2 (a) Express $5\sin x - 3\cos x$ in the form $R\sin(x-\alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. Give the exact value of R and give α correct to 2 decimal places. [3]

$5\sin x - 3\cos x = R\sin x\cos\alpha - R\cos x\sin\alpha$

$R\cos\alpha = 5$ ① $\quad R\sin\alpha = 3$ ②

②÷①: $\tan\alpha = \frac{3}{5}$

$\alpha = 0.54$
STO

①^2 + ②^2: $R^2 = 5^2 + 3^2$

$= 25 + 9$

$R = \sqrt{34}$

$\sqrt{34}\sin(x - 0.54)$

(b) Hence state the greatest and least possible values of $(5\sin x - 3\cos x)^2$. [2]

from (a): $5\sin x - 3\cos x = \sqrt{34}\sin(x-0.54)$

$\rightarrow (\sqrt{34}\sin(x-0.54))^2$

Max. of $\sqrt{34}\sin(x-0.54)$ is $\sqrt{34}$, minimum is $-\sqrt{34}$

greatest possible value $= (\sqrt{34})^2 = 34$

least possible value $= (0)^2 = 0$

4 **(a)** Express $4\cos x - \sin x$ in the form $R\cos(x + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. State the exact value of R and give α correct to 2 decimal places. [3]

$4\cos x - \sin x = R\cos x\cos\alpha - R\sin x\sin\alpha$

$R\cos\alpha = 4$ ① $R\sin\alpha = 1$ ②

② ÷ ①: $\tan\alpha = \frac{1}{4}$

$\alpha = 14.04°$ STO

①^2 + ②^2: $R^2 = 4^2 + 1^2$

$= 16 + 1$

$R = \sqrt{17}$

$\sqrt{17}\cos(x + 14.04°)$

(b) Hence solve the equation $4\cos 2x - \sin 2x = 3$ for $0° < x < 180°$. [5]

from (a): $4\cos x - \sin x = \sqrt{17}\cos(x + 14.04)$

so: $4\cos 2x - \sin 2x = \sqrt{17}\cos(2x + 14.04)$

$\sqrt{17}\cos(2x + 14.04) = 3$

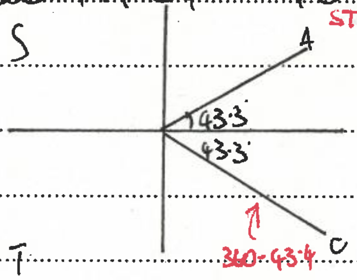
$\cos(2x + 14.04) = \frac{3}{\sqrt{17}}$

$2x + 14.04 = 43.31°$ STO

$0 < x < 180$

$0 < 2x < 360$

$14 < 2x + 14.04 < 374$



$2x + 14.04 = 43.31°, 316.68°$

$2x = 29.27.., 302.64..$

$x = 14.6°, 151.3°$

5 (a) Express $\sqrt{7}\sin x + 2\cos x$ in the form $R\sin(x+\alpha)$, where $R > 0$ and $0° < \alpha < 90°$. State the exact value of R and give α correct to 2 decimal places. [3]

$\sqrt{7}\sin x + 2\cos x = R\sin x\cos\alpha + R\cos x\sin\alpha$

$R\cos\alpha = \sqrt{7}$ ① $\quad R\sin\alpha = 2$ ②

② ÷ ①: $\tan\alpha = \frac{2}{\sqrt{7}}$

$\alpha = 37.09°$

STO

①^2 + ②^2: $R^2 = (\sqrt{7})^2 + 2^2$

$= 7 + 4$

$R = \sqrt{11}$

$\sqrt{11}\sin(x + 37.09°)$

(b) Hence solve the equation $\sqrt{7}\sin 2\theta + 2\cos 2\theta = 1$, for $0° < \theta < 180°$. [5]

