9 Let $f(x) = \dfrac{2 + 11x - 10x^2}{(1 + 2x)(1 - 2x)(2 + x)}$.

(a) Express $f(x)$ in partial fractions. [5]

$$\frac{2 + 11x - 10x^2}{(1+2x)(1-2x)(2+x)} \equiv \frac{A}{1+2x} + \frac{B}{1-2x} + \frac{C}{2+x}$$

$$2 + 11x - 10x^2 \equiv A(1-2x)(2+x) + B(1+2x)(2+x) + C(1+2x)(1-2x)$$

$\underline{x = -2:}$

$$2 + 11(-2) - 10(-2)^2 = 0 + 0 + C(-3)(5)$$

$$2 - 22 - 40 = -15C$$

$$-60 = -15C$$

$$\underline{C = 4}$$

$\underline{x = \tfrac{1}{2}:}$

$$2 + 11\left(\tfrac{1}{2}\right) - 10\left(\tfrac{1}{2}\right)^2 = 0 + B(2)\left(\tfrac{5}{2}\right) + 0$$

$$2 + \frac{11}{2} - \frac{5}{2} = 5B$$

$$5 = 5B$$

$$\underline{B = 1}$$

$\underline{x = -\tfrac{1}{2}:}$

$$2 + 11\left(\tfrac{-1}{2}\right) - 10\left(\tfrac{-1}{2}\right)^2 = A(2)\left(\tfrac{3}{2}\right) + 0 + 0$$

$$2 - \frac{11}{2} - \frac{5}{2} = 3A$$

$$-6 = 3A$$

$$\underline{A = -2}$$

$$f(x) = \underline{\frac{-2}{1+2x} + \frac{1}{1-2x} + \frac{4}{2+x}}$$

(b) Hence obtain the expansion of $\mathrm{f}(x)$ in ascending powers of x, up to and including the term in x^2. [5]

$$f(x) = \overset{A}{-2(1+2x)^{-1}} + \overset{B}{(1-2x)^{-1}} + \overset{C}{4(2+x)^{-1}}$$

A $$-2(1+2x)^{-1} = -2\left[1 + (-1)(2x) + \frac{(-1)(-2)}{2!}(2x)^2 + \dots\right]$$
$$= -2[1 - 2x + 4x^2]$$
$$= -2 + 4x - 8x^2$$

B $$(1-2x)^{-1} = 1 + (-1)(-2x) + \frac{(-1)(-2)}{2!}(-2x)^2 + \dots$$
$$= 1 + 2x + 4x^2$$

C $$4(2+x)^{-1} = 4 \times 2^{-1}\left(1+\frac{x}{2}\right)^{-1}$$
$$= 4 \times \frac{1}{2}\left(1+\frac{x}{2}\right)^{-1}$$
$$= 2\left(1+\frac{x}{2}\right)^{-1}$$
$$2\left(1+\frac{x}{2}\right)^{-1} = 2\left[1 + (-1)\left(\frac{x}{2}\right) + \frac{(-1)(-2)}{2!}\left(\frac{x}{2}\right)^2 + \dots\right]$$
$$= 2\left[1 - \frac{x}{2} + \frac{x^2}{4}\right]$$
$$= 2 - x + \frac{x^2}{2}$$

$$f(x) = \overbrace{-2 + 4x - 8x^2}^{A} + \overbrace{1 + 2x + 4x^2}^{B} + \overbrace{2 - x + \frac{x^2}{2}}^{C}$$

$$= 1 + 5x - \frac{7}{2}x^2$$

10 Let $f(x) = \dfrac{21 - 8x - 2x^2}{(1 + 2x)(3 - x)^2}$.

(a) Express $f(x)$ in partial fractions. [5]

$$\frac{21 - 8x - 2x^2}{(1+2x)(3-x)^2} \equiv \frac{A}{1+2x} + \frac{B}{3-x} + \frac{C}{(3-x)^2}$$

$$21 - 8x - 2x^2 \equiv A(3-x)^2 + B(1+2x)(3-x) + C(1+2x)$$

$x=3$:

$21 - 8(3) - 2(3)^2 = 0 + 0 + C(7)$

$21 - 24 - 18 = 7C$

$-21 = 7C$

$C = -3$

$x = \frac{-1}{2}$:

$21 - 8(\frac{-1}{2}) - 2(\frac{-1}{2})^2 = A(3 - -\frac{1}{2})^2 + 0 + 0$

$21 + 4 - \frac{1}{2} = (\frac{7}{2})^2 A$

$\frac{49}{2} = \frac{49}{4}A$

$A = 2$

$x = 0$:

$21 - 0 - 0 = A(3)^2 + B(1)(3) + C(1)$

$21 = 9A + 3B + C$

$21 = 9(2) + 3B + -3$

$21 = 15 + 3B$

$3B = 6$

$B = 2 \rightarrow \dfrac{2}{1+2x} + \dfrac{2}{3-x} - \dfrac{3}{(3-x)^2}$

(b) Hence obtain the expansion of f(x) in ascending powers of x, up to and including the term in x^2. [5]

Ⓐ $2(1+2x)^{-1} = 2\left[1 + (-1)(2x) + \frac{(-1)(-2)}{2!}(2x)^2 + \ldots\right]$

$= 2\left[1 - 2x + 4x^2 + \ldots\right]$

$= 2 - 4x + 8x^2$

Ⓑ $2(3-x)^{-1} = 2\times 3^{-1}\left(1-\frac{x}{3}\right)^{-1}$

$= \frac{2}{3}\left(1-\frac{x}{3}\right)^{-1}$

$= \frac{2}{3}\left[1 + (-1)\left(\frac{-x}{3}\right) + \frac{(-1)(-2)}{2!}\left(\frac{-x}{3}\right)^2 + \ldots\right]$

$= \frac{2}{3}\left[1 + \frac{x}{3} + \frac{x^2}{9} + \ldots\right]$

$= \frac{2}{3} + \frac{2}{9}x + \frac{2}{27}x^2$

Ⓒ $-3(3-x)^{-2} = -3\times 3^{-2}\left(1-\frac{x}{3}\right)^{-2}$

$= \frac{-1}{3}\left(1-\frac{x}{3}\right)^{-2}$

$= \frac{-1}{3}\left[1 + (-2)\left(\frac{-x}{3}\right) + \frac{(-2)(-3)}{2!}\left(\frac{-x}{3}\right)^2 + \ldots\right]$

$= \frac{-1}{3}\left[1 + \frac{2}{3}x + 3\times\frac{x^2}{9}\right]$

$= \frac{-1}{3}\left[1 + \frac{2}{3}x + \frac{1}{3}x^2\right]$

$= \frac{-1}{3} - \frac{2}{9}x - \frac{1}{9}x^2$

Ⓐ + Ⓑ + Ⓒ:

$2 - 4x + 8x^2 + \frac{2}{3} + \frac{2}{9}x + \frac{2}{27}x^2 - \frac{1}{3} - \frac{2}{9}x - \frac{1}{9}x^2$

$= \frac{7}{3} - 4x + \frac{215}{27}x^2$

10 Let $f(x) = \dfrac{24x + 13}{(1 - 2x)(2 + x)^2}$.

(a) Express $f(x)$ in partial fractions. [5]

$$\frac{24x+13}{(1-2x)(2+x)^2} \equiv \frac{A}{1-2x} + \frac{B}{2+x} + \frac{C}{(2+x)^2}$$

$$24x + 13 \equiv A(2+x)^2 + B(1-2x)(2+x) + C(1-2x)$$

$x = -2$:

$$24(-2) + 13 = 0 + 0 + C(5)$$

$$-35 = 5C$$

$$C = -7$$

$x = \frac{1}{2}$:

$$24\left(\tfrac{1}{2}\right) + 13 = A\left(\tfrac{5}{2}\right)^2 + 0 + 0$$

$$12 + 13 = \tfrac{25}{4}A$$

$$25 = \tfrac{25}{4}A$$

$$A = 4$$

$x = 0$:

$$24(0) + 13 = A(2)^2 + B(1)(2) + C(1)$$

$$13 = 4A + 2B + C$$

$$13 = 4(4) + 2B - 7$$

$$13 = 16 + 2B - 7$$

$$13 = 9 + 2B$$

$$2B = 4$$

$$B = 2$$

$$\rightarrow \frac{4}{1-2x} + \frac{2}{2+x} - \frac{7}{(2+x)^2}$$

(b) Hence obtain the expansion of f(x) in ascending powers of x, up to and including the term in x^2. [5]

