ :

$u = y \qquad v = e^{2x}$

$\frac{du}{dx} = \frac{dy}{dx} \qquad \frac{dv}{dx} = 2e^{2x}$

$\rightarrow 2ye^{2x} + e^{2x}\frac{dy}{dx}$

differentiate $\underset{u}{y^2}\,\underset{v}{e^x}$ :

$u = y^2 \qquad v = e^x$

$\frac{du}{dx} = 2y\frac{dy}{dx} \qquad \frac{dv}{dx} = e^x$

$\rightarrow y^2e^x + 2ye^x\frac{dy}{dx}$

whole thing:

$2ye^{2x} + e^{2x}\frac{dy}{dx} + y^2e^x + 2ye^x\frac{dy}{dx} = 0$ ①

Sub. $y=1$ into original eq^n:

$1e^{2x} + 1^2e^x = 6$

$e^{2x} + e^x - 6 = 0$

let $Y = e^x$: $Y^2 + Y - 6 = 0$

$(Y+3)(Y-2) = 0$

$Y = -3$ or $Y = 2$

$e^x = -3$ ✗ $\qquad e^x = 2$ ✓

Sub. $e^x = 2$ and $y = 1$ into ①:

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$$2ye^{2x} + e^{2x}\frac{dy}{dx} + y^2e^x + 2ye^x\frac{dy}{dx} = 0$$

$e^x = 2$ and $y = 1$:

Note: $e^{2x} = (e^x)^2$

so $e^{2x} = 2^2$

$= 4$

$$\rightarrow 2\times1\times4 + 4\frac{dy}{dx} + 1\times2 + 2\times1\times2\frac{dy}{dx} = 0$$

$$8 + 4\frac{dy}{dx} + 2 + 4\frac{dy}{dx} = 0$$

$$8\frac{dy}{dx} = -10$$

$$\underline{\frac{dy}{dx} = \frac{-5}{4}}$$

5 (a) It is given that the equation $e^{2x} = 5 + \cos 3x$ has only one root.

Show by calculation that this root lies in the interval $0.7 < x < 0.8$. [2]

Set $= 0$: $e^{2x} - 5 - \cos 3x = 0$

$x = 0.7$: $e^{1.4} - 5 - \cos(2.1) = -0.440$

$x = 0.8$: $e^{1.6} - 5 - \cos(2.4) = 0.690$

Sign change (and continuous function) between 0.7 and 0.8 shows there's a root between $x = 0.7$ and $x = 0.8$.

(b) Show that if a sequence of values in the interval $0.7 < x < 0.8$ given by the iterative formula

$$x_{n+1} = \frac{1}{2}\ln(5 + \cos 3x_n)$$

converges then it converges to the root of the equation in part **(a)**. [1]

$x = \frac{1}{2}\ln(5 + \cos 3x)$

$2x = \ln(5 + \cos 3x)$

$e^{2x} = 5 + \cos 3x$ QED

(c) Use this iterative formula to determine the root correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

$x_1 = 0.70000$

$x_2 = 0.75150$

$x_3 = 0.73719$

$x_4 = 0.74105$

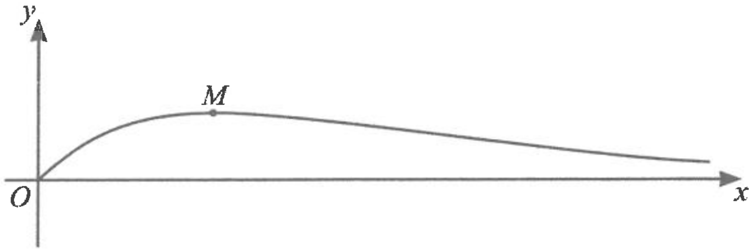
$x_5 = 0.74000$

$x_6 = 0.74028$

$x_7 = 0.74021$

$x_5, x_6$ and $x_7$ all round to $0.740$ to 3dp, so $x = 0.740$

**6**




The diagram shows the curve $y = xe^{-ax}$, where $a$ is a positive constant, and its maximum point $M$.

