1 A particle is projected vertically upwards from horizontal ground with a speed of $u \text{ m s}^{-1}$. The particle has height s m above the ground at times 3 seconds and 4 seconds after projection.

Find the value of u and the value of s. [3]

$t=3$:

↑+

$s = s$	$s = ut + \frac{1}{2}at^2$
$u = u$	
$v =$	$s = 3u - 5 \times 3^2$
$a = -10$	$s = 3u - 45$ ①
$t = 3$	

$t=4$:

↑+

$s = s$	$s = ut + \frac{1}{2}at^2$
$u = u$	
$v =$	$s = 4u - 5 \times 4^2$
$a = -10$	$s = 4u - 80$ ②
$t = 4$	

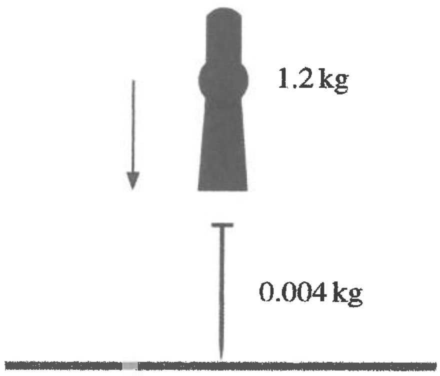
① = ②:

$3u - 45 = 4u - 80$

$-45 = u - 80$

$u = 35 \text{ ms}^{-1}$

2



A machine for driving a nail into a block of wood causes a hammerhead to drop vertically onto the top of a nail. The mass of the hammerhead is 1.2 kg and the mass of the nail is 0.004 kg (see diagram). The hammerhead hits the nail with speed $v\,\text{m s}^{-1}$ and remains in contact with the nail after the impact. The combined hammerhead and nail move immediately after the impact with speed $40\,\text{m s}^{-1}$.

(a) Calculate v, giving your answer as an exact fraction. [2]

initial   final

+ ↓ $v \downarrow$ H   HN $\downarrow 40$ $0 \downarrow$ N

$$M_H u_H + M_N u_N = M_{HN} V_{HN}$$
$$1.2 \times v + 0 = 1.204 \times 40$$
$$1.2v = 48.16$$
$$v = \frac{602}{15}\ \text{ms}^{-1}$$

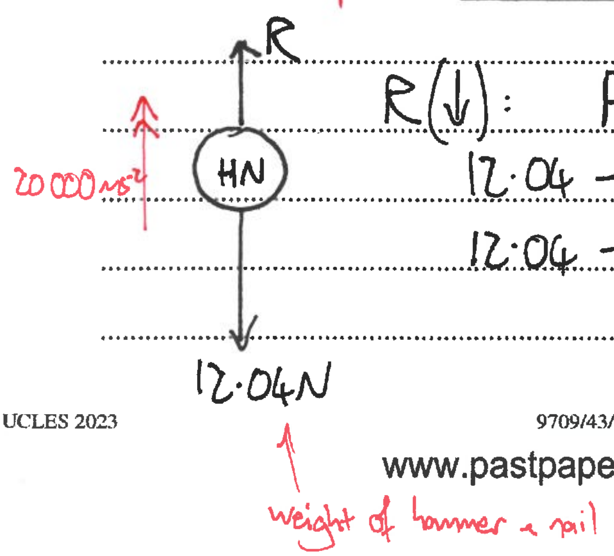
(b) The nail is driven 4 cm into the wood.

Find the constant force resisting the motion. [3]

+↓

$s = 0.04$
$u = 40$
$v = 0$
$a =$
$t =$

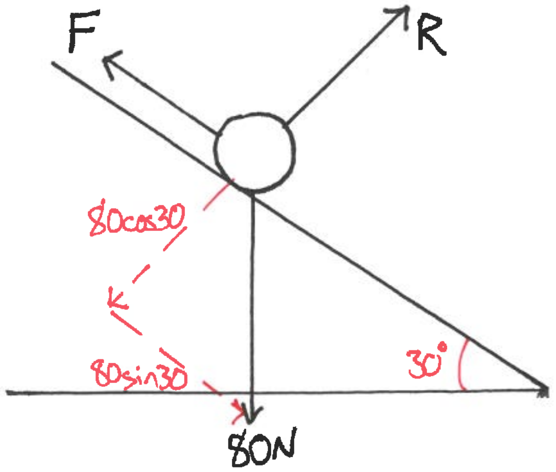
$$v^2 = u^2 + 2as$$
$$0^2 = 40^2 + 2 \times a \times 0.04$$
$$0 = 1600 + 0.08a$$
$$0.08a = -1600$$
$$a = -20\,000\ \text{ms}^{-2}$$



