1 Expand $(3+x)(1-2x)^{\frac{1}{2}}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [4]

$$(1-2x)^{\frac{1}{2}} = 1 + \tfrac{1}{2}\times -2x + \frac{(\frac{1}{2})(\frac{-1}{2})}{2!}(-2x)^2 + \dots$$

$$= 1 - x + \frac{-1}{8} \times 4x^2 + \dots$$

$$= 1 - x - \frac{1}{2}x^2 - \dots$$

$$(3+x)\left(1 - x - \frac{1}{2}x^2\right) :$$

$$= 3 - 3x - \frac{3}{2}x^2 + x - x^2 \left(- \frac{1}{2}x^3\right)$$

$$= \underline{3 - 2x - \frac{5}{2}x^2}$$

2 Solve the equation $\ln(x-5) = 7 - \ln x$. Give your answer correct to 2 decimal places. [4]

$\ln(x-5) + \ln x = 7$

$\ln[x(x-5)] = 7$

$x(x-5) = e^7$

$x^2 - 5x = e^7$

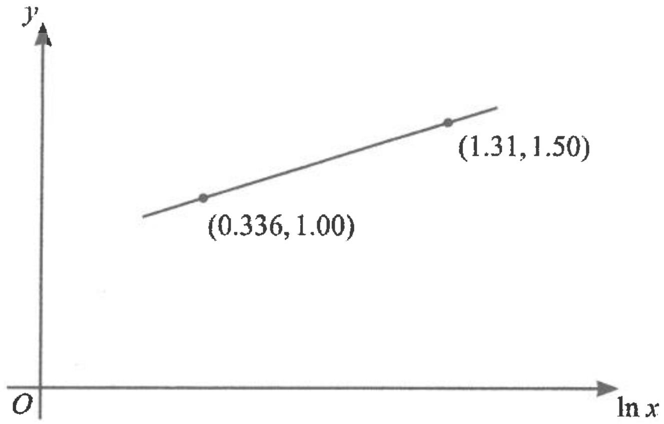
$x^2 - 5x - e^7 = 0$

$x = \dfrac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-e^7)}}{2(1)}$

$\underline{x = 35.71}$ or $x = -30.71$ ×

Gives ln of negative when substituted into Q.

3



The variables x and y satisfy the equation $a^y = bx$, where a and b are constants. The graph of y against $\ln x$ is a straight line passing through the points (0.336, 1.00) and (1.31, 1.50), as shown in the diagram.

Find the values of a and b. Give each value correct to the nearest integer. [4]

$a^y = bx$

$\ln a^y = \ln(bx)$

$y \ln a = \ln b + \ln x$

Sub. (0.336, 1.00):

$1.00 \ln a = \ln b + 0.336$

$\ln a = \ln b + 0.336$ (1)

Sub. (1.31, 1.50):

$1.50 \ln a = \ln b + 1.31$

$1.5 \ln a = \ln b + 1.31$ (2)

(2) - (1):

$0.5 \ln a = 0.974$

$\ln a = 1.948$

$a = e^{1.948}$

$a = 7.0146..$ STO

(1): $\ln(7.0146) = \ln b + 0.336$

$1.948 = \ln b + 0.336$

$\ln b = 1.612$

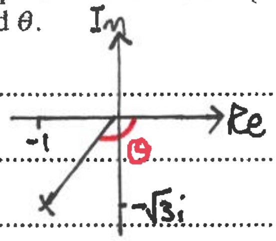
$b = e^{1.612}$

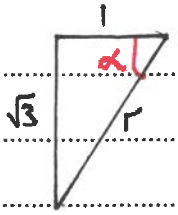
$b = 5.0128...$

$a = 7$, $b = 5$

4 The complex number u is given by $u = -1 - \mathrm{i}\sqrt{3}$.

(a) Express u in the form $r(\cos\theta + \mathrm{i}\sin\theta)$, where $r > 0$ and $-\pi < \theta \leqslant \pi$. Give the exact values of r and θ. [2]





$\tan\alpha = \frac{\sqrt{3}}{1}$

$\alpha = \frac{\pi}{3}$

$\theta = -\pi + \frac{\pi}{3}$

$= \frac{-2\pi}{3}$

$r = \sqrt{1^2 + (\sqrt{3})^2}$

$= \sqrt{4}$

$= 2$

→ $u = 2\left(\cos\left(\frac{-2\pi}{3}\right) + i\sin\left(\frac{-2\pi}{3}\right)\right)$

The complex number v is given by $v = 5\left(\cos\frac{1}{6}\pi + \mathrm{i}\sin\frac{1}{6}\pi\right)$.

