

6 (a) Using the expansions of $\sin(3x + 2x)$ and $\sin(3x - 2x)$, show that

$$\frac{1}{2}(\sin 5x + \sin x) \equiv \sin 3x \cos 2x.$$

[3]

2 Solve the equation $\cos(\theta - 60^\circ) = 3 \sin \theta$, for $0^\circ \leq \theta \leq 360^\circ$. [5]

4 Solve the equation $2 \cos x - \cos \frac{1}{2}x = 1$ for $0 \leq x \leq 2\pi$. [5]

4 Solve the equation $\tan(x + 45^\circ) = 2 \cot x$ for $0^\circ < x < 180^\circ$. [5]

5 (a) Show that $\frac{\cos 3x}{\sin x} + \frac{\sin 3x}{\cos x} = 2 \cot 2x$. [4]

(b) Hence solve the equation $\frac{\cos 3x}{\sin x} + \frac{\sin 3x}{\cos x} = 4$, for $0 < x < \pi$. [3]

5 Solve the equation $\sin \theta = 3 \cos 2\theta + 2$, for $0^\circ \leq \theta \leq 360^\circ$. [5]

3 Express the equation $\tan(\theta + 60^\circ) = 2 + \tan(60^\circ - \theta)$ as a quadratic equation in $\tan \theta$, and hence solve the equation for $0^\circ \leq \theta \leq 180^\circ$. [6]

3 By first expressing the equation $\tan(x + 45^\circ) = 2 \cot x + 1$ as a quadratic equation in $\tan x$, solve the equation for $0^\circ < x < 180^\circ$. [6]

5 The angles α and β lie between 0° and 180° and are such that

$$\tan(\alpha + \beta) = 2 \quad \text{and} \quad \tan \alpha = 3 \tan \beta.$$

Find the possible values of α and β .

[6]

5 (a) Given that

$$\sin(x + \frac{1}{6}\pi) - \sin(x - \frac{1}{6}\pi) = \cos(x + \frac{1}{3}\pi) - \cos(x - \frac{1}{3}\pi),$$

find the exact value of $\tan x$.

[4]

(b) Hence find the exact roots of the equation

$$\sin(x + \frac{1}{6}\pi) - \sin(x - \frac{1}{6}\pi) = \cos(x + \frac{1}{3}\pi) - \cos(x - \frac{1}{3}\pi)$$

for $0 \leq x \leq 2\pi$.

[2]

4 (a) Show that the equation $\sin 2\theta + \cos 2\theta = 2 \sin^2 \theta$ can be expressed in the form

$$\cos^2 \theta + 2 \sin \theta \cos \theta - 3 \sin^2 \theta = 0.$$

[2]

(b) Hence solve the equation $\sin 2\theta + \cos 2\theta = 2 \sin^2 \theta$ for $0^\circ < \theta < 180^\circ$.

[4]

5 (a) By first expanding $\tan(2\theta + 2\theta)$, show that the equation $\tan 4\theta = \frac{1}{2} \tan \theta$ may be expressed as $\tan^4 \theta + 2 \tan^2 \theta - 7 = 0$. [4]

(b) Hence solve the equation $\tan 4\theta = \frac{1}{2} \tan \theta$, for $0^\circ < \theta < 180^\circ$. [3]