7 Two lines have equations $\mathbf{r} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} + s\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} + t\begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix}$.

(a) Show that the lines are skew. [5]

(b) Find the acute angle between the directions of the two lines. [3]

6 Relative to the origin O, the points A, B and C have position vectors given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}, \qquad \overrightarrow{OB} = \begin{pmatrix} 4 \\ 3 \\ 2 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 3 \\ -2 \\ -4 \end{pmatrix}.$$

The quadrilateral $ABCD$ is a parallelogram.

(a) Find the position vector of D. [3]

(b) The angle between BA and BC is θ.

Find the exact value of $\cos\theta$. [3]

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(c) Hence find the area of $ABCD$, giving your answer in the form $p\sqrt{q}$, where p and q are integers. [4]

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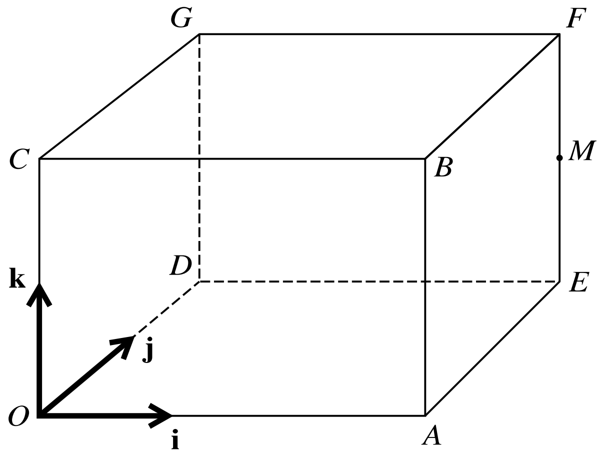

In the diagram, $OABCDEFG$ is a cuboid in which $OA = 3$ units, $OC = 2$ units and $OD = 2$ units. Unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to OA, OD and OC respectively. M is the midpoint of EF.

(a) Find the position vector of M. [1]

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The position vector of P is $\mathbf{i} + \mathbf{j} + 2\mathbf{k}$.

(b) Calculate angle PAM. [4]

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(c) Find the exact length of the perpendicular from P to the line passing through O and M. [5]

10 The equations of the lines l and m are given by

$$l: \ \mathbf{r} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \qquad \text{and} \qquad m: \ \mathbf{r} = \begin{pmatrix} 6 \\ -3 \\ 6 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 4 \\ c \end{pmatrix},$$

where c is a positive constant. It is given that the angle between l and m is 60°.

(a) Find the value of c. [4]

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(b) Show that the length of the perpendicular from $(6, -3, 6)$ to l is $\sqrt{11}$. [5]

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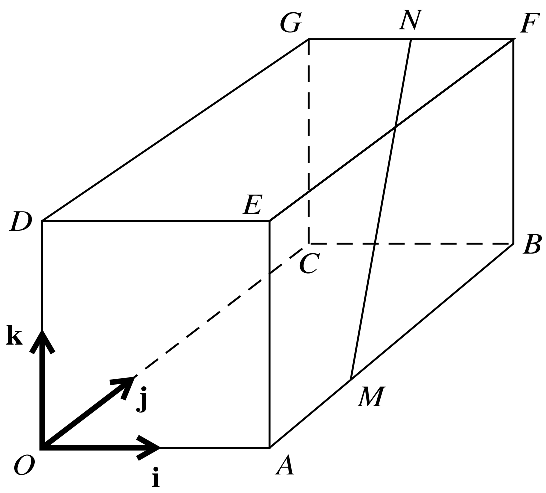

In the diagram, *OABCDEFG* is a cuboid in which $OA = 2$ units, $OC = 3$ units and $OD = 2$ units. Unit vectors **i**, **j** and **k** are parallel to *OA*, *OC* and *OD* respectively. The point *M* on *AB* is such that $MB = 2AM$. The midpoint of *FG* is *N*.

(a) Express the vectors $\overrightarrow{OM}$ and $\overrightarrow{MN}$ in terms of **i**, **j** and **k**. [3]

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(b) Find a vector equation for the line through *M* and *N*. [2]

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(c) Find the position vector of P, the foot of the perpendicular from D to the line through M and N. [4]

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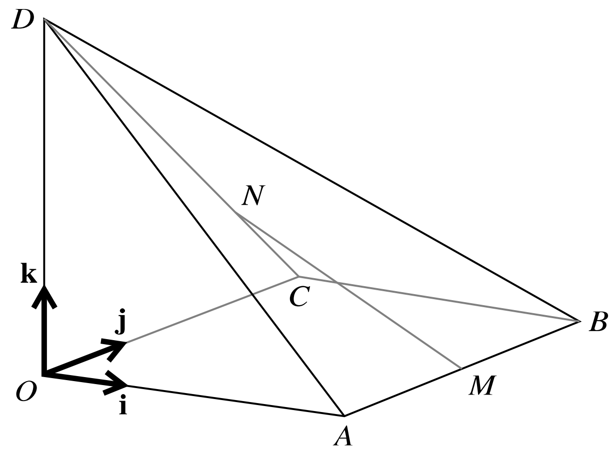

In the diagram, $OABCD$ is a pyramid with vertex D. The horizontal base $OABC$ is a square of side 4 units. The edge OD is vertical and $OD = 4$ units. The unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to OA, OC and OD respectively.

The midpoint of AB is M and the point N on CD is such that $DN = 3NC$.