$$R(\downarrow):\quad F = ma$$
$$12.04 - R = 1.204 \times -20\,000$$
$$12.04 - R = -24080$$
$$R = 12.04 + 24\,080$$
$$= 24092.04\ \text{N}$$

3 A block of mass 8 kg slides down a rough plane inclined at 30° to the horizontal, starting from rest. The coefficient of friction between the block and the plane is μ. The block accelerates uniformly down the plane at $2.4\,\mathrm{m\,s^{-2}}$.

(a) Draw a diagram showing the forces acting on the block. [1]



(b) Find the value of μ. [4]

$R(\nearrow)$: $R - 80\cos 30 = 0$

$R = 80\cos 30$

$= 40\sqrt{3}\ N$

$R(\searrow)$: $F = ma$

$80\sin 30 - F = 8 \times 2.4$

$40 - \mu R = 19.2$

$\mu R = 20.8$

$\mu \times 40\sqrt{3} = 20.8$

$\mu = 0.300$

(c) Find the speed of the block after it has moved 3 m down the plane. [1]

$s = 3$
$u = 0$
$v =$
$a = 2.4$
$t =$

$v^2 = u^2 + 2as$

$v^2 = 0^2 + 2 \times 2.4 \times 3$

$v^2 = 14.4$

$v = 3.79\,\mathrm{ms^{-1}}$

4 A car has mass 1600 kg.

(a) The car is moving along a straight horizontal road at a constant speed of $24\,\mathrm{m\,s^{-1}}$ and is subject to a constant resistance of magnitude 480 N.

Find, in kW, the rate at which the engine of the car is working. [2]

At constant speed, $a = 0$:

$R(\rightarrow): D - 480 = 0$

$D = 480\,\mathrm{N}$

$\text{Power} = D \times V$

$= 480 \times 24$

$= 11520\,\mathrm{W}$

$= 11.52\,\mathrm{kW}$

The car now moves down a hill inclined at an angle of θ to the horizontal, where $\sin\theta = 0.09$. The engine of the car is working at a constant rate of 12 kW. The speed of the car is $24\,\mathrm{m\,s^{-1}}$ at the top of the hill. Ten seconds later the car has travelled 280 m down the hill and has speed $32\,\mathrm{m\,s^{-1}}$.

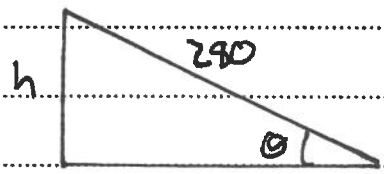
(b) Given that the resistance is not constant, use an energy method to find the total work done against the resistance during the ten seconds. [5]

Work done by engine = Power × time

$= 12000 \times 10$

$= 120\,000\,\mathrm{J}$

Height change:



$\sin\theta = \frac{h}{280}$

$h = 280\sin\theta$

$h = 280 \times 0.09$

$= 25.2\,\mathrm{m}$

$\text{Work}_{\text{in}} + KE_{\text{init}} + PE_{\text{init}} = KE_{\text{fin}} + PE_{\text{fin}} + \text{Work}_{\text{out}}$

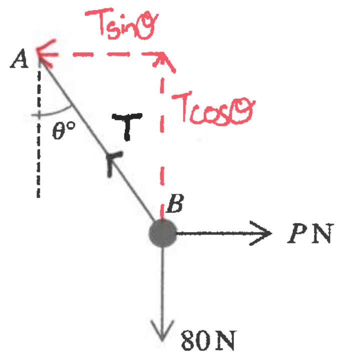
$120\,000 + \frac{1}{2} \times 1600 \times 24^2 + 0 = \frac{1}{2} \times 1600 \times 32^2 + 1600 \times 10 \times -25.2 + \text{Work}_{\text{out}}$