from (a) : $\sqrt{7}\sin x + 2\cos x = \sqrt{11}\sin(x + 37.09°)$

so : $\sqrt{7}\sin 2\theta + 2\cos 2\theta = \sqrt{11}\sin(2\theta + 37.09°)$

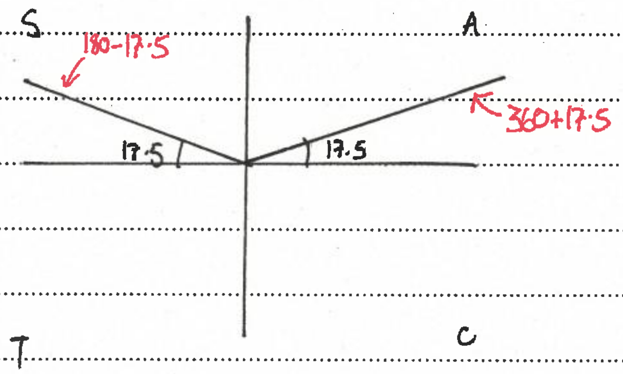
$\sqrt{11}\sin(2\theta + 37.09) = 1$

$0 < \theta < 180$
$0 < 2\theta < 360$
$37.09 < 2\theta + 37.09 < 397.09°$

$\sin(2\theta + 37.09) = \frac{1}{\sqrt{11}}$

$2\theta + 37.09 = 17.54...$

STO



$2\theta + 37.09 = 17.54...°, 162.45...°, 377.54...°$

$2\theta = -19.53.., 125.36...°, 340.46...°$

$\theta = -9.8°, 62.7°, 170.2°$

$\theta = 62.7°, 170.2°$

5 (a) Express $\sqrt{2}\cos x - \sqrt{5}\sin x$ in the form $R\cos(x + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give the exact value of R and the value of α correct to 3 decimal places. [3]

$\sqrt{2}\cos x - \sqrt{5}\sin x = R\cos x\cos\alpha - R\sin x\sin\alpha$

$R\cos\alpha = \sqrt{2}$ ① $\quad R\sin\alpha = \sqrt{5}$ ②

② ÷ ①: $\tan\alpha = \dfrac{\sqrt{5}}{\sqrt{2}}$

$\alpha = 57.688°$

STO

①^2 + ②^2: $R^2 = (\sqrt{2})^2 + (\sqrt{5})^2$

$= 2 + 5$

$R = \sqrt{7}$

$\underline{\sqrt{7}\cos(x + 57.688°)}$

(b) Hence solve the equation $\sqrt{2}\cos 2\theta - \sqrt{5}\sin 2\theta = 1$, for $0° < \theta < 180°$. [4]

from part (a): $\sqrt{2}\cos x - \sqrt{5}\sin x = \sqrt{7}\cos(x + 57.688)$

so: $\sqrt{2}\cos 2\theta - \sqrt{5}\sin 2\theta = \sqrt{7}\cos(2\theta + 57.688)$

$\sqrt{7}\cos(2\theta + 57.688) = 1$

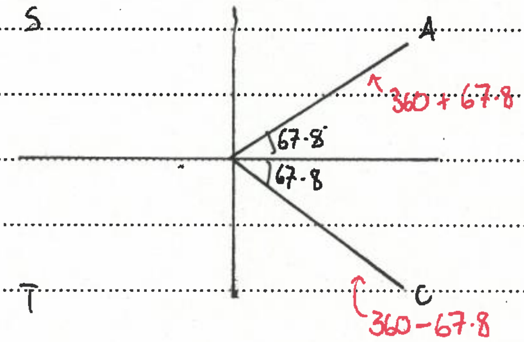
$\cos(2\theta + 57.688) = \frac{1}{\sqrt{7}}$

$0 < \theta < 180$

$0 < 2\theta < 360$

$57.7 < 2\theta + 57.688 < 417.7$

$2\theta + 57.688 = 67.79...$ STO



$2\theta + 57.688 = 67.79...,\ 292.20...,\ \cancel{427.79...}$

$2\theta = 10.10...,\ 234.51...$

$\theta = 5.05...,\ 117.25...$

$\theta = 5.1°,\ 117.3°$

6 **(a)** Express $5\sin\theta + 12\cos\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. [3]

$5\sin\theta + 12\cos\theta = R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$