(b) Express the complex number $\frac{v}{u}$ in the form $re^{\mathrm{i}\theta}$ where $r > 0$ and $-\pi < \theta \leqslant \pi$. [2]

$u = 2e^{-\frac{2\pi i}{3}}$ $\quad V = 5e^{\frac{\pi i}{6}}$

$\frac{V}{u} = \frac{5e^{\pi i/6}}{2e^{-\frac{2\pi i}{3}}}$

$r = \frac{5}{2}$ $\quad \theta = \frac{\pi}{6} - -\frac{2\pi}{3}$

$= \frac{5\pi}{6}$

→ $\frac{V}{u} = \frac{5}{2}e^{\frac{5\pi i}{6}}$

5 The equation of a curve is $y = \dfrac{e^{\sin x}}{\cos^2 x}$ for $0 \leqslant x \leqslant 2\pi$.

Find $\dfrac{dy}{dx}$ and hence find the x-coordinates of the stationary points of the curve. [7]

differentiate $e^{\sin x}$:

$y = e^t \qquad t = \sin x$

$\frac{dy}{dt} = e^t \qquad \frac{dt}{dx} = \cos x$

$\frac{dy}{dx} = e^t \cos x$

$= e^{\sin x} \cos x$

differentiate $\cos^2 x$:

$y = t^2 \qquad t = \cos x$

$\frac{dy}{dt} = 2t \qquad \frac{dt}{dx} = -\sin x$

$\frac{dy}{dx} = -2t\sin x$

$= -2\sin x \cos x$

differentiate $y = \frac{e^{\sin x}}{\cos^2 x}$ $\begin{matrix} u \\ v \end{matrix}$

$u = e^{\sin x} \qquad v = \cos^2 x$

$\frac{du}{dx} = e^{\sin x}\cos x \qquad \frac{dv}{dx} = -2\sin x \cos x$

$$\frac{dy}{dx} = \frac{\cos^2 x \times e^{\sin x}\cos x - e^{\sin x} \times -2\sin x\cos x}{(\cos^2 x)^2}$$

$$= \frac{\cos^3 x\, e^{\sin x} + 2\sin x\cos x\, e^{\sin x}}{\cos^4 x}$$

SP, so $\frac{dy}{dx} = 0$:

$$\frac{\cos^3 x\, e^{\sin x} + 2\sin x\cos x\, e^{\sin x}}{\cos^4 x} = 0$$

$\cos x\, e^{\sin x}\left(\cos^2 x + 2\sin x\right) = 0$

$\cos x = 0$ ① or $e^{\sin x} = 0$ ② or $\cos^2 x + 2\sin x = 0$ ③

①: $\cos x = 0$

When $\cos x = 0$, $y = \dfrac{e^{\sin x}}{\underbrace{\cos^2 x}_{=0}}$ is undefined, so this is not a stationary point.

②: $e^{\sin x} = 0$

No sol^s ($\ln 0$ undefined)

③: $\cos^2 x + 2\sin x = 0$

$1 - \sin^2 x + 2\sin x = 0$

$\sin^2 x - 2\sin x - 1 = 0$

let $Y = \sin x$:

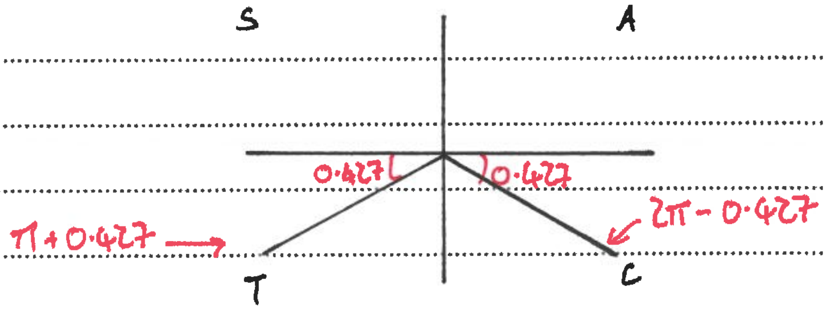
$Y^2 - 2Y - 1 = 0$

$Y = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$

$Y = 1 + \sqrt{2}$ or $Y = 1 - \sqrt{2}$

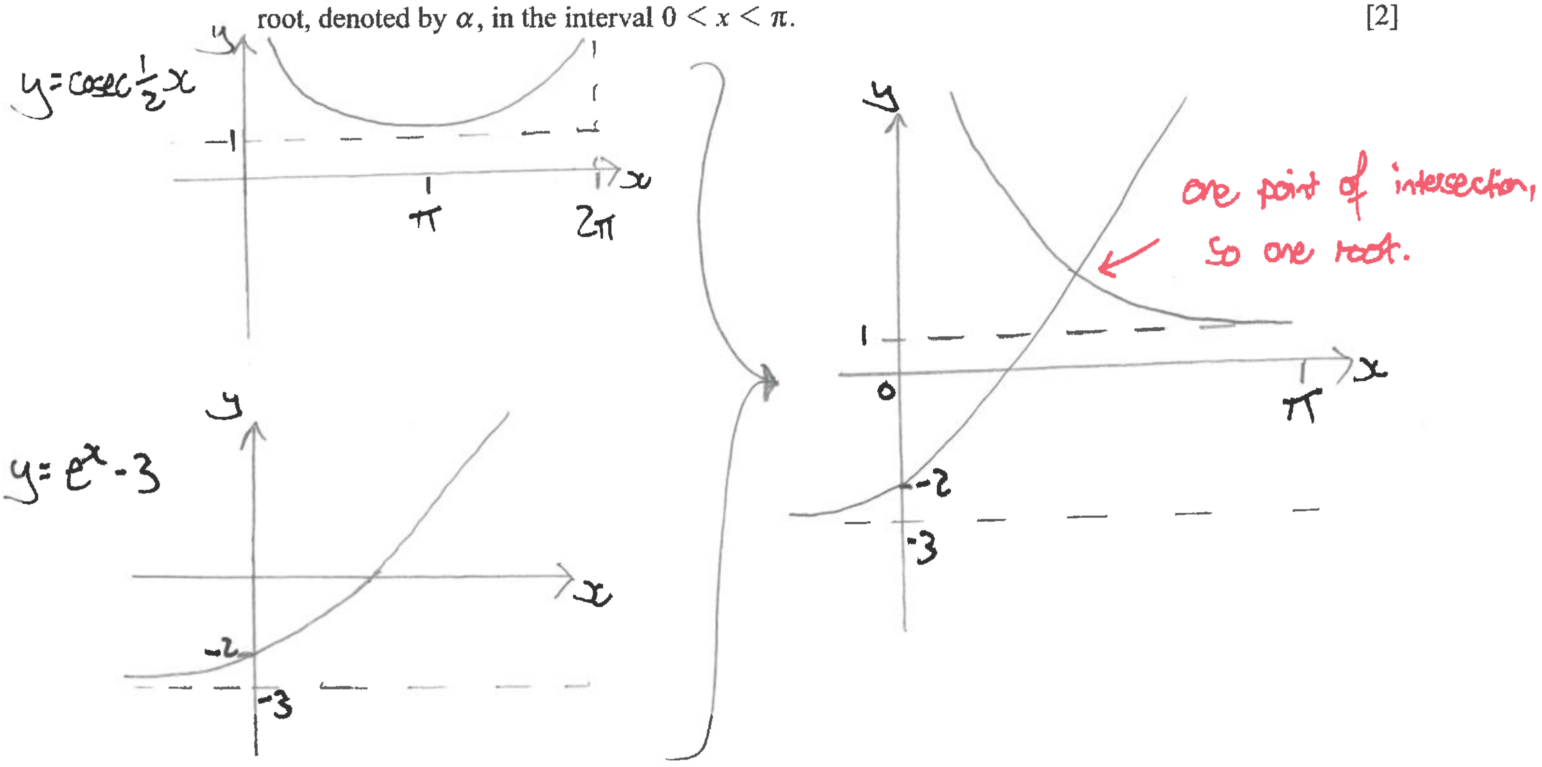
$\sin x = 1 + \sqrt{2}$ $\sin x = 1 - \sqrt{2}$