$120\,000 + 460\,800 = 819\,200 - 403\,200 + \text{Work}_{\text{out}}$

$580\,800 = 416\,000 + \text{Work}_{\text{out}}$

$\text{Work}_{\text{out}} = 164\,800\,\mathrm{J}$

5



A light string AB is fixed at A and has a particle of weight 80 N attached at B. A horizontal force of magnitude P N is applied at B such that the string makes an angle $\theta°$ to the vertical (see diagram).

(a) It is given that $P = 32$ and the system is in equilibrium.

Find the tension in the string and the value of θ. [4]

$R(\rightarrow)$: $32 - T\sin\theta = 0$

$T\sin\theta = 32$ (1)

$R(\uparrow)$: $T\cos\theta - 80 = 0$

$T\cos\theta = 80$ (2)

(1) ÷ (2): $\dfrac{T\sin\theta}{T\cos\theta} = \dfrac{32}{80}$

$\tan\theta = \dfrac{32}{80}$

$\theta = \underline{21.8°}$ STO

→ (1): $T\sin(21.8) = 32$

$T = \dfrac{32}{\sin(21.8)}$

$= \underline{86.2\text{N}}$

(b) It is given instead that the tension in the string is 120 N and that the particle attached at B still has weight 80 N.

Find the value of P and the value of θ. [4]

$R(\rightarrow): \quad P - 120\sin\theta = 0$

$P = 120\sin\theta$ ①

$R(\uparrow): \quad 120\cos\theta - 80 = 0$

$120\cos\theta = 80$

$\cos\theta = \frac{80}{120}$

$\theta = 48.2°$ STO

$\rightarrow$ ①: $P = 120\sin(48.2)$

$= 89.4\text{ N}$

6 A particle moves in a straight line. At time t s, the acceleration, $a \, \mathrm{m\,s^{-2}}$, of the particle is given by $a = 36 - 6t$. The velocity of the particle is $27 \, \mathrm{m\,s^{-1}}$ when $t = 2$.

(a) Find the values of t when the particle is at instantaneous rest. **[4]**

$V = \int a \, dt$

$= \int (36 - 6t) \, dt$

$V = 36t - 3t^2 + C$

$V = 27$ when $t = 2$:

$27 = 36(2) - 3(2)^2 + C$

$27 = 60 + C$

$C = -33$

$\rightarrow V = 36t - 3t^2 - 33$

$V = 0$:

$-3t^2 + 36t - 33 = 0 \quad \div -3$

$t^2 - 12t + 11 = 0$

$(t - 1)(t - 11) = 0$

$\underline{t = 1\text{s}}$ or $\underline{t = 11\text{s}}$

(b) Find the total distance the particle travels during the first 12 seconds. [4]

Stationary points at $t=1$ and $t=11$ mean that the particle changes direction at these times so we need to integrate separately from $0 \to 1s$, $1 \to 11s$, $11 \to 12s$.

$0 \to 1s$: $\int_0^1 (36t - 3t^2 - 33)\,dt = \left[18t^2 - t^3 - 33t\right]_0^1$

