1 A fair red spinner has edges numbered 1, 2, 2, 3. A fair blue spinner has edges numbered −3, −2, −1, −1. Each spinner is spun once and the number on the edge on which each spinner lands is noted. The random variable X denotes the sum of the resulting two numbers.

(a) Draw up the probability distribution table for X. [3]

	1	2	2	3
-3	-2	-1	-1	0
-2	-1	0	0	1
-1	0	1	1	2
-1	0	1	1	2

x	-2	-1	0	1	2
$P(X=x)$	$\frac{1}{16}$	$3/16$	$5/16$	$5/16$	$2/16$

(b) Given that $E(X) = 0.25$, find the value of $Var(X)$. [2]

$$Var(X) = (-2)^2 \times \frac{1}{16} + (-1)^2 \times \frac{3}{16} + 0^2 \times \frac{5}{16} + 1^2 \times \frac{5}{16} + 2^2 \times \frac{2}{16} - (E(X))^2$$

$$= \frac{4}{16} + \frac{3}{16} + 0 + \frac{5}{16} + \frac{8}{16} - (0.25)^2$$

$$= \frac{20}{16} - \frac{1}{16}$$

$$= \frac{19}{16}$$

2 In a certain country, the probability of more than 10 cm of rain on any particular day is 0.18, independently of the weather on any other day.

(a) Find the probability that in any randomly chosen 7-day period, more than 2 days have more than 10 cm of rain. [3]

$X \sim B(7, 0.18)$

$P(X > 2) = 1 - (P(0) + P(1) + P(2))$

$= 1 - ({}^7C_0 \times 0.18^0 \times 0.82^7 + {}^7C_1 \times 0.18^1 \times 0.82^6$
$+ {}^7C_2 \times 0.18^2 \times 0.82^5)$

$= \underline{0.115}$

(b) For 3 randomly chosen 7-day periods, find the probability that exactly two of these periods have at least one day with more than 10 cm of rain. [3]

Probability of at least one day of more than 10cm:

$P(X \geq 1) = P(X > 0)$
$= 1 - P(0)$
$= 1 - {}^7C_0 \times 0.18^0 \times 0.82^7$
$= 0.7507...$

← store

$Y \sim B(3, 0.7507)$

$P(Y = 2) = {}^3C_2 \times 0.7507^2 \times 0.2493$

$= \underline{0.421}$

3 At a summer camp an arithmetic test is taken by 250 children. The times taken, to the nearest minute, to complete the test were recorded. The results are summarised in the table.