✗ no sol^s $x = -0.427$



$x = 3.57,\ 5.87$

6 (a) By sketching a suitable pair of graphs, show that the equation $\operatorname{cosec}\frac{1}{2}x = e^x - 3$ has exactly one root, denoted by α, in the interval $0 < x < \pi$. [2]



(b) Verify by calculation that α lies between 1 and 2. [2]

Set $= 0$: $\operatorname{cosec}\frac{1}{2}x - e^x + 3 = 0$

$x = 1$: $\operatorname{cosec}\left(\frac{1}{2} \times 1\right) - e^1 + 3$

$\frac{1}{\sin\left(\frac{1}{2}\right)} - e^1 + 3 = 2.368$

$x = 2$: $\operatorname{cosec}\left(\frac{1}{2} \times 2\right) - e^2 + 3$

$\frac{1}{\sin(1)} - e^2 + 3 = -3.201$

Given there's a sign change between $x = 1$ and $x = 2$ (and the function is continuous between 1 and 2), there is a root.

(c) Show that if a sequence of values in the interval $0 < x < \pi$ given by the iterative formula

$$x_{n+1} = \ln(\operatorname{cosec}\tfrac{1}{2}x_n + 3)$$

converges, then it converges to α. [1]

$x = \ln(\operatorname{cosec}\frac{1}{2}x + 3)$

$e^x = \operatorname{cosec}\frac{1}{2}x + 3$

$e^x - 3 = \operatorname{cosec}\frac{1}{2}x$ QED

(d) Use this iterative formula with an initial value of 1.4 to determine α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

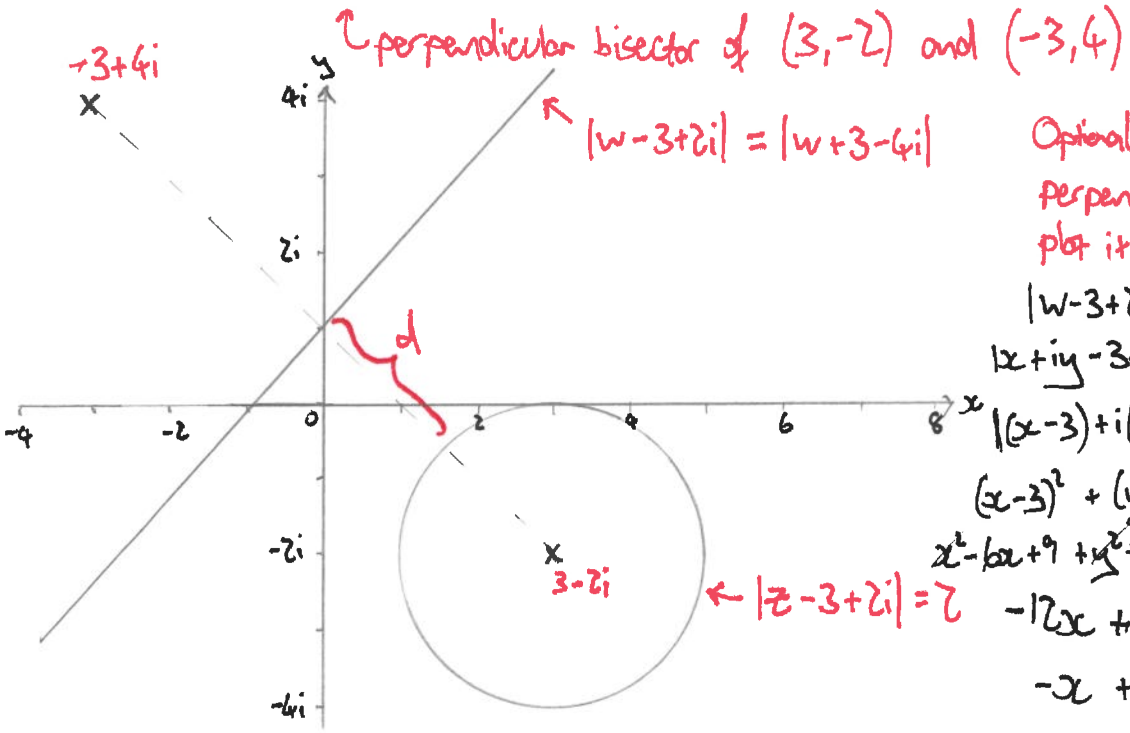
$x_1 = 1.4000$
$x_2 = 1.5156$
$x_3 = 1.4940$
$x_4 = 1.4978$
$x_5 = 1.4971$
$x_6 = 1.4972$