$= \left[18(1)^2 - (1)^3 - 33(1)\right] - [0]$

$= [18 - 1 - 33] - [0]$

$= -16m$

$1 \to 11s$: $\left[18t^2 - t^3 - 33t\right]_1^{11}$

$= \left[18(11)^2 - (11)^3 - 33(11)\right] - [-16]$

$= [2178 - 1331 - 363] - [-16]$

$= [484] - [-16]$

$= 500m$

$11 \to 12s$: $\left[18t^2 - t^3 - 33t\right]_{11}^{12}$

$= \left[18(12)^2 - (12)^3 - 33(12)\right] - [484]$

$= [2592 - 1728 - 396] - [484]$

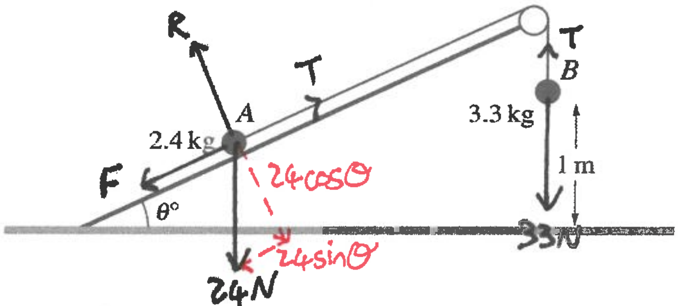
$= [468] - [484]$

$= -16m$

Distance $= 16 + 500 + 16$

$= 532m$

7



Particles A and B, of masses 2.4 kg and 3.3 kg respectively, are connected by a light inextensible string that passes over a smooth pulley which is fixed to the top of a rough plane. The plane makes an angle of $\theta°$ with horizontal ground. Particle A is on the plane and the section of the string between A and the pulley is parallel to a line of greatest slope of the plane. Particle B hangs vertically below the pulley and is 1 m above the ground (see diagram). The coefficient of friction between the plane and A is μ.

(a) It is given that $\theta = 30$ and the system is in equilibrium with A on the point of moving directly up the plane.

Show that $\mu = 1.01$ correct to 3 significant figures. [5]

B: $R(\downarrow)$: $F = ma$

$33 - T = 0$

$T = 33N$

A: $R(\nearrow)$: $F = ma$

$T - 24\sin\theta - F = 0$

$33 - 24\sin 30 - \mu R = 0$

$33 - 12 - \mu R = 0$

$21 - \mu R = 0$

$\mu R = 21$

$\mu \times 24\cos 30 = 21$

$\mu = 1.01$ QED

(b) It is given instead that $\theta = 20$ and $\mu = 1.01$. The system is released from rest with the string taut.

Find the total distance travelled by A before coming to instantaneous rest. You may assume that A does not reach the pulley and that B remains at rest after it hits the ground. [8]

B: R(↓): $F = ma$

$33 - T = 3.3a$ (1)

A: R(↗): $F = ma$

$T - 24\sin 20 - F = 2.4a$

$T - 24\sin 20 - 1.01 \times 24\cos 20 = 2.4a$

$T - 24\sin 20 - 24.24\cos 20 = 2.4a$ (2)

(1) + (2):

$33 - 24\sin 20 - 24.24\cos 20 = 5.7a$

$2.0133\ldots = 5.7a$

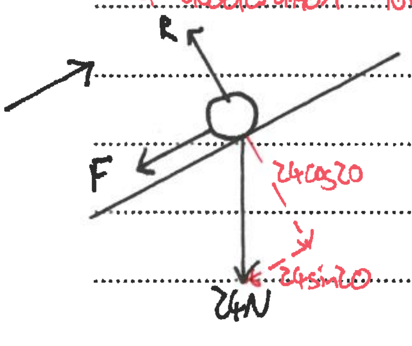
$a = \underline{0.353\ ms^{-2}}$ STO

First 1m (before B hits ground):

+↗

$s = 1$	$v^2 = u^2 + 2as$
$u = 0$	$v^2 = 0^2 + 2(0.353) \times 1$
$v =$	$v^2 = 0.706$
$a = 0.353$	$v = \underline{0.8405}$ STO
$t =$	

Find acceleration now that B has hit ground and string has gone slack:



R(↗): $F = ma$

$-24\sin 20 - F = 2.4a$

$-24\sin 20 - 1.01 \times 24\cos 20 = 2.4a$

$-30.987 = 2.4a$

$a = \underline{-12.911\ ms^{-2}}$ STO

Find distance travelled up the plane now that string has gone slack:

+↗

s:	$v^2 = u^2 + 2as$
$u = 0.8405$	$0^2 = 0.8405^2 + 2(-12.911) \times s$
$v = 0$	$0 = 0.7064 - 25.822s$
$a = -12.911$	$25.822s = 0.7064$
t:	$s = \underline{0.0274\text{m}}$

Total distance travelled $= 1 + 0.0274$

before B hits floor ↗ ↖ after B hits floor

$= \underline{1.03\text{m}}$