A $4(1-2x)^{-1} = 4\left[1 + (-1)(-2x) + \frac{(-1)(-2)}{2!}(-2x)^2 + \dots\right]$

$= 4\left[1 + 2x + 4x^2 + \dots\right]$

$= 4 + 8x + 16x^2$

B $2(2+x)^{-1} = 2 \times 2^{-1}\left(1+\frac{x}{2}\right)^{-1}$

$= \left(1+\frac{x}{2}\right)^{-1}$

$= 1 + (-1)\left(\frac{x}{2}\right) + \frac{(-1)(-2)}{2!}\left(\frac{x}{2}\right)^2$

$= 1 - \frac{x}{2} + \frac{x^2}{4}$

C $-7(2+x)^{-2} = -7 \times 2^{-2}\left(1+\frac{x}{2}\right)^{-2}$

$= \frac{-7}{4}\left[1 + (-2)\left(\frac{x}{2}\right) + \frac{(-2)(-3)}{2!}\left(\frac{x}{2}\right)^2 + \dots\right]$

$= \frac{-7}{4}\left[1 - x + \frac{3}{4}x^2 + \dots\right]$

$= \frac{-7}{4} + \frac{7}{4}x - \frac{21}{16}x^2$

A + B + C:

$4 + 8x + 16x^2 + 1 - \frac{1}{2}x + \frac{1}{4}x^2 - \frac{7}{4} + \frac{7}{4}x - \frac{21}{16}x^2$

$= \frac{13}{4} + \frac{37}{4}x + \frac{239}{16}x^2$

(c) State the set of values of x for which the expansion in **(b)** is valid. [1]

$|-2x| < 1$ and $\left|\frac{x}{2}\right| < 1$

$|x| < \frac{1}{2}$ $\quad$ $|x| < 2$

so $|x| < \frac{1}{2}$

9 Let $f(x) = \dfrac{8 + 5x + 12x^2}{(1-x)(2+3x)^2}$.

(a) Express $f(x)$ in partial fractions. [5]

$$\frac{8+5x+12x^2}{(1-x)(2+3x)^2} \equiv \frac{A}{1-x} + \frac{B}{(2+3x)} + \frac{C}{(2+3x)^2}$$

$$8 + 5x + 12x^2 \equiv A(2+3x)^2 + B(1-x)(2+3x) + C(1-x)$$

$x=1$:

$$8 + 5 + 12 = A(5)^2 + 0 + 0$$
$$25 = 25A$$
$$A = 1$$

$x = \frac{-2}{3}$:

$$8 + 5\left(\frac{-2}{3}\right) + 12\left(\frac{-2}{3}\right)^2 = 0 + 0 + C\left(1 - \frac{-2}{3}\right)$$
$$8 - \frac{10}{3} + 12 \times \frac{4}{9} = \frac{5}{3}C$$
$$8 - \frac{10}{3} + \frac{16}{3} = \frac{5}{3}C$$
$$10 = \frac{5}{3}C$$
$$C = 6$$

$x=0$:

A=1 C=6

$$8 + 0 + 0 = A(2)^2 + B(1)(2) + C(1)$$
$$8 = 1 \times 4 + 2B + 6 \times 1$$
$$8 = 4 + 2B + 6$$
$$8 = 10 + 2B$$
$$-2 = 2B$$
$$B = -1$$

$$\rightarrow \frac{1}{1-x} + \frac{-1}{2+3x} + \frac{6}{(2+3x)^2}$$

(b) Hence obtain the expansion of $\mathrm{f}(x)$ in ascending powers of x, up to and including the term in x^2. [5]