x_4, x_5 and x_6 all round to 1.50 to 2dp,
so $x = 1.50$

(e) State the minimum number of calculated iterations needed with this initial value to determine α correct to 2 decimal places. [1]

This is debatable. Usually having two consecutive answers which round to the same thing is sufficient, but this doesn't always give the correct answer.

7 (a) On a single Argand diagram sketch the loci given by the equations $|z-3+2i|=2$ and $|w-3+2i|=|w+3-4i|$ where z and w are complex numbers. [4]



Optional: Find equation of perpendicular bisector to help plot it:

$|w-3+2i| = |w+3-4i|$

$|x+iy-3+2i| = |x+iy+3-4i|$

$|(x-3)+i(y+2)| = |(x+3)+i(y-4)|$

$(x-3)^2+(y+2)^2 = (x+3)^2+(y-4)^2$

$x^2-6x+9+\cancel{y^2}+4y+4 = x^2+6x+9+\cancel{y^2}-8y+16$

$-12x+12y-12=0$

$-x+y-1=0$

$y=x+1$

(b) Hence find the least value of $|z-w|$ for points on these loci. Give your answer in an exact form. [2]

least value is shortest distance, d as shown on diagram.

d = distance between (0,1) and (3,-2) subtract radius.

distance between (0,1) and (3,-2):

$\sqrt{(3-0)^2+(-2-1)^2} = \sqrt{9+9}$

$= \sqrt{18}$

$= 3\sqrt{2}$

radius of circle $= 2$

$d = 3\sqrt{2}-2$

8 Use the substitution $u = 1 - \sin x$ to find the exact value of

$$\int_{\pi}^{\frac{3}{2}\pi} \frac{\sin 2x}{\sqrt{1-\sin x}}\,dx.$$

Give your answer in the form $a + b\sqrt{2}$ where a and b are rational numbers to be determined. [7]

$\int_{\pi}^{\frac{3\pi}{2}} \frac{\sin 2x}{\sqrt{1-\sin x}}\, dx$

$u = 1 - \sin x$

$\frac{du}{dx} = -\cos x$

$du = -\cos x\, dx$

$\boxed{dx = \frac{du}{-\cos x}}$

$\int_{\pi}^{\frac{3\pi}{2}} \frac{\sin 2x}{\sqrt{1-\sin x}} \times \frac{du}{-\cos x}$

$u = 1 - \sin\left(\frac{3\pi}{2}\right)$
$= 2$

$u = 1 - \sin(\pi)$
$= 1$

$2\sin x \cos x$

$\int_{1}^{2} \frac{\sin 2x}{\sqrt{1-\sin x}} \times \frac{du}{-\cos x}$

$= \int_{1}^{2} \frac{2\sin x \cancel{\cos x}}{\sqrt{1-\sin x}} \times \frac{du}{-\cancel{\cos x}}$

$\sin x = 1 - u$

$= \int_{1}^{2} \frac{-2\sin x}{\sqrt{1-\sin x}}\, du$

$= \int_{1}^{2} \frac{-2(1-u)}{\sqrt{u}}\, du$

$= \int_{1}^{2} \frac{2u-2}{u^{\frac{1}{2}}}\, du$

$= \int_{1}^{2} \left(2u^{\frac{1}{2}} - 2u^{-\frac{1}{2}}\right) du$

$= \left[\frac{2}{3} \times 2u^{\frac{3}{2}} - 2 \times 2u^{\frac{1}{2}}\right]_{1}^{2}$

$= \left[\frac{4}{3}u^{3/2} - 4u^{\frac{1}{2}}\right]_1^2$

$= \left[\frac{4}{3}(2)^{3/2} - 4(2)^{\frac{1}{2}}\right] - \left[\frac{4}{3}(1)^{3/2} - 4(1)^{\frac{1}{2}}\right]$

