7 The variables x and y satisfy the differential equation

$$e^{2x}\frac{dy}{dx} = 4xy^2,$$

and it is given that $y = 1$ when $x = 0$.

Solve the differential equation, obtaining an expression for y in terms of x. [7]

7 The variables x and θ satisfy the differential equation

$$\frac{x}{\tan\theta}\frac{dx}{d\theta} = x^2 + 3.$$

It is given that $x = 1$ when $\theta = 0$.

Solve the differential equation, obtaining an expression for x^2 in terms of θ. [7]

7 The variables x and y satisfy the differential equation

$$\cos 2x \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4\tan 2x}{\sin^2 3y},$$

where $0 \leqslant x < \frac{1}{4}\pi$. It is given that $y = 0$ when $x = \frac{1}{6}\pi$.

Solve the differential equation to obtain the value of x when $y = \frac{1}{6}\pi$. Give your answer correct to 3 decimal places. [8]

6 The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = xe^{y-x},$$

and $y = 0$ when $x = 0$.

(a) Solve the differential equation, obtaining an expression for y in terms of x. [7]

(b) Find the value of y when $x = 1$, giving your answer in the form $a - \ln b$, where a and b are integers. [1]

7 A curve is such that the gradient at a general point with coordinates (x, y) is proportional to $\frac{y}{\sqrt{x+1}}$. The curve passes through the points with coordinates $(0, 1)$ and $(3, \mathrm{e})$.

By setting up and solving a differential equation, find the equation of the curve, expressing y in terms of x. [7]

6 The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = \frac{1 + 4y^2}{e^x}.$$

It is given that $y = 0$ when $x = 1$.

(a) Solve the differential equation, obtaining an expression for y in terms of x. [7]

(b) State what happens to the value of y as x tends to infinity. [1]

8 The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = \frac{y^2 + 4}{x(y + 4)}$$

for $x > 0$. It is given that $x = 4$ when $y = 2\sqrt{3}$.

Solve the differential equation to obtain the value of x when $y = 2$. [8]

9 The variables x and y satisfy the differential equation

$$(x+1)(3x+1)\frac{dy}{dx} = y,$$

and it is given that $y = 1$ when $x = 1$.

Solve the differential equation and find the exact value of y when $x = 3$, giving your answer in a simplified form. [9]

7 The variables x and t satisfy the differential equation

$$e^{3t}\frac{dx}{dt} = \cos^2 2x,$$

for $t \geqslant 0$. It is given that $x = 0$ when $t = 0$.

(a) Solve the differential equation and obtain an expression for x in terms of t. [7]

(b) State what happens to the value of x when t tends to infinity. [1]

10 The variables x and t satisfy the differential equation $\frac{dx}{dt} = x^2(1 + 2x)$, and $x = 1$ when $t = 0$.

Using partial fractions, solve the differential equation, obtaining an expression for t in terms of x. [11]

8 At time t days after the start of observations, the number of insects in a population is N. The variation in the number of insects is modelled by a differential equation of the form $\frac{dN}{dt} = kN^{\frac{3}{2}} \cos 0.02t$, where k is a constant and N is a continuous variable. It is given that when $t = 0$, $N = 100$.

(a) Solve the differential equation, obtaining a relation between N, k and t. [5]

(b) Given also that $N = 625$ when $t = 50$, find the value of k. [2]

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(c) Obtain an expression for N in terms of t, and find the greatest value of N predicted by this model. [2]

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10 A gardener is filling an ornamental pool with water, using a hose that delivers 30 litres of water per minute. Initially the pool is empty. At time t minutes after filling begins the volume of water in the pool is V litres. The pool has a small leak and loses water at a rate of $0.01V$ litres per minute.

The differential equation satisfied by V and t is of the form $\frac{dV}{dt} = a - bV$.

(a) Write down the values of the constants a and b. [1]

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(b) Solve the differential equation and find the value of t when $V = 1000$. [6]

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(c) Obtain an expression for V in terms of t and hence state what happens to V as t becomes large. [2]

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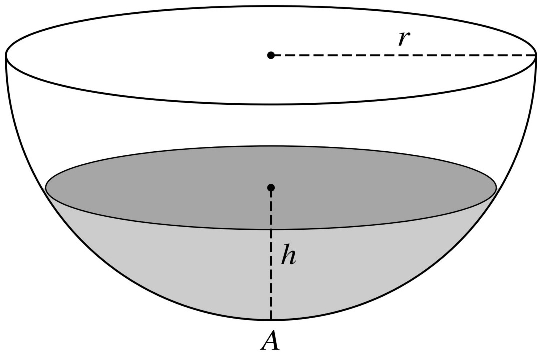

A tank containing water is in the form of a hemisphere. The axis is vertical, the lowest point is A and the radius is r, as shown in the diagram. The depth of water at time t is h. At time $t = 0$ the tank is full and the depth of the water is r. At this instant a tap at A is opened and water begins to flow out at a rate proportional to $\sqrt{h}$. The tank becomes empty at time $t = 14$.

The volume of water in the tank is V when the depth is h. It is given that $V = \frac{1}{3}\pi(3rh^2 - h^3)$.

(a) Show that h and t satisfy a differential equation of the form

$$\frac{dh}{dt} = -\frac{B}{2rh^{\frac{1}{2}} - h^{\frac{3}{2}}},$$

where B is a positive constant. [4]

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(b) Solve the differential equation and obtain an expression for t in terms of h and r. [8]