A B C

$$f(x) = \frac{1}{1-x} + \frac{-1}{2+3x} + \frac{6}{(2+3x)^2}$$

A: $(1-x)^{-1} = 1 + (-1)(-x) + \frac{(-1)(-2)}{2!}(-x)^2$

$= 1 + x + x^2$

B: $-1(2+3x)^{-1} = -1 \times 2^{-1}\left(1+\frac{3x}{2}\right)^{-1} = -1 \times \frac{1}{2}\left(1+\frac{3x}{2}\right)^{-1}$

$\frac{-1}{2}\left(1+\frac{3x}{2}\right)^{-1} = \frac{-1}{2}\left[1 + (-1)\left(\frac{3x}{2}\right) + \frac{(-1)(-2)}{2!}\left(\frac{3x}{2}\right)^2 + \ldots\right]$

$= \frac{-1}{2}\left[1 - \frac{3x}{2} + \frac{9x^2}{4}\right]$

$= \frac{-1}{2} + \frac{3x}{4} - \frac{9x^2}{8}$

C: $6(2+3x)^{-2} = 6 \times 2^{-2}\left(1+\frac{3x}{2}\right)^{-2} = 6 \times \frac{1}{4}\left(1+\frac{3x}{2}\right)^{-2}$

$\frac{3}{2}\left(1+\frac{3x}{2}\right)^{-2} = \frac{3}{2}\left[1 + (-2)\left(\frac{3x}{2}\right) + \frac{(-2)(-3)}{2!}\left(\frac{3x}{2}\right)^2 + \ldots\right]$

$= \frac{3}{2}\left[1 - 3x + \frac{27}{4}x^2\right]$

$= \frac{3}{2} - \frac{9}{2}x + \frac{81}{8}x^2$

A + B + C: $1 + x + x^2 - \frac{1}{2} + \frac{3x}{4} - \frac{9x^2}{8} + \frac{3}{2} - \frac{9}{2}x + \frac{81}{8}x^2$

$= 2 - \frac{11}{4}x + 10x^2$

9 Let $f(x) = \dfrac{14 - 3x + 2x^2}{(2+x)(3+x^2)}$.

(a) Express $f(x)$ in partial fractions. [5]

$$\frac{14 - 3x + 2x^2}{(2+x)(3+x^2)} \equiv \frac{A}{2+x} + \frac{Bx + C}{3 + x^2}$$

$$14 - 3x + 2x^2 \equiv A(3 + x^2) + (Bx + C)(2 + x)$$

$x = -2$:

$$14 - 3(-2) + 2(-2)^2 = A(3 + (-2)^2) + 0$$

$$14 + 6 + 8 = 7A$$

$$28 = 7A$$

$$A = 4$$

$x = 0$:

$$14 - 0 + 0 = A(3 + 0) + (B \times 0 + C)(2 + 0)$$

$$14 = 3A + 2C$$

$A = 4$

$$14 = 12 + 2C$$

$$2 = 2C$$

$$C = 1$$

$x = 1$:

$$14 - 3 + 2 = A(3 + 1) + (B + C)(3)$$

$$13 = 4A + 3B + 3C$$

$A=4$ $C=1$

$$13 = 16 + 3B + 3$$

$$13 = 19 + 3B$$

$$-6 = 3B$$

$$B = -2 \rightarrow \frac{4}{2+x} + \frac{-2x + 1}{3 + x^2}$$

(b) Hence obtain the expansion of $\mathrm{f}(x)$ in ascending powers of x, up to and including the term in x^2. [5]

$$f(x) = \overset{A}{\frac{4}{2+x}} + \overset{B}{\frac{-2x+1}{3+x^2}}$$

A: $4(2+x)^{-1} = 4\times 2^{-1}\left(1+\frac{x}{2}\right)^{-1} = 4\times\frac{1}{2}\left(1+\frac{x}{2}\right)^{-1} = 2\left(1+\frac{x}{2}\right)^{-1}$

$$2\left(1+\frac{x}{2}\right)^{-1} = 2\left[1+(-1)\left(\frac{x}{2}\right)+\frac{(-1)(-2)}{2!}\left(\frac{x}{2}\right)^2+\ldots\right]$$
$$= 2\left[1-\frac{x}{2}+\frac{x^2}{4}\right]$$
$$= 2-x+\frac{x^2}{2}$$