**(a)** Find the exact coordinates of $M$. **[4]**

$u \quad v$

$y = xe^{-ax}$

$u = x \qquad v = e^{-ax}$

$\frac{du}{dx} = 1 \qquad \frac{dv}{dx} = -ae^{-ax}$

$\frac{dy}{dx} = -axe^{-ax} + e^{-ax}$

$= e^{-ax}(1 - ax)$

SP, so $\frac{dy}{dx} = 0$:

$e^{-ax}(1 - ax) = 0$

$e^{-ax} = 0$ or $1 - ax = 0$

X No sol^ns $\qquad ax = 1$

$x = \frac{1}{a}$

Sub. $x = \frac{1}{a}$ into y:

$y = \frac{1}{a} e^{-a \times \frac{1}{a}}$

$= \frac{1}{a} e^{-1}$

$\rightarrow \left(\frac{1}{a}, \frac{1}{a}e^{-1}\right)$

**(b)** Find the exact value of $\int_0^{\frac{2}{a}} xe^{-ax}\,dx$. [5]

I L A T E

$u = x \qquad v = \frac{-1}{a}e^{-ax}$

$\frac{du}{dx} = 1 \qquad \frac{dv}{dx} = e^{-ax}$

$\rightarrow x \times \frac{-1}{a}e^{-ax} - \int_0^{2/a} \frac{-1}{a}e^{-ax}\,dx$

$= \frac{-x}{a}e^{-ax} + \int_0^{2/a} \frac{1}{a}e^{-ax}\,dx$

$= \left[\frac{-x}{a}e^{-ax} - \frac{1}{a^2}e^{-ax}\right]_0^{2/a}$

$= \left[\frac{-\frac{2}{a}}{a}e^{-a\times\frac{2}{a}} - \frac{1}{a^2}e^{-a\times\frac{2}{a}}\right] - \left[0 - \frac{1}{a^2}e^{0}\right]$

$= \left[\frac{-2}{a^2}e^{-2} - \frac{1}{a^2}e^{-2}\right] - \left[-\frac{1}{a^2}\right]$

$= \frac{-3}{a^2}e^{-2} + \frac{1}{a^2}$

$= \underline{\frac{1}{a^2}(1 - 3e^{-2})}$

7 (a) Show that $\cos^4\theta - \sin^4\theta \equiv \cos 2\theta$. [3]

Change LHS to either just $\cos\theta$ or $\sin\theta$:

LHS: $\cos^4\theta - (\sin^2\theta)^2$

$1 - \cos^2\theta$

$= \cos^4\theta - (1 - \cos^2\theta)^2$

$= \cos^4\theta - (1 - 2\cos^2\theta + \cos^4\theta)$

$= \cancel{\cos^4\theta} - 1 + 2\cos^2\theta - \cancel{\cos^4\theta}$

$= 2\cos^2\theta - 1$

$= \underline{\underline{\cos 2\theta}}$ QED

**(b)** Hence find the exact value of $\int_{-\frac{1}{8}\pi}^{\frac{1}{8}\pi} (\underbrace{\cos^4\theta - \sin^4\theta}_{\cos 2\theta} + 4\sin^2\theta\cos^2\theta)\,d\theta$. [6]

$$= \int_{-\frac{\pi}{8}}^{\frac{\pi}{8}} (\cos 2\theta + 4\sin^2\theta\cos^2\theta)\,d\theta$$

$2\sin\theta\cos\theta = \sin 2\theta$

$(2\sin\theta\cos\theta)^2 = \sin^2 2\theta$

$4\sin^2\theta\cos^2\theta = \sin^2 2\theta$

$$= \int_{-\frac{\pi}{8}}^{\frac{\pi}{8}} (\cos 2\theta + \sin^2 2\theta)\,d\theta$$

