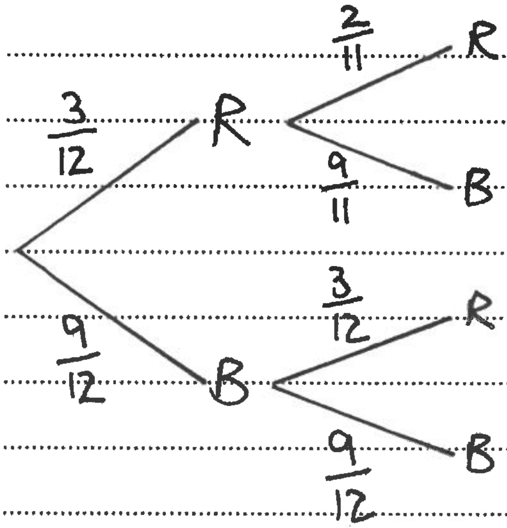
1 A bag contains 9 blue marbles and 3 red marbles. One marble is chosen at random from the bag. If this marble is blue, it is replaced back into the bag. If this marble is red, it is **not** returned to the bag. A second marble is now chosen at random from the bag.

(a) Find the probability that both the marbles chosen are red. [1]



$P(R \cap R) = \frac{3}{12} \times \frac{2}{11}$

$= \frac{1}{22}$

(b) Find the probability that the first marble chosen is blue given that the second marble chosen is red. [3]

$P(B_1 \cap R_2) = P(B_1 | R_2) \times P(R_2)$

$P(B_1 | R_2) = \frac{P(B_1 \cap R_2)}{P(R_2)}$

$P(B_1 \cap R_2) = \frac{9}{12} \times \frac{3}{12}$

$= \frac{3}{16}$

$P(R_2) = \frac{3}{12} \times \frac{2}{11} + \frac{9}{12} \times \frac{3}{12}$

$= \frac{1}{22} + \frac{3}{16}$

$= \frac{41}{176}$

$P(B_1 | R_2) = \frac{\frac{3}{16}}{\frac{41}{176}} = \frac{33}{41}$

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2 Sam is a member of a soccer club. She is practising scoring goals. The probability that Sam will score a goal on any attempt is 0.7, independently of all other attempts.

(a) Sam makes 10 attempts at scoring goals.

Find the probability that Sam will score goals on fewer than 8 of these attempts. [3]

$G \sim B(10, 0.7)$

$P(G < 8) = 1 - (P(8) + P(9) + P(10))$

$= 1 - ({}^{10}C_8 \times 0.7^8 \times 0.3^2 + {}^{10}C_9 \times 0.7^9 \times 0.3^1 + {}^{10}C_{10} \times 0.7^{10})$

$= 1 - 0.3828$

$= \underline{0.617}$

(b) Find the probability that Sam's first successful attempt will be before her 5th attempt. [2]

$G \sim Geo(0.7)$

$P(G < 5) = P(G \leq 4)$

$= 1 - q^4$

probability of 4 failures

$= 1 - 0.3^4$

$= \underline{0.9919}$

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(c) Wei is a member of the same soccer club. He is also practising scoring goals. The probability that Wei will score a goal on any attempt is 0.6, independently of all other attempts.

Wei is going to keep making attempts until he scores 3 goals.

Find the probability that he scores his third goal on his 7th attempt. [3]

We can't do a binomial distribution with all 7 attempts because the last attempt must be a goal.
Instead, we'll do a binomial distribution for the first 6 attempts (in which he scores 2 goals):

$$W \sim B(6, 0.6)$$

Two goals in 6 attempts:

$$P(W = 2) = {}^6C_2 \times 0.6^2 \times 0.4^4$$

$$= 15 \times 0.36 \times 0.0256$$

$$= 0.13824$$

7th attempt is a goal:

$$\underbrace{0.13824}_{\text{2 goals in first 6 attempts}} \times \underbrace{0.6}_{\text{goal in seventh attempt}} = \underline{0.0829}$$

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3 The times taken, in minutes, by 150 students to complete a puzzle are summarised in the table.