B: $(-2x+1)(3+x^2)^{-1} = (-2x+1)\times 3^{-1}\left(1+\frac{x^2}{3}\right)^{-1} = (-2x+1)\times\frac{1}{3}\left(1+\frac{x^2}{3}\right)^{-1}$

$$(-2x+1)\times\frac{1}{3}\left(1+\frac{x^2}{3}\right)^{-1} = (-2x+1)\times\frac{1}{3}\left[1+(-1)\left(\frac{x^2}{3}\right)+\ldots\right]$$
$$= (-2x+1)\times\frac{1}{3}\left(1-\frac{x^2}{3}\right)$$
$$= (-2x+1)\times\left(\frac{1}{3}-\frac{x^2}{9}\right)$$

$> x^2$

$$= \frac{-2x}{3}+\cancel{\frac{2x^3}{9}}+\frac{1}{3}-\frac{x^2}{9}$$

A+B: $2-x+\frac{x^2}{2}+\frac{1}{3}-\frac{2x}{3}-\frac{x^2}{9}$

$$= \frac{7}{3}-\frac{5x}{3}+\frac{7x^2}{18}$$

10 Let $f(x) = \dfrac{2x^2 + 7x + 8}{(1+x)(2+x)^2}$.

(a) Express $f(x)$ in partial fractions. [5]

$$\frac{2x^2+7x+8}{(1+x)(2+x)^2} \equiv \frac{A}{1+x} + \frac{B}{2+x} + \frac{C}{(2+x)^2}$$

$$2x^2 + 7x + 8 \equiv A(2+x)^2 + B(1+x)(2+x) + C(1+x)$$

$x = -1$:

$$2(-1)^2 + 7(-1) + 8 = A(1)^2 + 0 + 0$$

$$2 - 7 + 8 = A$$

$$A = 3$$

$x = -2$:

$$2(-2)^2 + 7(-2) + 8 = 0 + 0 + C(-1)$$

$$2 \times 4 - 14 + 8 = -C$$

$$2 = -C$$

$$C = -2$$

$x = 0$:

$$0 + 0 + 8 = A(2)^2 + B(1)(2) + C(1)$$

$$8 = 4A + 2B + C$$

$A=3$ $C=-2$

$$8 = 12 + 2B - 2$$

$$8 = 10 + 2B$$

$$-2 = 2B$$

$$B = -1$$

$$\rightarrow \frac{3}{1+x} + \frac{-1}{2+x} + \frac{-2}{(2+x)^2}$$

(b) Hence obtain the expansion of $\mathrm{f}(x)$ in ascending powers of x, up to and including the term in x^2. [5]

$$f(x) = \frac{3}{1+x} + \frac{-1}{2+x} + \frac{-2}{(2+x)^2}$$

A: $3(1+x)^{-1} = 3\left[1 + (-1)(x) + \frac{(-1)(-2)}{2!}(x)^2 + \ldots\right]$

$= 3[1 - x + x^2]$

$= \underline{3 - 3x + 3x^2}$

B: $-1(2+x)^{-1} = -1 \times 2^{-1}\left(1+\frac{x}{2}\right)^{-1} = -1 \times \frac{1}{2}\left(1+\frac{x}{2}\right)^{-1}$

$\frac{-1}{2}\left(1+\frac{x}{2}\right)^{-1} = \frac{-1}{2}\left[1 + (-1)\left(\frac{x}{2}\right) + \frac{(-1)(-2)}{2!}\left(\frac{x}{2}\right)^2 + \ldots\right]$