$\cos 2\theta = 1 - 2\sin^2\theta$

$\cos 4\theta = 1 - 2\sin^2 2\theta$

$\sin^2 2\theta = \frac{1}{2} - \frac{1}{2}\cos 4\theta$

$$= \int_{-\frac{\pi}{8}}^{\frac{\pi}{8}} \left(\cos 2\theta + \frac{1}{2} - \frac{1}{2}\cos 4\theta\right)d\theta$$

$$= \left[\frac{1}{2}\sin 2\theta + \frac{1}{2}\theta - \frac{1}{8}\sin 4\theta\right]_{-\frac{\pi}{8}}^{\frac{\pi}{8}}$$

$$= \left[\frac{1}{2}\sin\left(\frac{\pi}{4}\right) + \frac{1}{2}\left(\frac{\pi}{8}\right) - \frac{1}{8}\sin\left(\frac{\pi}{2}\right)\right] - \left[\frac{1}{2}\sin\left(\frac{-\pi}{4}\right) + \frac{1}{2}\left(\frac{-\pi}{8}\right) - \frac{1}{8}\sin\left(\frac{-\pi}{2}\right)\right]$$

$$= \left[\frac{1}{2} \times \frac{\sqrt{2}}{2} + \frac{\pi}{16} - \frac{1}{8}\right] - \left[\frac{1}{2} \times \frac{-\sqrt{2}}{2} - \frac{\pi}{16} + \frac{1}{8}\right]$$

$$= \frac{\sqrt{2}}{4} + \frac{\pi}{16} - \frac{1}{8} + \frac{\sqrt{2}}{4} + \frac{\pi}{16} - \frac{1}{8}$$

$$= \underline{\frac{\sqrt{2}}{2} + \frac{\pi}{8} - \frac{1}{4}}$$

8 The points $A$, $B$ and $C$ have position vectors $\overrightarrow{OA} = -2\mathbf{i} + \mathbf{j} + 4\mathbf{k}$, $\overrightarrow{OB} = 5\mathbf{i} + 2\mathbf{j}$ and $\overrightarrow{OC} = 8\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$, where $O$ is the origin. The line $l_1$ passes through $B$ and $C$.

**(a)** Find a vector equation for $l_1$. [3]

direction vector: $\overrightarrow{BC} = \underline{c} - \underline{b}$

$$= \begin{pmatrix} 8 \\ 5 \\ -3 \end{pmatrix} - \begin{pmatrix} 5 \\ 2 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 3 \\ 3 \\ -3 \end{pmatrix}$$

$$l_1 : \underline{r} = \begin{pmatrix} 5 \\ 2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 3 \\ -3 \end{pmatrix}$$

point on line (A)

Other solutions are possible.

The line $l_2$ has equation $\mathbf{r} = -2\mathbf{i} + \mathbf{j} + 4\mathbf{k} + \mu(3\mathbf{i} + \mathbf{j} - 2\mathbf{k})$.

**(b)** Find the coordinates of the point of intersection of $l_1$ and $l_2$. [4]

$$\begin{pmatrix} 5 + 3\lambda \\ 2 + 3\lambda \\ -3\lambda \end{pmatrix} = \begin{pmatrix} -2 + 3\mu \\ 1 + \mu \\ 4 - 2\mu \end{pmatrix}$$

$\underline{i}$: $5 + 3\lambda = -2 + 3\mu$ $\quad$ $\underline{j}$: $2 + 3\lambda = 1 + \mu$

$7 = 3\mu - 3\lambda$ ① $\quad$ $1 = \mu - 3\lambda$ ②

① − ②: $6 = 2\mu$

$\mu = 3$

sub. into $l_2$: $\underline{r} = \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix} + 3 \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$

$$= \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix} + \begin{pmatrix} 9 \\ 3 \\ -6 \end{pmatrix}$$

$$= \begin{pmatrix} 7 \\ 4 \\ -2 \end{pmatrix}$$

coordinates: $\underline{(7, 4, -2)}$

**(c)** The point $D$ on $l_2$ is such that $AB = BD$.

Find the position vector of $D$. ↖ this means $|AB| = |BD|$ **[5]**

$|AB|$:

$$|AB| = \sqrt{(5--2)^2 + (2-1)^2 + (-4)^2}$$
$$= \sqrt{49 + 1 + 16}$$
$$= \underline{\sqrt{66}}$$