(a) Find a vector equation for the line through M and N. [5]

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(b) Show that the length of the perpendicular from O to MN is $\frac{1}{3}\sqrt{82}$. [4]

9 With respect to the origin O, the position vectors of the points A, B and C are given by

$$\overrightarrow{OA} = \begin{pmatrix} 0 \\ 5 \\ 2 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 4 \\ -3 \\ -2 \end{pmatrix}.$$

The midpoint of AC is M and the point N lies on BC, between B and C, and is such that $BN = 2NC$.

(a) Find the position vectors of M and N. [3]

(b) Find a vector equation for the line through M and N. [2]

(c) Find the position vector of the point Q where the line through M and N intersects the line through A and B. [4]

9 With respect to the origin O, the vertices of a triangle ABC have position vectors

$$\overrightarrow{OA} = 2\mathbf{i} + 5\mathbf{k}, \quad \overrightarrow{OB} = 3\mathbf{i} + 2\mathbf{j} + 3\mathbf{k} \quad \text{and} \quad \overrightarrow{OC} = \mathbf{i} + \mathbf{j} + \mathbf{k}.$$

(a) Using a scalar product, show that angle ABC is a right angle. [3]

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(b) Show that triangle ABC is isosceles. [2]

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(c) Find the exact length of the perpendicular from O to the line through B and C. [4]

8 With respect to the origin O, the position vectors of the points A, B, C and D are given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 4 \\ -1 \\ 1 \end{pmatrix}, \quad \overrightarrow{OC} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OD} = \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix}.$$

(a) Show that $AB = 2CD$. [3]

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(b) Find the angle between the directions of $\overrightarrow{AB}$ and $\overrightarrow{CD}$. [3]

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(c) Show that the line through A and B does not intersect the line through C and D. [4]

8 With respect to the origin O, the points A and B have position vectors given by $\overrightarrow{OA} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$. The line l has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$.

(a) Find the acute angle between the directions of AB and l. [4]

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(b) Find the position vector of the point P on l such that $AP = BP$. [5]

10 With respect to the origin O, the points A and B have position vectors given by $\overrightarrow{OA} = 6\mathbf{i} + 2\mathbf{j}$ and $\overrightarrow{OB} = 2\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$. The midpoint of OA is M. The point N lying on AB, between A and B, is such that $AN = 2NB$.

(a) Find a vector equation for the line through M and N. [5]

The line through M and N intersects the line through O and B at the point P.

(b) Find the position vector of P. [3]

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(c) Calculate angle OPM, giving your answer in degrees. [3]

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9 The lines l and m have vector equations

$$\mathbf{r} = -\mathbf{i} + 3\mathbf{j} + 4\mathbf{k} + \lambda(2\mathbf{i} - \mathbf{j} - \mathbf{k}) \quad \text{and} \quad \mathbf{r} = 5\mathbf{i} + 4\mathbf{j} + 3\mathbf{k} + \mu(a\mathbf{i} + b\mathbf{j} + \mathbf{k})$$

respectively, where a and b are constants.

(a) Given that l and m intersect, show that $2b - a = 4$. [4]

(b) Given also that l and m are perpendicular, find the values of a and b. [4]

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(c) When a and b have these values, find the position vector of the point of intersection of l and m. [2]

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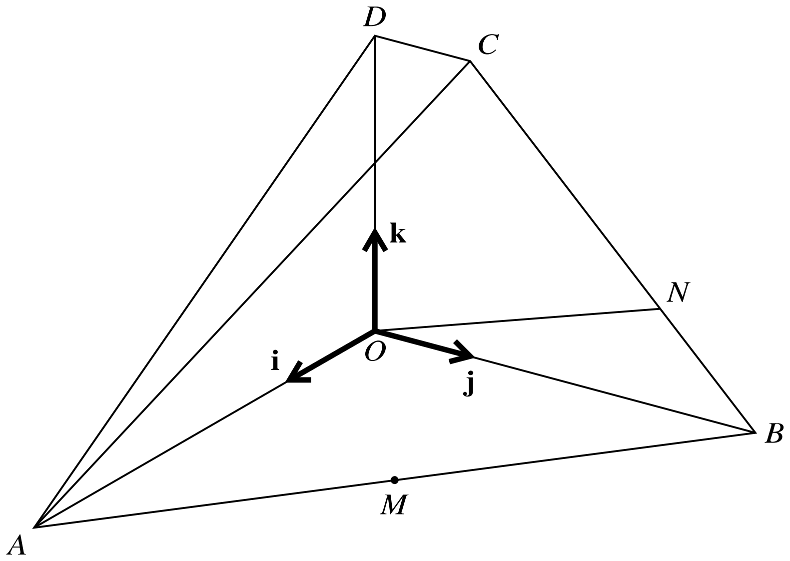

In the diagram, $OABCD$ is a solid figure in which $OA = OB = 4$ units and $OD = 3$ units. The edge OD is vertical, DC is parallel to OB and $DC = 1$ unit. The base, OAB, is horizontal and angle $AOB = 90°$. Unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to OA, OB and OD respectively. The midpoint of AB is M and the point N on BC is such that $CN = 2NB$.

(a) Express vectors $\overrightarrow{MD}$ and $\overrightarrow{ON}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. [4]

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(b) Calculate the angle in degrees between the directions of $\overrightarrow{MD}$ and $\overrightarrow{ON}$. [3]

(c) Show that the length of the perpendicular from M to ON is $\sqrt{\frac{22}{5}}$. [4]