Time taken (t minutes)	$0 \leqslant t < 20$	$20 \leqslant t < 30$	$30 \leqslant t < 35$	$35 \leqslant t < 40$	$40 \leqslant t < 50$	$50 \leqslant t < 70$
Frequency	8	23	35	52	20	12
Class width	20	10	5	5	10	20
f.d.	0·4	2·3	7	10·4	2	0·6

(a) Draw a histogram to represent this information. [4]

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(b) Calculate an estimate for the mean time for these students to complete the puzzle. [3]

Mid-point (t)	Frequency (f)	$f \times t$
10	8	80
25	23	575
32.5	35	1137.5
37.5	52	1950
45	20	900
60	12	720
	$\Sigma f = 150$	$\Sigma ft = 5362.5$

$$\bar{t} = \frac{5362.5}{150}$$

$$= 35.75$$

(c) In which class interval does the lower quartile of the times lie? [1]

Time	0-20	20-30	30-35	35-40 ...
Cumulative freq	8	31	66	118

Q_1: $\frac{150}{4} = 37.5^{th}$ value

Q_1 lies in $30 \leq t < 35$

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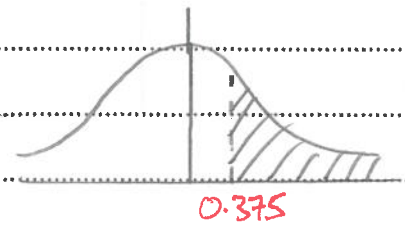
4 A company sells small and large bags of rice. The masses of the small bags of rice are normally distributed with mean 1.20 kg and standard deviation 0.16 kg.

(a) In a random sample of 500 of these small bags of rice, how many would you expect to have a mass greater than 1.26 kg? [4]

$P(S > 1.26) = P\left(Z > \frac{1.26 - 1.20}{0.16}\right)$

$= P(Z > 0.375)$



$= 1 - \Phi(0.375)$

$= 1 - 0.6462$

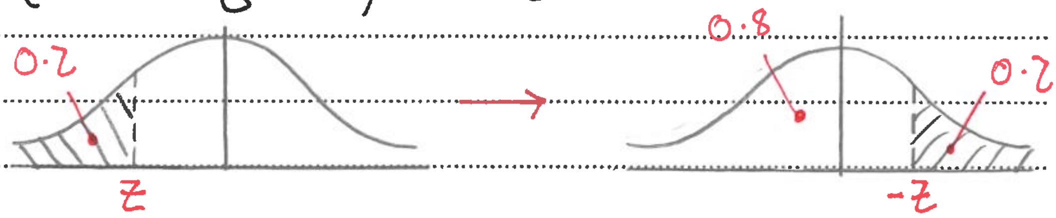
$= \underline{\underline{0.3538}}$

The masses of the large bags of rice are normally distributed with mean 2.50 kg and standard deviation σ kg. 20% of these large bags of rice have a mass less than 2.40 kg.

(b) Find the value of σ. [3]

$P(L < 2.4) = 0.2$

$P\left(Z < \frac{2.4 - 2.5}{\sigma}\right) = 0.2$



$0.8 = \Phi(0.842)$

$z = -0.842$

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$$\frac{2.4-2.5}{\sigma} = -0.842$$

$$2.4-2.5 = -0.842\sigma$$

$$-0.1 = -0.842\sigma$$

$$\sigma = 0.119$$

A random sample of 80 large bags of rice is chosen.

(c) Use a suitable approximation to find the probability that fewer than 22 of these large bags of rice have a mass less than 2.40 kg. [5]

$L \sim B(80, 0.2)$ probability that mass < 2.4kg

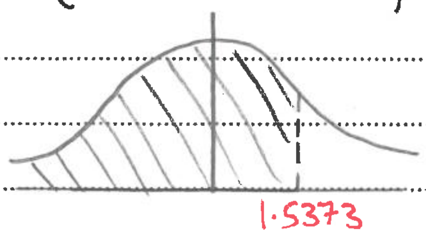
$\mu = 80 \times 0.2 = 16$ $\qquad$ $\sigma^2 = 16 \times 0.8 = 12.8$

