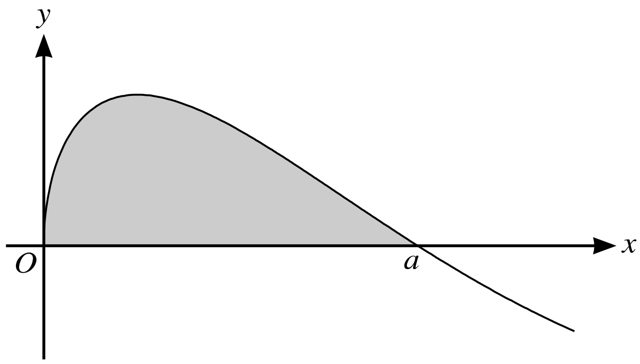
8



The diagram shows part of the curve $y = \sin\sqrt{x}$. This part of the curve intersects the x-axis at the point where $x = a$.

(a) State the exact value of a. [1]

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(b) Using the substitution $u = \sqrt{x}$, find the exact area of the shaded region in the first quadrant bounded by this part of the curve and the x-axis. [7]

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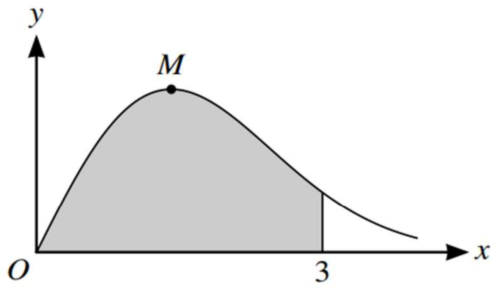


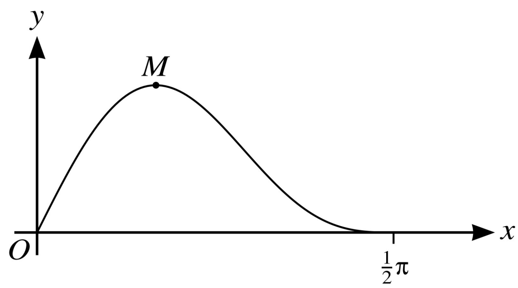
The diagram shows the curve $y = xe^{-\frac{1}{4}x^2}$, for $x \geqslant 0$, and its maximum point M.

(b) Using the substitution $x = \sqrt{u}$, or otherwise, find by integration the exact area of the shaded region bounded by the curve, the x-axis and the line $x = 3$. [5]

9 Let $f(x) = \dfrac{1}{(9-x)\sqrt{x}}$.

(b) Using the substitution $u = \sqrt{x}$, show that $\int_0^4 f(x)\,dx = \frac{1}{3}\ln 5$. [6]

10



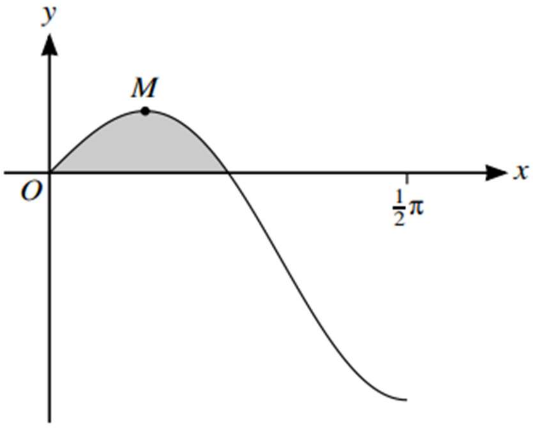
The diagram shows the curve $y = \sin 2x \cos^2 x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$, and its maximum point M.

(a) Using the substitution $u = \sin x$, find the exact area of the region bounded by the curve and the x-axis. [5]

7 **(a)** Use the substitution $u = \cos x$ to show that

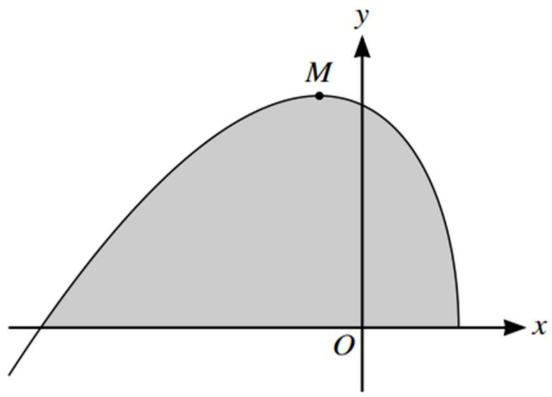
$$\int_0^{\pi} \sin 2x\, e^{2\cos x}\, dx = \int_{-1}^{1} 2u e^{2u}\, du.$$ [4]

(b) Hence find the exact value of $\int_0^{\pi} \sin 2x \, e^{2\cos x} \, dx$. [4]



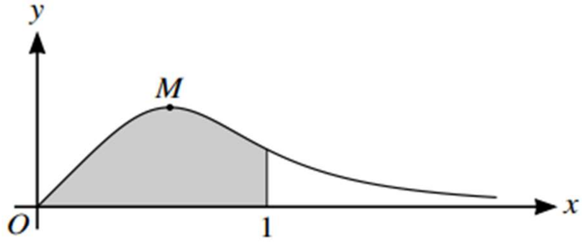
The diagram shows the curve $y = \sin x \cos 2x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$, and its maximum point M.

(b) Using the substitution $u = \cos x$, find the area of the shaded region enclosed by the curve and the x-axis in the first quadrant, giving your answer in a simplified exact form. [5]



The diagram shows the curve $y = (x + 5)\sqrt{3 - 2x}$ and its maximum point M.

(b) Using the substitution $u = 3 - 2x$, find by integration the area of the shaded region bounded by the curve and the x-axis. Give your answer in the form $a\sqrt{13}$, where a is a rational number. [5]



The diagram shows the curve $y = \frac{x}{1 + 3x^4}$, for $x \geqslant 0$, and its maximum point M.

(b) Using the substitution $u = \sqrt{3}x^2$, find by integration the exact area of the shaded region bounded by the curve, the x-axis and the line $x = 1$. [5]

6 Let $I = \int_0^3 \frac{27}{(9+x^2)^2}\,dx$.

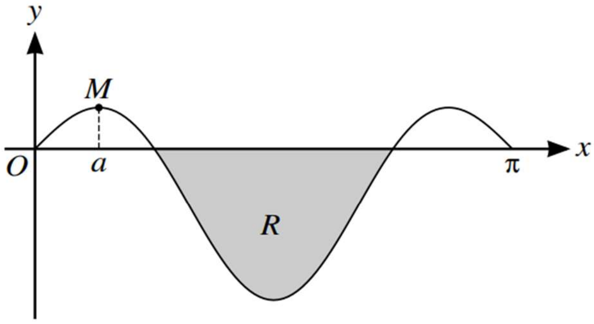
(a) Using the substitution $x = 3\tan\theta$, show that $I = \int_0^{\frac{1}{4}\pi} \cos^2\theta\,d\theta$. [4]

(b) Hence find the exact value of I. [4]

4 Using the substitution $u = \sqrt{x}$, find the exact value of

$$\int_3^{\infty} \frac{1}{(x+1)\sqrt{x}}\,dx.$$

[6]



The diagram shows the curve $y = \sin x \cos 2x$, for $0 \leqslant x \leqslant \pi$, and a maximum point M, where $x = a$. The shaded region between the curve and the x-axis is denoted by R.

(b) Find the exact area of the region R, giving your answer in simplified form. [4]