$= \frac{-1}{2}\left[1 - \frac{x}{2} + \frac{x^2}{4}\right]$

$= \underline{\frac{-1}{2} + \frac{x}{4} - \frac{x^2}{8}}$

C: $-2(2+x)^{-2} = -2 \times 2^{-2}\left(1+\frac{x}{2}\right)^{-2} = -2 \times \frac{1}{4}\left(1+\frac{x}{2}\right)^{-2}$

$\frac{-1}{2}\left(1+\frac{x}{2}\right)^{-2} = \frac{-1}{2}\left[1 + (-2)\left(\frac{x}{2}\right) + \frac{(-2)(-3)}{2!}\left(\frac{x}{2}\right)^2 + \ldots\right]$

$= \frac{-1}{2}\left[1 - x + \frac{3}{4}x^2\right]$

$= \underline{\frac{-1}{2} + \frac{1}{2}x - \frac{3}{8}x^2}$

A + B + C: $3 - 3x + 3x^2 - \frac{1}{2} + \frac{x}{4} - \frac{x^2}{8} - \frac{1}{2} + \frac{1}{2}x - \frac{3}{8}x^2$

$= \underline{2 - \frac{9}{4}x + \frac{5}{2}x^2}$

9 Let $f(x) = \dfrac{17x^2 - 7x + 16}{(2 + 3x^2)(2 - x)}$.

(a) Express $f(x)$ in partial fractions. [5]

$$\frac{17x^2 - 7x + 16}{(2+3x^2)(2-x)} \equiv \frac{Ax + B}{(2+3x^2)} + \frac{C}{2-x}$$

$$17x^2 - 7x + 16 \equiv (Ax+B)(2-x) + C(2+3x^2)$$

$x=2$:

$$17(2)^2 - 7(2) + 16 = 0 + C(2+3(2)^2)$$
$$68 - 14 + 16 = C(2+12)$$
$$70 = 14C$$
$$C = 5$$

$x=0$:

$$17(0)^2 - 7(0) + 16 = (0+B)(2) + C(2)$$
$$16 = 2B + 2C$$
$$16 = 2B + 10$$
$$2B = 6$$
$$B = 3$$

$x=1$:

$$17(1)^2 - 7(1) + 16 = (A+B)(1) + C(5)$$
$$17 - 7 + 16 = A + B + 5C$$
$$26 = A + 3 + 25$$
$$26 = A + 28$$
$$A = -2$$

$$\rightarrow \frac{-2x+3}{(2+3x^2)} + \frac{5}{2-x}$$

(b) Hence obtain the expansion of $\mathrm{f}(x)$ in ascending powers of x, up to and including the term in x^3. [5]

A $(-2x+3)(2+3x^2)^{-1} = (-2x+3)\times 2^{-1}\left(1+\frac{3x^2}{2}\right)^{-1}$

$= \frac{1}{2}(-2x+3)\left(1+\frac{3x^2}{2}\right)^{-1}$

$= \frac{1}{2}(-2x+3)\left[1+(-1)\left(\frac{3x^2}{2}\right)+\dots\right]$

$= \frac{1}{2}(-2x+3)\left(1-\frac{3x^2}{2}\right]$

$= \frac{1}{2}\left(-2x+3x^3+3-\frac{9}{2}x^2\right)$

$= \frac{3}{2}-x-\frac{9}{4}x^2+\frac{3}{2}x^3$

B $5(2-x)^{-1} = 5\times 2^{-1}\left(1-\frac{x}{2}\right)^{-1}$

$= \frac{5}{2}\left(1-\frac{x}{2}\right)^{-1}$

$= \frac{5}{2}\left[1+(-1)\left(\frac{-x}{2}\right)+\frac{(-1)(-2)}{2!}\left(\frac{-x}{2}\right)^2+\frac{(-1)(-2)(-3)}{3!}\left(\frac{-x}{2}\right)^3+\dots\right]$

$= \frac{5}{2}\left[1+\frac{x}{2}+\frac{x^2}{4}+\frac{x^3}{8}\right]$

$= \frac{5}{2}+\frac{5}{4}x+\frac{5}{8}x^2+\frac{5}{16}x^3$

A + B: $\frac{3}{2}-x-\frac{9}{4}x^2+\frac{3}{2}x^3+\frac{5}{2}+\frac{5}{4}x+\frac{5}{8}x^2+\frac{5}{16}x^3$