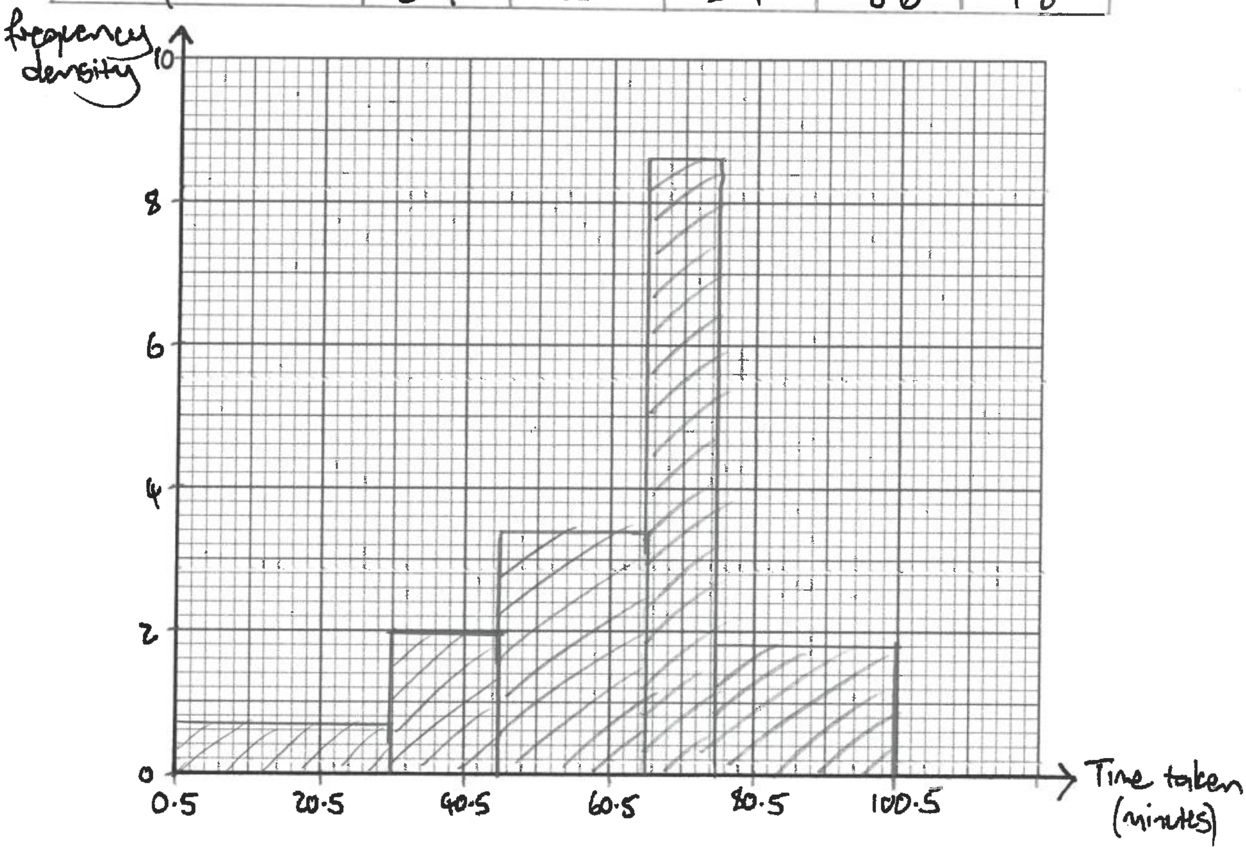
0.5 30.5 45.5 65.5 75.5 100.5

Time taken, in minutes	1 – 30	31 – 45	46 – 65	66 – 75	76 – 100
Frequency	21	30	68	86	45

(a) Draw a histogram to represent this information. [4]

30.5 - 0.5

Class width	30	15	20	10	25
f.d.	0.7	2	3.4	8.6	1.8



(b) State which class interval contains the median. [1]

Time	1-30	31-45	46-65	66-75	76-100
cumulative freq.	21	51	119	205	250

Q_2: $\frac{250}{2} = 125$

125 (Q_2)

Q_2 lies in 66-75

(c) Given that an estimate of the mean time is 61.05 minutes, state what feature of the distribution accounts for the median and the mean being different. [1]

The distribution is negatively skewed
(median > mean)

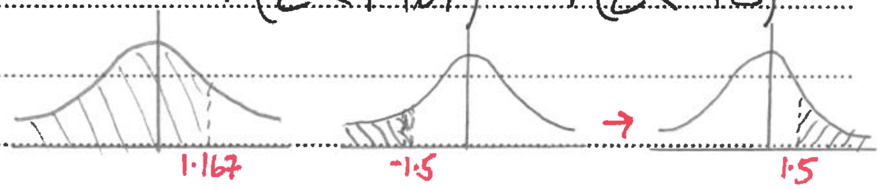
4 The weights of male leopards in a particular region are normally distributed with mean 55 kg and standard deviation 6 kg.

(a) Find the probability that a randomly chosen male leopard from this region weighs between 46 and 62 kg. [4]

$P(46 < W < 62) = P\left(\frac{46-55}{6} < Z < \frac{62-55}{6}\right)$

$= P(-1.5 < Z < 1.167)$

$= P(Z < 1.167) - P(Z < -1.5)$



$= \Phi(1.167) - (1 - \Phi(1.5))$

$= 0.8784 - (1 - 0.9332)$

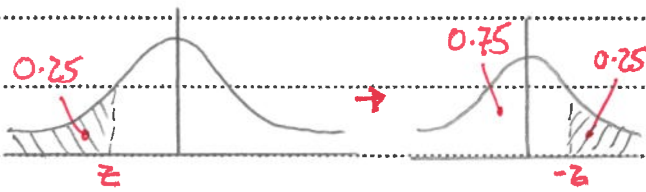
$= 0.8116$

The weights of female leopards in this region are normally distributed with mean 42 kg and standard deviation σ kg. It is known that 25% of female leopards in the region weigh less than 36 kg.

(b) Find the value of σ. [3]

$P(F < 36) = 0.25$

$P\left(Z < \frac{36-42}{\sigma}\right) = 0.25$



$0.75 = \Phi(0.674)$

critical value

$\therefore z = -0.674$

$\frac{36-42}{\sigma} = -0.674$

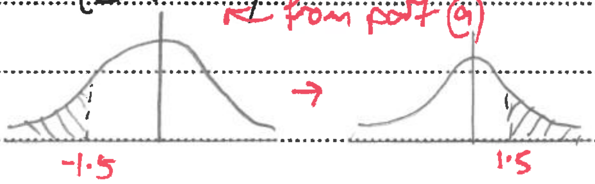
$-6 = -0.674\sigma$

$\sigma = 8.90$

The distributions of the weights of male and female leopards are independent of each other. A male leopard and a female leopard are each chosen at random.