$L \sim N(16, 12.8)$

$P(L < 22) \rightarrow P(L < 21.5)$ (continuity correction)

$$P\left(L < \frac{21.5-16}{\sqrt{12.8}}\right)$$

$$= P(L < 1.5373)$$



$$= \Phi(1.5373)$$

$$= 0.9378$$

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5 Anil is taking part in a tournament. In each game in this tournament, players are awarded 2 points for a win, 1 point for a draw and 0 points for a loss. For each of Anil's games, the probabilities that he will win, draw or lose are 0.5, 0.3 and 0.2 respectively. The results of the games are all independent of each other.

The random variable X is the total number of points that Anil scores in his first 3 games in the tournament.

(a) Show that $P(X = 2) = 0.114$. [2]

For 2 points, either 1 Win, 2 Losses or 2 Draws, 1 Loss.

① WLL: (in any order)

$^3C_1 \times 0.5^1 \times 0.2^2 = 0.06$

② DDL: (in any order)

$^3C_2 \times 0.3^2 \times 0.2^1 = 0.054$

$0.06 + 0.054 = \underline{0.114}$ QED

(b) Complete the probability distribution table for X. [3]

x	0	1	2	3	4	5	6
$P(X = x)$	0.008	0.036	0.114	0.207	0.285	0.225	0.125

0: LLL:

$0.2 \times 0.2 \times 0.2 = 0.008$

1: DLL (any order):

$^3C_1 \times 0.3^1 \times 0.2^2 = 0.036$

5: WWD (any order):

$^3C_2 \times 0.5^2 \times 0.3^1 = 0.225$

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(c) Find the value of Var(X). [3]

$E(X) = 0 \times 0.008 + 1 \times 0.036 + 2 \times 0.114 + 3 \times 0.207 + 4 \times 0.285 + \ldots$

$\ldots + 5 \times 0.225 + 6 \times 0.125$

$= \underline{3.9}$

$Var(X) = 0^2 \times 0.008 + 1^2 \times 0.036 + 2^2 \times 0.114 + 3^2 \times 0.207 + 4^2 \times 0.285 + \ldots$

$\ldots + 5^2 \times 0.225 + 6^2 \times 0.125 - (E(X))^2$

$= 0 + 0.036 + 0.456 + 1.863 + 4.56 + \ldots$

$\ldots + 5.625 + 4.5 - (3.9)^2$

$= 17.04 - 15.21$

$= \underline{1.83}$

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6 A new village social club has 10 members of whom 6 are men and 4 are women. The club committee will consist of 5 members.

(a) In how many ways can the committee of 5 members be chosen if it must include at least 2 men and at least 1 woman? [4]

4M, 1W: $^6C_4 \times {}^4C_1 = 60$

4 men from 6   1 woman from 4

3M, 2W: $^6C_3 \times {}^4C_2 = 120$

2M, 3W: $^6C_2 \times {}^4C_3 = 60$

$60 + 120 + 60 = \underline{\underline{240}}$

The 10 members of the club stand in a line for a photograph.

(b) How many different arrangements are there of the 10 members if all the men stand together and all the women stand together? [2]

Arrange all the men: $6!$

Arrange all the women: $4!$

Either men on left, women on right, or vice-versa: $2!$

$6! \times 4! \times 2! = \underline{\underline{34\,560}}$

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For a second photograph, the members stand in two rows, with 6 on the back row and 4 on the front row. Olly and his sister Petra are two of the members of the club.

(c) How many different arrangements are there of the 10 members in which Olly and Petra stand next to each other on the front row? [4]

Front row:

Pick two more people from the remaining 8 (excluding O and P) to be in front row with O and P: $^8C_2$

Treat O and P as one object and arrange with other two people:

OP

$$^8C_2 \times 3! \times 2! = 336$$

could be OP or PO

Back row:

Pick the remaining six people and permutate them:

$$^6P_6 = 720$$

Altogether: $336 \times 720 = \underline{\underline{241\,920}}$

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