$= \left[\frac{4}{3} \times 2\sqrt{2} - 4\sqrt{2}\right] - \left[\frac{4}{3} - 4\right]$

$= \left[\frac{8}{3}\sqrt{2} - 4\sqrt{2}\right] - \left[-\frac{8}{3}\right]$

$= \left[-\frac{4}{3}\sqrt{2}\right] - \left[-\frac{8}{3}\right]$

$= \underline{\frac{8}{3} - \frac{4}{3}\sqrt{2}}$

9 The equations of two straight lines l_1 and l_2 are

$$l_1: \quad \mathbf{r} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k} + \lambda(2\mathbf{i} - \mathbf{j} + a\mathbf{k}) \quad \text{and} \quad l_2: \quad \mathbf{r} = -\mathbf{i} - \mathbf{j} - \mathbf{k} + \mu(3\mathbf{i} - 2\mathbf{j} - 2\mathbf{k}),$$

where a is a constant.

The lines l_1 and l_2 are perpendicular.

(a) Show that $a = 4$. [1]

$l_1: \underline{r} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ a \end{pmatrix}$ $\quad l_2: \underline{r} = \begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -2 \\ -2 \end{pmatrix}$

If perpendicular, the scalar product of the direction vectors $= 0$:

$$\begin{pmatrix} 2 \\ -1 \\ a \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -2 \\ -2 \end{pmatrix} = 0$$

$$2 \times 3 + -1 \times -2 + a \times -2 = 0$$

$$6 + 2 - 2a = 0$$

$$8 - 2a = 0$$

$$2a = 8$$

$$\underline{a = 4} \quad \text{QED}$$

The lines l_1 and l_2 also intersect.

(b) Find the position vector of the point of intersection. [4]

$$\begin{pmatrix} 1 + 2\lambda \\ -2 - \lambda \\ 3 + 4\lambda \end{pmatrix} = \begin{pmatrix} -1 + 3\mu \\ -1 - 2\mu \\ -1 - 2\mu \end{pmatrix}$$

$\cancel{i}$: $1 + 2\lambda = -1 + 3\mu$

$2 = 3\mu - 2\lambda$ ①

$\cancel{j}$: $-2 - \lambda = -1 - 2\mu$

$-1 = -2\mu + \lambda$ $\times 2$

$-2 = -4\mu + 2\lambda$ ②

①+②: $0 = -\mu$

$\underline{\mu = 0}$

Sub. into l_2: $\underline{r} = \begin{pmatrix} -1 + 0 \\ -1 - 0 \\ -1 - 0 \end{pmatrix} = \underline{\begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix}}$

The point A has position vector $-5\mathbf{i}+\mathbf{j}-9\mathbf{k}$.

(c) Show that A lies on l_1. [2]

$$\begin{pmatrix}-5\\1\\-9\end{pmatrix} = \begin{pmatrix}1+2\lambda\\-2-\lambda\\3+4\lambda\end{pmatrix}$$

$\underline{\underline{i}}$: $-5 = 1+2\lambda$ $\qquad$ $\underline{\underline{j}}$: $1 = -2-\lambda$

$-6 = 2\lambda$ $\qquad$ $3 = -\lambda$

$\underline{\lambda = -3}$ $\qquad$ $\underline{\lambda = -3}$

$\underline{\underline{k}}$: $-9 = 3+4\lambda$

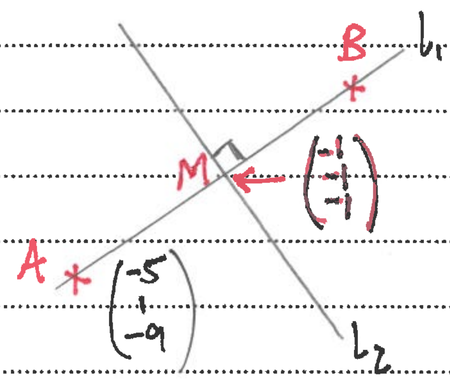
$-12 = 4\lambda$

$\underline{\lambda = -3}$

$\lambda=-3$ for all three components, therefore A lies on l_1.

The point B is the image of A after a reflection in the line l_2.

