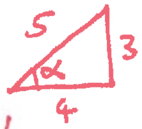


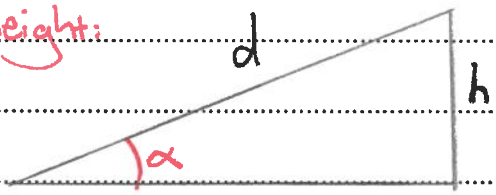
$\sin\alpha = \frac{3}{5}$

$\cos\alpha = \frac{4}{5}$

1 A particle of mass 1.6 kg is projected with a speed of $20\,\mathrm{m\,s^{-1}}$ up a line of greatest slope of a smooth plane inclined at α to the horizontal, where $\tan\alpha = \frac{3}{4}$.

Use an energy method to find the distance the particle moves up the plane before coming to instantaneous rest. [3]

Change in height:



$\sin\alpha = \frac{h}{d}$

$\frac{3}{5} = \frac{h}{d}$

$h = \frac{3}{5}d$

$\text{Work}_{\text{in}} + KE_{\text{init}} + PE_{\text{init}} = KE_{\text{fin}} + PE_{\text{fin}} + \text{Work}_{\text{out}}$

$0 + \frac{1}{2} \times 1{\cdot}6 \times 20^2 + 0 = 0 + 1{\cdot}6 \times 10 \times h + 0$

$320 = 16h$

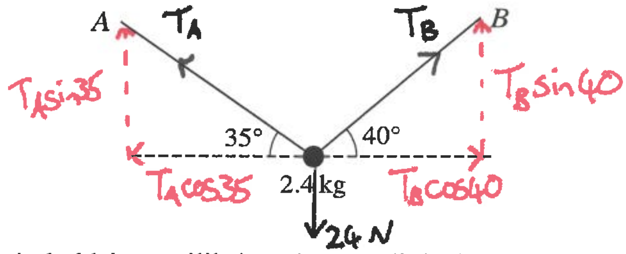
$h = 20\,\text{m}$

$h = \frac{3}{5}d$: $20 = \frac{3}{5}d$

$3d = 100$

$d = \frac{100}{3}\ \text{m}$

2



A particle of mass 2.4 kg is held in equilibrium by two light inextensible strings, one of which is attached to point A and the other attached to point B. The strings make angles of 35° and 40° with the horizontal (see diagram).

Find the tension in each of the two strings. [5]

$R(\rightarrow)$: $T_B\cos40 - T_A\cos35 = 0$

$T_B\cos40 = T_A\cos35$

$T_B = \dfrac{T_A\cos35}{\cos40}$ (1)

$R(\uparrow)$: $T_A\sin35 + T_B\sin40 - 24 = 0$ (2)

Sub. (1) into (2):

$T_A\sin35 + \dfrac{T_A\cos35}{\cos40}\sin40 - 24 = 0$

$T_A\left(\sin35 + \dfrac{\cos35\sin40}{\cos40}\right) = 24$

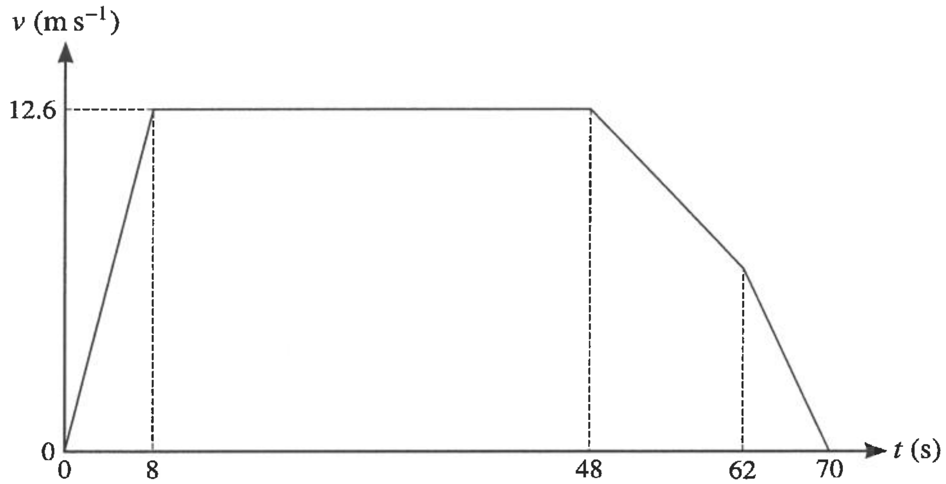
$T_A = \dfrac{24}{\sin35 + \dfrac{\cos35\sin40}{\cos40}}$

$= \underline{19.0\text{ N}}$ STO

$\rightarrow$ (1): $T_B = \dfrac{19.0\cos35}{\cos40}$

$= \underline{20.4\text{ N}}$

3



The diagram shows the velocity-time graph for the motion of a bus. The bus starts from rest and accelerates uniformly for 8 seconds until it reaches a speed of 12.6 m s^{-1}. The bus maintains this speed for 40 seconds. It then decelerates uniformly in two stages. Between 48 and 62 seconds the bus decelerates at a m s^{-2} and between 62 and 70 seconds it decelerates at $2a$ m s^{-2} until coming to rest.