$12\cos\theta + 5\sin\theta = R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$

$R\cos\alpha = 12$ ① $\qquad R\sin\alpha = 5$ ②

② ÷ ①: $\dfrac{R\sin\alpha}{R\cos\alpha} = \dfrac{5}{12}$

$\tan\alpha = \dfrac{5}{12}$

$\underline{\alpha = 0.395}$ STO

①^2 + ②^2: $R^2 = 12^2 + 5^2$

$R^2 = 144 + 25$

$R^2 = 169$

$\underline{R = 13}$

→ $\underline{13\cos(\theta - 0.395)}$

(b) Hence solve the equation $5 \sin 2x + 12 \cos 2x = 6$ for $0 \leqslant x \leqslant \pi$. **[4]**

from (a): $5\sin\theta + 12\cos\theta = 13\cos(\theta - 0.395)$

replace θ with $2x$:

$5\sin 2x + 12\cos 2x = 13\cos(2x - 0.395)$

$\rightarrow 13\cos(2x - 0.395) = 6$

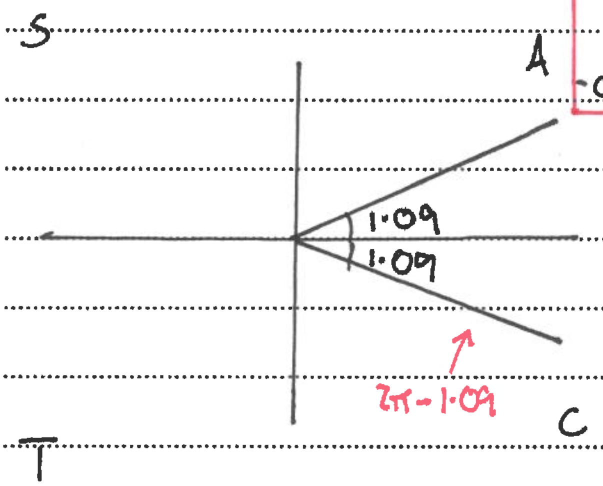
$\cos(2x - 0.395) = \frac{6}{13}$

$2x - 0.395 = 1.09$ STO

$0 \leqslant x \leqslant \pi$

$0 \leqslant 2x \leqslant 2\pi$

$-0.395 \leqslant 2x - 0.395 \leqslant 2\pi - 0.395$



$2x - 0.395 = 1.09, 5.19$

$2x = 1.486, 5.587$

$x = 0.743, 2.79$

6 **(a)** By first expanding $\cos(x - 60°)$, show that the expression

$$2\cos(x - 60°) + \cos x$$

can be written in the form $R\cos(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give the exact value of R and the value of α correct to 2 decimal places. [5]

$\cos(x-60) = \cos x\cos 60 + \sin x \sin 60$

$= \frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x$

$2\cos(x-60) + \cos x = 2\left[\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x\right] + \cos x$

$= \cos x + \sqrt{3}\sin x + \cos x$

$= 2\cos x + \sqrt{3}\sin x$

$2\cos x + \sqrt{3}\sin x = R\cos x\cos\alpha + R\sin x\sin\alpha$

$R\cos\alpha = 2$ ① $\quad R\sin\alpha = \sqrt{3}$ ②

② ÷ ①: $\tan\alpha = \frac{\sqrt{3}}{2}$

$\alpha = 40.89°$ STO

$①^2 + ②^2$: $R^2 = 2^2 + (\sqrt{3})^2$

$= 4 + 3$

$R = \sqrt{7}$ $\rightarrow$ $\sqrt{7}\cos(x - 40.89)$

(b) Hence find the value of x in the interval $0° < x < 360°$ for which $2\cos(x - 60°) + \cos x$ takes its least possible value. [2]

least value of $\sqrt{7}\cos(x-40.89)$ is $-\sqrt{7}$:

$\sqrt{7}\cos(x-40.89) = -\sqrt{7}$

$\cos(x-40.89) = -1$

$x - 40.89 = 180$

$x = 220.9°$

6 (a) Express $3\cos x + 2\cos(x - 60°)$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. State the exact value of R and give α correct to 2 decimal places. [4]

$$2\cos(x-60) = 2[\cos x\cos 60 + \sin x\sin 60]$$
$$= 2[\tfrac{1}{2}\cos x + \tfrac{\sqrt{3}}{2}\sin x]$$
$$= \cos x + \sqrt{3}\sin x$$

$$\rightarrow 3\cos x + 2\cos(x-60) = 3\cos x + \cos x + \sqrt{3}\sin x$$
$$= 4\cos x + \sqrt{3}\sin x$$

$$\rightarrow 4\cos x + \sqrt{3}\sin x = R\cos x\cos\alpha + R\sin x\sin\alpha$$

$$R\cos\alpha = 4 \quad (1) \qquad R\sin\alpha = \sqrt{3} \quad (2)$$

$$(2) \div (1): \frac{R\sin\alpha}{R\cos\alpha} = \frac{\sqrt{3}}{4}$$
$$\tan\alpha = \frac{\sqrt{3}}{4}$$
$$\underline{\alpha = 23.41°}$$

$$(1)^2 + (2)^2: R^2 = 4^2 + (\sqrt{3})^2$$
$$= 16 + 3$$
$$= 19$$
$$R = \sqrt{19}$$

$$\rightarrow \underline{\sqrt{19}\cos(x - 23.41)}$$

(b) Hence solve the equation

$$3\cos 2\theta + 2\cos(2\theta - 60°) = 2.5$$

for $0° < \theta < 180°$. [4]

replace x with 2θ in part (a):

$$\sqrt{19}\cos(2\theta - 23.41) = 2.5$$

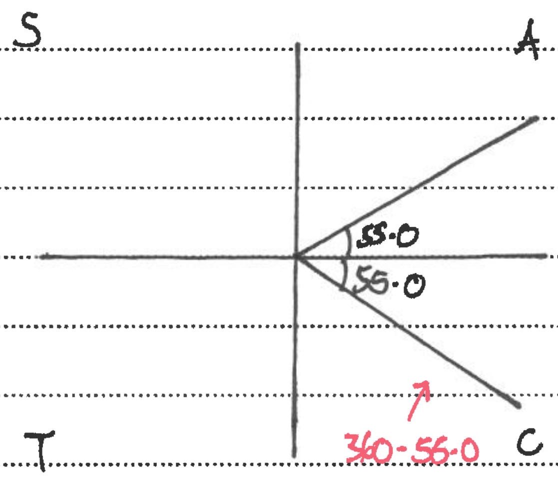
$$\cos(2\theta - 23.41) = \frac{2.5}{\sqrt{19}}$$

$$2\theta - 23.41 = 55.0 \quad \text{STO}$$

$$0 < \theta < 180$$

$$0 < 2\theta < 360$$

$$-23.41 < 2\theta - 23.41 < 336.59$$



$$2\theta - 23.4 = 55.0, \; 305.0$$

$$2\theta = 78.4, \; 328.4$$

$$\theta = 39.2°, \; 164.2°$$

6 **(a)** Express $\sqrt{6}\cos\theta + 3\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. State the exact value of R and give α correct to 2 decimal places. [3]

$\sqrt{6}\cos\theta + 3\sin\theta = R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$

$R\cos\alpha = \sqrt{6}$ ① $\quad R\sin\alpha = 3$ ②

② ÷ ①: $\tan\alpha = \frac{3}{\sqrt{6}}$

$\alpha = 50.77°$

STO

①² + ②²: $R^2 = (\sqrt{6})^2 + 3^2$

$= 6 + 9$

$R = \sqrt{15}$

$\sqrt{15}\cos(\theta - 50.77°)$

(b) Hence solve the equation $\sqrt{6}\cos\frac{1}{3}x + 3\sin\frac{1}{3}x = 2.5$, for $0° < x < 360°$. [4]