D lies on $l_2$, so use equation of line to represent $\underline{d}$:

$$\underline{d} = \begin{pmatrix} -2+3\mu \\ 1+\mu \\ 4-2\mu \end{pmatrix}$$

$$\vec{BD} = \underline{d} - \underline{b} = \begin{pmatrix} -2+3\mu \\ 1+\mu \\ 4-2\mu \end{pmatrix} - \begin{pmatrix} 5 \\ 2 \\ 0 \end{pmatrix}$$
$$= \begin{pmatrix} -7+3\mu \\ -1+\mu \\ 4-2\mu \end{pmatrix}$$

$$|\vec{BD}| = \sqrt{(-7+3\mu)^2 + (-1+\mu)^2 + (4-2\mu)^2}$$
$$= \sqrt{49 - 42\mu + 9\mu^2 + 1 - 2\mu + \mu^2 + 16 - 16\mu + 4\mu^2}$$
$$= \sqrt{14\mu^2 - 60\mu + 66}$$

$|BD| = \sqrt{66}$ :

$$\sqrt{14\mu^2 - 60\mu + 66} = \sqrt{66}$$
$$14\mu^2 - 60\mu + 66 = 66$$
$$14\mu^2 - 60\mu = 0$$
$$7\mu^2 - 30\mu = 0$$
$$\mu(7\mu - 30) = 0$$
$$\mu = 0 \quad \text{or} \quad \mu = \frac{30}{7} \checkmark$$

× ↖ this gives point A

sub. into $l_2$: $$\underline{d} = \begin{pmatrix} -2 + \frac{90}{7} \\ 1 + 30/7 \\ 4 - 60/7 \end{pmatrix} = \underline{\begin{pmatrix} \frac{76}{7} \\ 37/7 \\ -32/7 \end{pmatrix}}$$

**9** The complex numbers $z$ and $\omega$ are defined by $z = 1 - \mathrm{i}$ and $\omega = -3 + 3\sqrt{3}\mathrm{i}$.

**(a)** Express $z\omega$ in the form $a + b\mathrm{i}$, where $a$ and $b$ are real and in exact surd form. [1]

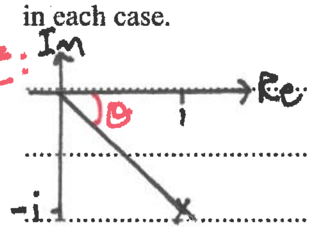
$zw = (1 - i)(-3 + 3\sqrt{3}i)$

$= -3 + 3\sqrt{3}i + 3i + 3\sqrt{3}$

$= (3\sqrt{3} - 3) + (3\sqrt{3} + 3)i$

**(b)** Express $z$ and $\omega$ in the form $re^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$. Give the exact values of $r$ and $\theta$ in each case. [4]

z:




$r = \sqrt{1^2 + (-1)^2}$

$= \sqrt{2}$

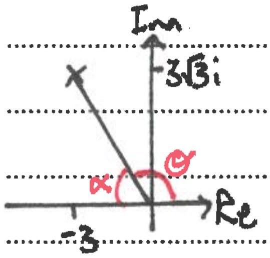
$\tan\theta = \frac{1}{1}$

$\theta = \frac{\pi}{4}$

below x-axis, so $\theta = \frac{-\pi}{4}$

$z = \sqrt{2}e^{\frac{-i\pi}{4}}$

W:




$r = \sqrt{(-3)^2 + (3\sqrt{3})^2}$

$= \sqrt{9 + 27}$

$= 6$

$\tan\alpha = \frac{3\sqrt{3}}{3}$

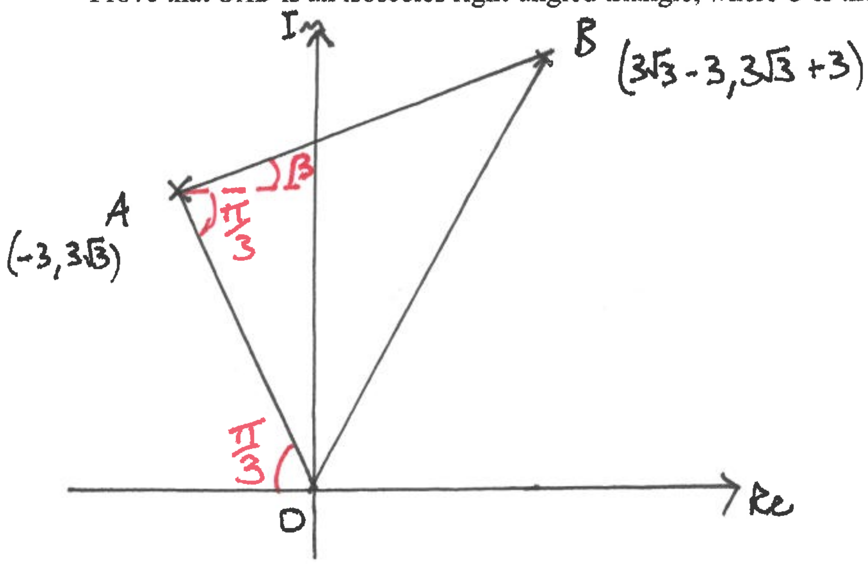
$\alpha = \frac{\pi}{3}$

$\theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$W = 6e^{\frac{2\pi i}{3}}$

**(c)** On an Argand diagram, the points representing $\omega$ and $z\omega$ are $A$ and $B$ respectively.

Prove that $OAB$ is an isosceles right-angled triangle, where $O$ is the origin. [2]



$|\vec{OA}| = 6$ (from part (b)) Looks more likely that $|\vec{AB}| = |\vec{OA}|$:

$\vec{AB} = \underline{b} - \underline{a} = zw - w$

$= (3\sqrt{3} - 3) + (3\sqrt{3} + 3)i - (-3 + 3\sqrt{3}i)$

$= \underline{3\sqrt{3} + 3i}$

$|\vec{AB}| = \sqrt{(3\sqrt{3})^2 + 3^2}$

$= \underline{6} = |\vec{OA}|$, therefore isosceles.

Angle OAB $= \frac{\pi}{3} + \beta$ (see diagram)

$\beta = \arg(\vec{AB})$

$= \tan^{-1}\left(\frac{3}{3\sqrt{3}}\right)$

$= \frac{\pi}{6}$

$\frac{\pi}{3} + \frac{\pi}{6} = \underline{\underline{\frac{\pi}{2}}}$ QED

**(d)** Using your answers to part **(b)**, prove that $\tan\frac{5}{12}\pi = \frac{\sqrt{3}+1}{\sqrt{3}-1}$. [3]

Through a bit of trial and error, $\frac{5\pi}{12} = \frac{2\pi}{3} + \frac{-\pi}{4}$:

$\frac{5\pi}{12} = \arg(w) + \arg(z)$

$\frac{5\pi}{12} = \arg(wz)$ $(= \arg(zw))$

$\frac{5\pi}{12} = \tan^{-1}\left(\frac{3\sqrt{3} + 3}{3\sqrt{3} - 3}\right)$

$\tan\left(\frac{5\pi}{12}\right) = \frac{3\sqrt{3} + 3}{3\sqrt{3} - 3}$

$\underline{\underline{\tan\left(\frac{5\pi}{12}\right) = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}}}$ QED