(a) Find the distance covered by the bus in the first 8 seconds. [1]

distance = area : $\frac{1}{2} \times 8 \times 12.6$

$= 50.4\text{m}$

(b) Find the value of a. [3]

48→62s : $s =$ | $v = u + at$
$u = 12.6$ | $= 12.6 + -a(14)$
$v =$ | $= 12.6 - 14a$
$a = -a$
$t = 14$

Continued..

62→70s: $s =$

$u = 12.6 - 14a$

$v = 0$

$a = -2a$

$t = 8$

$v = u + at$

$0 = 12.6 - 14a + (-2a) \times 8$

$0 = 12.6 - 14a - 16a$

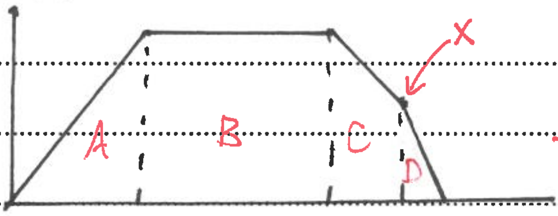
$30a = 12.6$

$a = 0.42\ \text{ms}^{-2}$

(c) Find the average speed of the bus for the whole journey. [4]

average speed $= \dfrac{\text{total distance}}{\text{total time}}$

distance:



$A = 50.4\text{m}$ (part (a))

B: $40 \times 12.6 = 504\text{m}$

velocity at X:

$v = 12.6 - 14(0.42)$ (from part (b))

$= 6.72\ \text{ms}^{-1}$

C:

$\frac{1}{2}(12.6 + 6.72) \times 14 = 135.24\text{m}$

D:

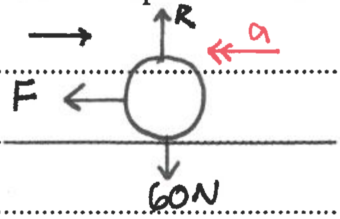
$\frac{1}{2}(8)(6.72) = 26.88$

Total distance: $50.4 + 504 + 135.24 + 26.88 = 716.52$

average speed $= \dfrac{716.52}{70} = 10.236\ \text{ms}^{-1}$

4 Two particles P and Q, of masses 6 kg and 2 kg respectively, lie at rest 12.5 m apart on a rough horizontal plane. The coefficient of friction between each particle and the plane is 0.4. Particle P is projected towards Q with speed $20\,\text{m s}^{-1}$.

(a) Show that the speed of P immediately before the collision with Q is $10\sqrt{3}\,\text{m s}^{-1}$. [3]



$R(\rightarrow)$: $F = ma$

$-\mu R = 6a$

$-0.4 \times 60 = 6a$

$-24 = 6a$

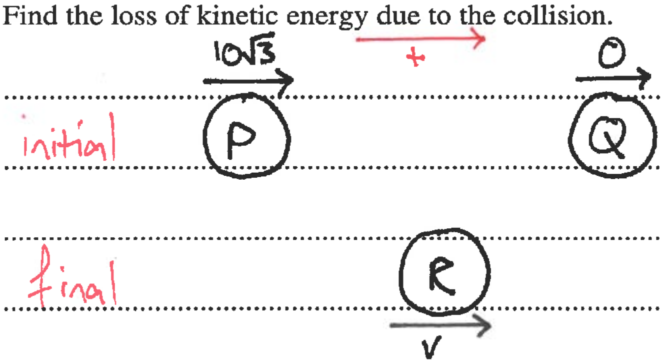
$a = -4\,\text{ms}^{-2}$

$s = 12.5$	$v^2 = u^2 + 2as$
$u = 20$	$= 20^2 + 2(-4)(12.5)$
$v =$	$= 400 - 100$
$a = -4$	$v^2 = 300$
$t =$	$v = \sqrt{300}$

$= 10\sqrt{3}\,\text{ms}^{-1}$ QED

In the collision P and Q coalesce to form particle R.

(b) Find the loss of kinetic energy due to the collision. [4]



$M_P U_P + M_Q U_Q = M_R V$

$6 \times 10\sqrt{3} + 0 = 8V$

$60\sqrt{3} = 8V$

$V = 7.5\sqrt{3}\,\text{ms}^{-1}$

Continued ...