(d) Find the position vector of B. [2]



$\underline{\vec{OB} = \vec{OM} + \vec{MB}}$; $\vec{MB} = \vec{AM}$

$\vec{OM} = \begin{pmatrix}-1\\-1\\-1\end{pmatrix}$

$\vec{AM} = \underline{m} - \underline{a}$

$= \begin{pmatrix}-1\\-1\\-1\end{pmatrix} - \begin{pmatrix}-5\\1\\-9\end{pmatrix}$

$= \begin{pmatrix}4\\-2\\8\end{pmatrix}$

$\vec{OB} = \begin{pmatrix}-1\\-1\\-1\end{pmatrix} + \begin{pmatrix}4\\-2\\8\end{pmatrix} = \underline{\begin{pmatrix}3\\-3\\7\end{pmatrix}}$

10 **(a)** Given that $2x = \tan y$, show that $\frac{dy}{dx} = \frac{2}{1+4x^2}$. [3]

differentiate:

$$2 = \sec^2 y \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{2}{\sec^2 y} \leftarrow \sec^2 y = 1 + \tan^2 y$$

$$\frac{dy}{dx} = \frac{2}{1 + \tan^2 y} \leftarrow \tan y = 2x \text{ (from Q.)}$$

$$\frac{dy}{dx} = \frac{2}{1 + (2x)^2}$$

$$\frac{dy}{dx} = \frac{2}{1 + 4x^2} \quad \text{QED}$$

(b) Hence find the exact value of $\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} x \tan^{-1}(2x)\,dx$. [7]

$$\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \underbrace{x}_{dv} \underbrace{\tan^{-1}(2x)}_{u}\, dx$$

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$u = \tan^{-1}(2x)$ $\quad\quad V = \frac{1}{2}x^2$

or: $\tan(u) = 2x$ $\quad\quad \frac{dv}{dx} = x$

$\frac{du}{dx} = \frac{2}{1+4x^2} \leftarrow$ from part (a)

$$\text{Integral} = \frac{1}{2}x^2 \times \tan^{-1}(2x) - \int \frac{1}{\cancel{2}} x^2 \times \frac{\cancel{2}}{1+4x^2}\, dx$$

$$= \frac{1}{2}x^2 \tan^{-1}(2x) - \int \frac{x^2}{1+4x^2}\, dx$$

$\dfrac{x^2}{1+4x^2}$ ← improper fraction, so use algebraic division:

$$\begin{array}{r|l} & \tfrac{1}{4} \\ 4x^2+0x+1 & x^2+0x+0 \\ & x^2+0x+\tfrac{1}{4} \\ \hline & -\tfrac{1}{4} \end{array}$$

$$\rightarrow \frac{x^2}{1+4x^2} = \frac{1}{4} + \frac{-\frac{1}{4}}{1+4x^2}$$

Back to whole integral:

$$= \frac{1}{2}x^2\tan^{-1}(2x) - \int\left(\frac{1}{4} + \frac{-\frac{1}{4}}{1+4x^2}\right)dx \quad \div 4$$

$$= \frac{1}{2}x^2\tan^{-1}(2x) - \int\left(\frac{1}{4} + \frac{-\frac{1}{16}}{\frac{1}{4}+x^2}\right)dx$$

$\int \frac{1}{x^2+a^2}$ with $a^2 = \frac{1}{4}$ so $a = \frac{1}{2}$

$$= \frac{1}{2}x^2\tan^{-1}(2x) - \int\left(\frac{1}{4} - \frac{1}{16}\times\frac{1}{x^2+\frac{1}{4}}\right)dx$$

$$= \left[\frac{1}{2}x^2\tan^{-1}(2x)\right]_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} - \left[\frac{1}{4}x - \frac{1}{16}\times\frac{1}{\left(\frac{1}{2}\right)}\tan^{-1}\left(\frac{x}{\left(\frac{1}{2}\right)}\right)\right]_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}}$$

$$= \left[\frac{1}{2}x^2\tan^{-1}(2x) - \frac{1}{4}x + \frac{1}{8}\tan^{-1}(2x)\right]_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}}$$