**10** **(a)** By writing $y = \sec^3\theta$ as $\dfrac{1}{\cos^3\theta}$, show that $\dfrac{dy}{d\theta} = 3\sin\theta\sec^4\theta$. [2]

$$y = (\cos\theta)^{-3}$$

$$\frac{dy}{d\theta} = -3(\cos\theta)^{-4} \times -\sin\theta$$

$$= 3\sin\theta\cos^{-4}\theta$$

$$= 3\sin\theta \times \frac{1}{\cos^4\theta}$$

$$= 3\sin\theta\sec^4\theta$$

**(b)** The variables $x$ and $\theta$ satisfy the differential equation

$$(x^2+9)\sin\theta\frac{d\theta}{dx} = (x+3)\cos^4\theta.$$

It is given that $x = 3$ when $\theta = \frac{1}{3}\pi$.

Solve the differential equation to find the value of $\cos\theta$ when $x = 0$. Give your answer correct to 3 significant figures. [8]

$$\sin\theta\frac{d\theta}{dx} = \frac{(x+3)\cos^4\theta}{x^2+9}$$

$$\int\frac{\sin\theta}{\cos^4\theta}\,d\theta = \int\frac{x+3}{x^2+9}\,dx$$

$$\int\sin\theta\sec^4\theta\,d\theta = \int\frac{x+3}{x^2+9}\,dx$$

$$\frac{1}{3}\int 3\sin\theta\sec^4\theta\,d\theta = \int\frac{x}{x^2+9}\,dx + \int\frac{3}{x^2+9}\,dx$$

$$\frac{1}{3}\int 3\sin\theta\sec^4\theta\,d\theta = \frac{1}{2}\int\frac{2x}{x^2+9}\,dx + 3\int\frac{1}{x^2+9}\,dx$$

$\int\frac{1}{x^2+a^2}$ with $a^2=9$ so: $a=3$

from part (a)

$$\frac{1}{3} \times \frac{1}{\cos^3\theta} = \frac{1}{2}\ln|x^2+9| + 3\times\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + C$$

$$\frac{1}{3\cos^3\theta} = \frac{1}{2}\ln|x^2+9| + \tan^{-1}\left(\frac{x}{3}\right) + C$$

$x=3$, $\theta = \frac{\pi}{3}$:

$$\frac{1}{3\cos^3\left(\frac{\pi}{3}\right)} = \frac{1}{2}\ln|3^2+9| + \tan^{-1}\left(\frac{3}{3}\right) + C$$

$$\frac{1}{3\times\left(\frac{1}{2}\right)^3} = \frac{1}{2}\ln 18 + \tan^{-1}(1) + C$$

$$\frac{1}{3/8} = \frac{1}{2}\ln 18 + \frac{\pi}{4} + C$$

$$\frac{8}{3} = \frac{1}{2}\ln 18 + \frac{\pi}{4} + C$$

$$C = \frac{8}{3} - \frac{1}{2}\ln 18 - \frac{\pi}{4}$$

$$\frac{1}{3\cos^3\theta} = \frac{1}{2}\ln|x^2+9| + \tan^{-1}\left(\frac{x}{3}\right) + \frac{8}{3} - \frac{1}{2}\ln 18 - \frac{\pi}{4}$$

$x=0$:

$$\frac{1}{3\cos^3\theta} = \frac{1}{2}\ln 9 + 0 + \frac{8}{3} - \frac{1}{2}\ln 18 - \frac{\pi}{4}$$

$$\frac{1}{\cos^3\theta} = \frac{3}{2}\ln 9 + 8 - \frac{3}{2}\ln 18 - \frac{3\pi}{4}$$

$$\frac{1}{\cos\theta} = \left(\frac{3}{2}\ln 9 + 8 - \frac{3}{2}\ln 18 - \frac{3\pi}{4}\right)^{\frac{1}{3}}$$

$$\cos\theta = \left(\frac{3}{2}\ln 9 + 8 - \frac{3}{2}\ln 18 - \frac{3\pi}{4}\right)^{-\frac{1}{3}}$$

$$= \underline{0.601}$$