Initial KE (only P moving):

$KE_{init} = \frac{1}{2} \times 6 \times (10\sqrt{3})^2$

$= \frac{1}{2} \times 6 \times 300$

$= \underline{900 J}$

Final KE (only R moving):

$KE_{fin} = \frac{1}{2} \times 8 \times (7.5\sqrt{3})^2$

$= \frac{1}{2} \times 8 \times 168.75$

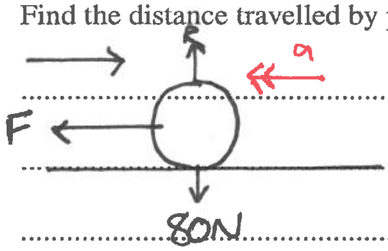
$= \underline{675 J}$

Loss of KE $= 900 - 675$

$= \underline{225 J}$

The coefficient of friction between R and the plane is 0.4.

(c) Find the distance travelled by particle R before coming to rest. [2]



$R(\rightarrow)$: $F = ma$

$-\mu R = 8a$

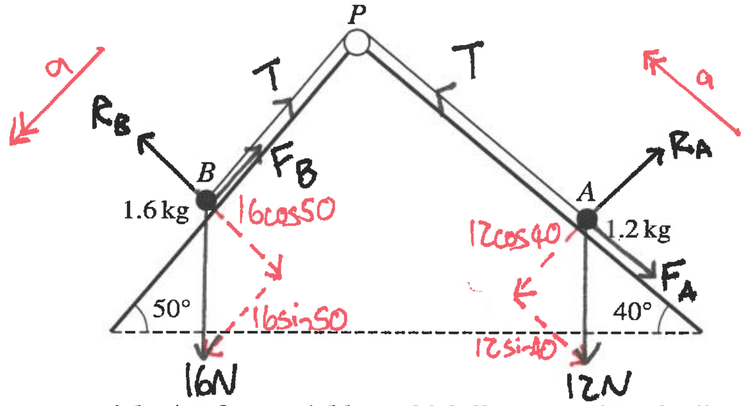
$-0.4 \times 80 = 8a$

$-32 = 8a$

$\underline{a = -4 \text{ ms}^{-2}}$

$s =$	$v^2 = u^2 + 2as$
$u = 7.5\sqrt{3}$	$0^2 = (7.5\sqrt{3})^2 + 2(-4)s$
$v = 0$	$0 = 168.75 - 8s$
$a = -4$	$8s = 168.75$
$t =$	$\underline{s = 21.1 m}$

5



The diagram shows a particle A, of mass 1.2 kg, which lies on a plane inclined at an angle of 40° to the horizontal and a particle B, of mass 1.6 kg, which lies on a plane inclined at an angle of 50° to the horizontal. The particles are connected by a light inextensible string which passes over a small smooth pulley P fixed at the top of the planes. The parts AP and BP of the string are taut and parallel to lines of greatest slope of the respective planes. The two planes are rough, with the same coefficient of friction, μ, between the particles and the planes.

Find the value of μ for which the system is in limiting equilibrium. [7]

Find which way the system wants to move by comparing 16sin50 and 12sin40:

16sin50 > 12sin40, so B wants to move down the slope and A wants to move up. So friction at B is up and friction at A is down.

B:

$R(\swarrow)$: $16\sin 50 - T - F_B = 0$

$16\sin 50 - T - \mu \times 16\cos 50 = 0$ ①

A:

$R(\nwarrow)$: $T - 12\sin 40 - F_A = 0$

$T - 12\sin 40 - \mu \times 12\cos 40 = 0$ ②

continued..

① + ②:

$16\sin 50 - 16\mu\cos 50 - 12\sin 40 - 12\mu\cos 40 = 0$

$16\sin 50 - 12\sin 40 = 16\mu\cos 50 + 12\mu\cos 40$

$\mu(16\cos 50 + 12\cos 40) = 16\sin 50 - 12\sin 40$

$$\mu = \frac{16\sin 50 - 12\sin 40}{16\cos 50 + 12\cos 40}$$

$\mu = 0.233$

6 A car of mass 1300 kg is moving on a straight road.

(a) On a horizontal section of the road, the car has a constant speed of $30\,\text{m}\,\text{s}^{-1}$ and there is a constant force of 650 N resisting the motion.