$$= \left[\frac{1}{2}\left(\frac{\sqrt{3}}{2}\right)^2\tan^{-1}(\sqrt{3}) - \frac{1}{4}\left(\frac{\sqrt{3}}{2}\right) + \frac{1}{8}\tan^{-1}(\sqrt{3})\right] - \left[\frac{1}{2}\left(\frac{1}{2}\right)^2\tan^{-1}(1) - \frac{1}{4}\left(\frac{1}{2}\right) + \frac{1}{8}\tan^{-1}(1)\right]$$

$$= \left[\frac{3}{8}\times\frac{\pi}{3} - \frac{\sqrt{3}}{8} + \frac{1}{8}\times\frac{\pi}{3}\right] - \left[\frac{1}{8}\times\frac{\pi}{4} - \frac{1}{8} + \frac{1}{8}\times\frac{\pi}{4}\right]$$

$$= \left[\frac{4\pi}{24} - \frac{\sqrt{3}}{8}\right] - \left[\frac{2\pi}{32} - \frac{1}{8}\right]$$

$$= \underline{\frac{5\pi}{48} - \frac{\sqrt{3}}{8} + \frac{1}{8}}$$

11 In a field there are 300 plants of a certain species, all of which can be infected by a particular disease. At time t after the first plant is infected there are x infected plants. The rate of change of x is proportional to the product of the number of plants infected and the number of plants that are **not** yet infected. The variables x and t are treated as continuous, and it is given that $\frac{dx}{dt} = 0.2$ and $x = 1$ when $t = 0$. *

(a) Show that x and t satisfy the differential equation

$$1495 \frac{dx}{dt} = x(300 - x).$$

[2]

Infected plants $= x$ Plants not infected $: 300 - x$

* $\frac{dx}{dt} \propto x(300-x)$

$\frac{dx}{dt} = kx(300-x)$

$\frac{dx}{dt} = 0.2, x = 1, t = 0:$

$0.2 = k \times 1 \times (300-1)$

$0.2 = 299k$

$k = \frac{1}{1495}$

$\rightarrow \frac{dx}{dt} = \frac{1}{1495} x(300-x)$

$1495 \frac{dx}{dt} = x(300-x)$ QED

(b) Using partial fractions, solve the differential equation and obtain an expression for t in terms of a single logarithm involving x. [9]

$\int \frac{1495}{x(300-x)} dx = \int 1 \, dt$

$\div 1495$ will make partial fractions a bit easier:

$\int \frac{1}{x(300-x)} dx = \int \frac{1}{1495} dt$

$\frac{1}{x(300-x)} \equiv \frac{A}{x} + \frac{B}{300-x}$

$1 \equiv A(300-x) + Bx$

$x = 300: \quad 1 = 0 + 300B$

$B = \frac{1}{300}$

$x = 0: \quad 1 = 300A + 0$

$A = \frac{1}{300}$

$\rightarrow \int \left(\frac{\frac{1}{300}}{x} + \frac{\frac{1}{300}}{300-x} \right) dx = \int \frac{1}{1495} dt$

$\frac{1}{300} \int \left(\frac{1}{x} + \frac{1}{300-x} \right) dx = \int \frac{1}{1495} dt$

$\frac{1}{300} \int \left(\frac{1}{x} - \frac{-1}{300-x} \right) dx = \int \frac{1}{1495} dt$

$\frac{1}{300} \left[\ln|x| - \ln|300-x| \right] = \frac{1}{1495} t + C$

$x = 1,\ t = 0:$

$\frac{1}{300} \left[\ln(1) - \ln(299) \right] = 0 + C$

$\frac{1}{300} \times -\ln 299 = C$

$C = \frac{-\ln 299}{300}$

$\rightarrow \frac{1}{300} \left(\ln x - \ln(300-x) \right) = \frac{1}{1495} t - \frac{\ln 299}{300}$

$\ln x - \ln(300-x) = \frac{300}{1495} t - \ln 299$

$\ln\left(\frac{x}{300-x} \right) = \frac{300}{1495} t - \ln 299$

Cont...

$$\ln\left(\frac{x}{300-x}\right) + \ln 299 = \frac{300}{1495}t$$

$$\ln\left(\frac{299x}{300-x}\right) = \frac{300}{1495}t$$

$$\ln\left(\frac{299x}{300-x}\right) = \frac{60}{299}t$$

$$t = \frac{299}{60}\ln\left(\frac{299x}{300-x}\right)$$