$= 4+\frac{1}{4}x-\frac{13}{8}x^2+\frac{29}{16}x^3$

(c) State the set of values of x for which the expansion in **(b)** is valid. Give your answer in an exact form. [1]

$\left|\frac{3x^2}{2}\right| < 1$ and $\left|\frac{-x}{2}\right| < 1$

$|3x^2| < 2$ $\quad |x| < 2$

$|x^2| < \frac{2}{3}$

$|x| < \sqrt{\frac{2}{3}}$

so $|x| < \sqrt{\frac{2}{3}}$

7 Let $f(x) = \dfrac{5x^2 + 8x - 3}{(x-2)(2x^2+3)}$.

(a) Express $f(x)$ in partial fractions. [5]

$$\frac{5x^2+8x-3}{(x-2)(2x^2+3)} \equiv \frac{A}{x-2} + \frac{Bx+C}{2x^2+3}$$

$$5x^2 + 8x - 3 \equiv A(2x^2+3) + (Bx+C)(x-2)$$

$x=2$:

$$5(2)^2 + 8(2) - 3 = A(2(2)^2+3) + 0$$

$$20 + 16 - 3 = A(8+3)$$

$$33 = 11A$$

$$A = 3$$

$x=0$:

$A=3$

$$0 + 0 - 3 = A(3) + C(-2)$$

$$-3 = 9 - 2C$$

$$-12 = -2C$$

$$C = 6$$

$x=1$:

$A=3$ $C=6$

$$5 + 8 - 3 = A(2+3) + (B+C)(-1)$$

$$10 = 3 \times 5 + (B+6) \times -1$$

$$10 = 15 - B - 6$$

$$10 = 9 - B$$

$$1 = -B$$

$$B = -1$$

$$\rightarrow \frac{3}{x-2} + \frac{-x+6}{2x^2+3}$$

(b) Hence obtain the expansion of $\mathrm{f}(x)$ in ascending powers of x, up to and including the term in x^2. [5]

A B

$$f(x) = \frac{3}{x-2} + \frac{-x+6}{2x^2+3}$$

A: $3(x-2)^{-1} = 3(-2+x)^{-1} = 3\times(-2)^{-1}\left(1-\frac{x}{2}\right)^{-1} = 3\times\frac{-1}{2}\left(1-\frac{x}{2}\right)^{-1}$

$\frac{-3}{2}\left(1-\frac{x}{2}\right)^{-1} = \frac{-3}{2}\left[1+(-1)\left(\frac{-x}{2}\right)+\frac{(-1)(-2)}{2!}\left(\frac{-x}{2}\right)^2+\ldots\right]$

$= \frac{-3}{2}\left[1+\frac{x}{2}+\frac{x^2}{4}\right]$

$= \frac{-3}{2} - \frac{3x}{4} - \frac{3x^2}{8}$

B: $(-x+6)(2x^2+3)^{-1} = (-x+6)(3+2x^2)^{-1} = (-x+6)\times 3^{-1}\left(1+\frac{2x^2}{3}\right)^{-1}$

$(-x+6)\times\frac{1}{3}\left(1+\frac{2x^2}{3}\right)^{-1} = (-x+6)\times\frac{1}{3}\left[1+(-1)\left(\frac{2x^2}{3}\right)+\ldots\right]$

$= (-x+6)\times\frac{1}{3}\left[1-\frac{2x^2}{3}\right]$

$= (-x+6)\times\left(\frac{1}{3}-\frac{2x^2}{9}\right)$

$> x^2$

$= -\frac{1}{3}x + \cancel{\frac{2x^3}{9}} + 2 - \frac{4x^2}{3}$

A + B: $\frac{-3}{2} - \frac{3x}{4} - \frac{3x^2}{8} + 2 - \frac{x}{3} - \frac{4x^2}{3}$

$= \frac{1}{2} - \frac{13}{12}x - \frac{41}{24}x^2$