from (a) : $\sqrt{6}\cos\theta + 3\sin\theta = \sqrt{15}\cos(\theta - 50.77°)$

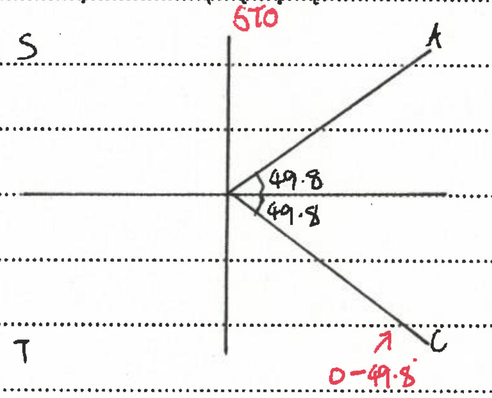
so : $\sqrt{6}\cos\left(\frac{x}{3}\right) + 3\sin\left(\frac{x}{3}\right) = \sqrt{15}\cos\left(\frac{x}{3} - 50.77°\right)$

$\sqrt{15}\cos\left(\frac{x}{3} - 50.77\right) = 2.5$

$$\boxed{\begin{aligned} 0 &< x < 360 \\ 0 &< \tfrac{x}{3} < 120 \\ -50.8 &< \tfrac{x}{3} - 50.77 < 69.2 \end{aligned}}$$

$\cos\left(\frac{x}{3} - 50.77\right) = \frac{2.5}{\sqrt{15}}$

$\frac{x}{3} - 50.77 = 49.79\ldots$



$\frac{x}{3} - 50.77 = 49.79\ldots, -49.79\ldots$

$\frac{x}{3} = 100.56\ldots, 0.97\ldots$

$x = 301.69\ldots, 2.91\ldots$

$x = 2.9°, 301.7°$

7 (a) Show that the equation $\sqrt{5}\sec x + \tan x = 4$ can be expressed as $R\cos(x+\alpha) = \sqrt{5}$, where $R > 0$ and $0° < \alpha < 90°$. Give the exact value of R and the value of α correct to 2 decimal places. [4]

$\sqrt{5}\sec x + \tan x = 4$

$\frac{\sqrt{5}}{\cos x} + \frac{\sin x}{\cos x} = 4$

$\sqrt{5} + \sin x = 4\cos x$

$4\cos x - \sin x = \sqrt{5}$

$4\cos x - \sin x = R\cos x\cos\alpha + R\sin x\sin\alpha$

$R\cos\alpha = 4$ ① $\quad R\sin\alpha = 1$ ②

② ÷ ①: $\tan\alpha = \frac{1}{4}$

$\alpha = 14.04°$

STO

①^2 + ②^2: $R^2 = 4^2 + 1^2$

$= 16 + 1$

$R = \sqrt{17}$

$\sqrt{17}\cos(x + 14.04°) = \sqrt{5}$

(b) Hence solve the equation $\sqrt{5}\sec 2x + \tan 2x = 4$, for $0° < x < 180°$. [4]

from (a): $\sqrt{5}\sec x + \tan x = 4 \Leftrightarrow \sqrt{17}\cos(x + 14.04) = \sqrt{5}$

so: $\sqrt{5}\sec 2x + \tan 2x = 4 \Leftrightarrow \sqrt{17}\cos(2x + 14.04) = \sqrt{5}$

$$\sqrt{17}\cos(2x + 14.04) = \sqrt{5}$$

$$\cos(2x + 14.04) = \frac{\sqrt{5}}{\sqrt{17}}$$

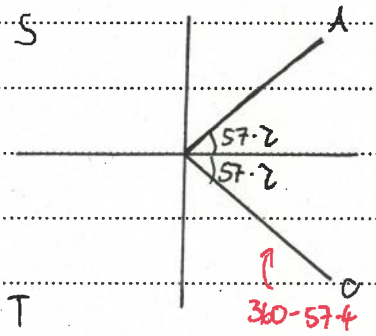
$0 < x < 180$

$0 < 2x < 360$

$14 < 2x + 14.04 < 374$

$2x + 14.04 = 57.15...°$

STO



$$2x + 14.04 = 57.15...,\ 302.84...$$

$$2x = 43.12...,\ 288.80...$$

$$x = 21.6°,\ 144.4°$$