(c) Find the probability that both the weights of these leopards are less than 46 kg. [4]

Male: $P(W < 46) = P(Z < -1.5)$ ← from part (a)



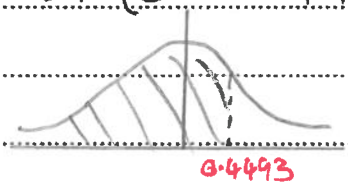
$= 1 - \Phi(1.5)$

$= 1 - 0.9332$

$= \underline{0.0668}$

Female: $P(F < 46) = P\left(Z < \frac{46 - 42}{8.90\ldots}\right)$

$= P(Z < 0.4493)$



$= \Phi(0.449)$

$= \underline{0.6732}$

$P(\text{both weigh} < 46) = 0.0668 \times 0.6732$

$= \underline{\underline{0.0450}}$

5 A group of 12 people consists of 3 boys, 4 girls and 5 adults.

(a) In how many ways can a team of 5 people be chosen from the group if exactly one adult is included? [2]

$${}^5C_1 \times {}^7C_4 = \underline{175}$$

pick one adult from 5

pick 4 children from 7

(b) In how many ways can a team of 5 people be chosen from the group if the team includes at least 2 boys and at least 1 girl? [4]

2B, 1G: B B G ___ ___ ${}^3C_2 \times {}^4C_1 \times {}^5C_2 = 120$

pick 2 boys from 3 pick one girl pick 2 adults

3B, 1G: B B B G ___ ${}^3C_3 \times {}^4C_1 \times {}^5C_1 = 20$

2B, 2G: B B G G ___ ${}^3C_2 \times {}^4C_2 \times {}^5C_1 = 90$

3B, 2G: B B B G G ${}^3C_3 \times {}^4C_2 = 6$

2B, 3G: B B G G G ${}^3C_2 \times {}^4C_3 = 12$

$120 + 20 + 90 + 6 + 12 = \underline{248}$

The same group of 12 people stand in a line.

(c) How many different arrangements are there in which the 3 boys stand together and an adult is at each end of the line? [4]

Pick two adults for the ends and permute: ${}^5P_2$

Treat the three boys as one object and permutate with remaining people:

A __ __ __ __ __ __ __ __ A

fixed (B B B) fixed

one object

$8!$

$\rightarrow {}^5P_2 \times 8! \times 3! = 4\,838\,400$

permutate the three boys

6 A factory produces chocolates in three flavours: lemon, orange and strawberry in the ratio 3 : 5 : 7 respectively. Nell checks the chocolates on the production line by choosing chocolates randomly one at a time.

(a) Find the probability that the first chocolate with lemon flavour that Nell chooses is the 7th chocolate that she checks. [1]

$P(\text{lemon}) = \frac{3}{15} = \frac{1}{5}$

$L \sim Geo\left(\frac{1}{5}\right)$

$P(L=7) = \left(\frac{1}{5}\right) \times \left(\frac{4}{5}\right)^6$

$= 0.0524$

(b) Find the probability that the first chocolate with lemon flavour that Nell chooses is after she has checked at least 6 chocolates. [2]

After 6 chocolates $= P(L>6)$

$= q^6$ ← probability of 6 failures

$= \left(\frac{4}{5}\right)^6$

$= 0.262$

'Surprise' boxes of chocolates each contain 15 chocolates: 3 are lemon, 5 are orange and 7 are strawberry.

Petra has a box of Surprise chocolates. She chooses 3 chocolates at random from the box. She eats each chocolate before choosing the next one.

(c) Find the probability that none of Petra's 3 chocolates has orange flavour. [2]

Probability is not constant, so can't use binomial or geometric distributions.

pick 3 from 10 (lemon and strawberry) → $\dfrac{{}^{10}C_3 \times ({}^{5}C_0)}{{}^{15}C_3}$ ← no oranges ← total number of possibilities

$= \frac{24}{91}$

(d) Find the probability that each of Petra's 3 chocolates has a different flavour. [3]

1 lemon 1 orange 1 strawberry

$$\frac{{}^{3}C_{1} \times {}^{5}C_{1} \times {}^{7}C_{1}}{{}^{15}C_{3}}$$

$$= \frac{3}{13}$$

(e) Find the probability that at least 2 of Petra's 3 chocolates have strawberry flavour given that none of them has orange flavour. [4]

P(S) P(O')

$$P(S \cap O') = P(S|O') \times P(O')$$

$$P(S|O') = \frac{P(S \cap O')}{P(O')}$$

$P(S \cap O')$:

$SSS = \frac{{}^{7}C_{3}}{{}^{15}C_{3}} = \frac{1}{13}$

$\left.\begin{array}{l} SSL \\ SLS \\ LSS \end{array}\right\} + = \frac{{}^{7}C_{2} \times {}^{3}C_{1}}{{}^{15}C_{3}} = \frac{9}{65}$

$$P(S \cap O') = \frac{1}{13} + \frac{9}{65}$$

$$= \frac{14}{65}$$

$$P(S|O') = \frac{\frac{14}{65}}{\frac{24}{91}}$$

from part (c)

$$= \frac{49}{60}$$