(i) Calculate, in kW, the power developed by the engine of the car. [2]

At constant speed, $a = 0$:

$R(\rightarrow)$: $D - 650 = 0$

$D = 650\,\text{N}$

$\text{Power} = Dv$

$= 650 \times 30$

$= \underline{19\,500\,\text{W}}$

$= \underline{19.5\,\text{kW}}$

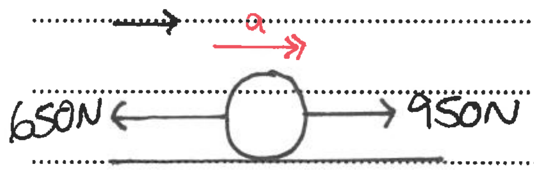
(ii) Given that this power is suddenly increased **by** 9 kW, find the instantaneous acceleration of the car. [3]

New Power $= 28\,500\,\text{W}$

$\text{Power} = Dv$

$28\,500 = D \times 30$

$D = 950\,\text{N}$



$R(\rightarrow)$: $F = ma$

$950 - 650 = 1300a$

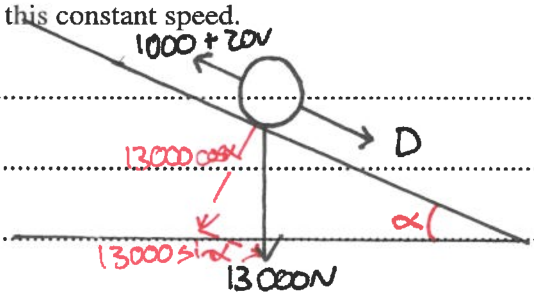
$300 = 1300a$

$a = \underline{0.231\,\text{ms}^{-2}}$

$\sin\alpha = 0.08$

(b) On a section of the road inclined at $\sin^{-1} 0.08$ to the horizontal, the resistance to the motion of the car is $(1000 + 20v)$ N when the speed of the car is $v\,\text{m s}^{-1}$. The car travels downwards along this section of the road at constant speed with the engine working at 11.5 kW.

Find this constant speed. [4]



$a = 0$ (constant speed)

$R(\searrow)$: $F = ma$

$13000\sin\alpha + D - (1000 + 20V) = ma$

$13000(0.08) + D - 1000 - 20V = 0$

$1040 + D - 1000 - 20V = 0$

$D - 20V + 40 = 0$ ①

$\text{Power} = DV$

$11500 = DV$

$D = \dfrac{11500}{V}$ ②

Sub. ② into ①:

$\dfrac{11500}{V} - 20V + 40 = 0$ $\times V$

$11500 - 20V^2 + 40V = 0$ $\div 20$

$V^2 - 2V + 575 = 0$

$(V - 25)(V + 23) = 0$

$V = 25\,\text{ms}^{-1}$ ✓ or $V = -23$ ✗

7 A particle moves in a straight line starting from a point O before coming to instantaneous rest at a point X. At time t s after leaving O, the velocity $v\,\mathrm{m\,s^{-1}}$ of the particle is given by

$$v = 7.2t^2 \qquad 0 \leqslant t \leqslant 2, \quad ①$$

equal when $t=8$

$$v = 30.6 - 0.9t \qquad 2 \leqslant t \leqslant 8, \quad ②$$

$$v = \frac{1600}{t^2} + kt \qquad 8 \leqslant t, \quad ③$$

where k is a constant. It is given that there is no instantaneous change in velocity at $t = 8$.

Find the distance OX. [9]

No change in velocity at $t=8$: ② = ③:

$$30.6 - 0.9(8) = \frac{1600}{8^2} + 8k$$

$$23.4 = 25 + 8k$$

$$8k = -1.6$$

$$k = -0.2$$

Find when particle is at rest:

①: $7.2t^2 = 0$

$t = 0$

②: $30.6 - 0.9t = 0$

$30.6 = 0.9t$

$t = 34$ ✗ outside $2 \leqslant t \leqslant 8$

③: $\frac{1600}{t^2} - 0.2t = 0$

$\frac{1600}{t^2} = 0.2t$

$1600 = 0.2t^3$

$t^3 = 8000$

$t = 20$s → point X is when $t=20$s.

No other stationary points between $t=0$ and $t=20$, so we can begin integrating to find displacement:

①: $\int_0^2 7.2t^2\,dt = \left[2.4t^3\right]_0^2$

$= \left[2.4(2)^3\right] - \left[0\right]$

$= \underline{19.2\text{m}}$

②: $\int_2^8 (30.6 - 0.9t)\,dt = \left[30.6t - 0.45t^2\right]_2^8$

$= \left[30.6(8) - 0.45(8)^2\right] - \left[30.6(2) - 0.45(2)^2\right]$

$= \left[216\right] - \left[59.4\right]$

$= \underline{156.6\text{m}}$

③: $\int_8^{20} (1600t^{-2} - 0.2t)\,dt = \left[-1600t^{-1} - 0.1t^2\right]_8^{20}$

$= \left[-1600(20)^{-1} - 0.1(20)^2\right] - \left[-1600(8)^{-1} - 0.1(8)^2\right]$

$= \left[-80 - 40\right] - \left[-200 - 6.4\right]$

$= \left[-120\right] - \left[-206.4\right]$

$= \underline{86.4\text{m}}$

Distance $= 19.2 + 156.6 + 86.4$

$= \underline{262.2\text{m}}$