2 Solve the equation $\ln(2x^2 - 3) = 2\ln x - \ln 2$, giving your answer in an exact form. [3]

$$\ln(2x^2-3) = \ln x^2 - \ln 2$$

$$\ln(2x^2-3) = \ln\left(\frac{x^2}{2}\right)$$

$$2x^2 - 3 = \frac{x^2}{2}$$

×2 ×2 ×2

$$4x^2 - 6 = x^2$$
$$3x^2 - 6 = 0$$
$$x^2 - 2 = 0$$
$$x^2 = 2$$
$$x = \pm\sqrt{2}$$

$\underline{\underline{x = \sqrt{2}}}$ $\qquad x = -\sqrt{2}$ x

gives $2\ln(-\sqrt{2})$ in $2\ln x$ and logs of negatives don't exist.

2 Solve the equation $\ln 3 + \ln(2x+5) = 2\ln(x+2)$. Give your answer in a simplified exact form. [4]

$$\ln 3 + \ln(2x+5) = 2\ln(x+2)$$

$$\ln\left(3(2x+5)\right) = \ln(x+2)^2$$

$$3(2x+5) = (x+2)^2$$

$$6x + 15 = x^2 + 4x + 4$$

$$x^2 - 2x - 11 = 0$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-11)}}{2(1)}$$

$x = 1 + 2\sqrt{3}$ or $x = 1 - 2\sqrt{3}$

gives a negative log in $2\ln(x+2)$ so not a valid solution.

1 Solve the equation $\ln(2x-1) = 2\ln(x+1) - \ln x$. Give your answer correct to 3 decimal places. [4]

$$\ln(2x-1) = \ln(x+1)^2 - \ln x$$

$$\ln(2x-1) = \ln\left(\frac{(x+1)^2}{x}\right)$$

$$2x-1 = \frac{(x+1)^2}{x}$$

$$x(2x-1) = (x+1)^2$$

$$2x^2 - x = x^2 + 2x + 1$$

$$x^2 - 3x - 1 = 0$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{3+\sqrt{13}}{2} \quad \text{or} \quad x = \frac{3-\sqrt{13}}{2}$$

$x = 3.303$ $\quad$ $x = -0.303$ ✗

gives a negative in all three logarithms + logs of negatives do not exist.

1 It is given that $x = \ln(2y - 3) - \ln(y + 4)$.

Express y in terms of x. [3]

$$x = \ln\left(\frac{2y-3}{y+4}\right)$$

$$e^x = \frac{2y-3}{y+4}$$

$$(y+4)e^x = 2y-3$$

$$ye^x + 4e^x = 2y - 3$$

$$ye^x - 2y = -4e^x - 3$$

$$y(e^x - 2) = -4e^x - 3$$

$$y = \frac{-4e^x - 3}{e^x - 2}$$

or $$y = \frac{4e^x + 3}{2 - e^x}$$

4 Solve the equation

$$\log_{10}(2x+1) = 2\log_{10}(x+1) - 1.$$

Give your answers correct to 3 decimal places. [6]

$$\log_{10}(2x+1) - 2\log_{10}(x+1) = -1$$

$$\log_{10}(2x+1) - \log_{10}(x+1)^2 = -1$$

$$\log_{10}\left(\frac{2x+1}{(x+1)^2}\right) = -1$$

$$\frac{2x+1}{(x+1)^2} = 10^{-1}$$

$$\frac{2x+1}{(x+1)^2} = \frac{1}{10}$$

$$10(2x+1) = (x+1)^2$$

$$20x + 10 = x^2 + 2x + 1$$

$$x^2 - 18x - 9 = 0$$

$$x = \frac{-(-18) \pm \sqrt{(-18)^2 - 4(1)(-9)}}{2(1)}$$

$x = 9 + 3\sqrt{10}$ or $x = 9 - 3\sqrt{10}$

$x = 18.487$ $\qquad$ $x = -0.487$

3 **(a)** Show that the equation

$$\ln(1 + e^{-x}) + 2x = 0$$

can be expressed as a quadratic equation in e^x. [2]

$$\ln(1 + e^{-x}) = -2x$$

$$1 + e^{-x} = e^{-2x}$$

$$1 + \frac{1}{e^x} = \frac{1}{e^{2x}}$$

$$\times e^{2x} \quad \times e^{2x} \quad \times e^{2x}$$

$$e^{2x} + e^x = 1$$

$$e^{2x} + e^x - 1 = 0$$

$$(e^x)^2 + e^x - 1 = 0 \quad \text{QED}$$

(b) Hence solve the equation $\ln(1 + e^{-x}) + 2x = 0$, giving your answer correct to 3 decimal places. [4]

let $y = e^x$: $\quad y^2 + y - 1 = 0$

$$y = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(-1)}}{2(1)}$$

$$y = \frac{-1+\sqrt{5}}{2} \quad \text{or} \quad y = \frac{-1-\sqrt{5}}{2}$$

$$e^x = \frac{-1+\sqrt{5}}{2} \quad \text{or} \quad e^x = \frac{-1-\sqrt{5}}{2}$$

$$x = \ln\left(\frac{-1+\sqrt{5}}{2}\right) \quad \text{or} \quad x = \ln\left(\frac{-1-\sqrt{5}}{2}\right)$$

$$= -0.481$$

no sol^n

3 Solve the equation $2^{3x-1} = 5(3^{-x})$. Give your answer in the form $\frac{\ln a}{\ln b}$, where a and b are integers. [4]

$$2^{3x-1} = 5(3^{-x})$$

$$\ln 2^{3x-1} = \ln(5(3^{-x}))$$

$$(3x-1)\ln 2 = \ln 5 + \ln 3^{-x}$$

$$3x\ln 2 - \ln 2 = \ln 5 - x\ln 3$$

$$3x\ln 2 + x\ln 3 = \ln 5 + \ln 2$$

$$x(3\ln 2 + \ln 3) = \ln 10$$

$$x(\ln 2^3 + \ln 3) = \ln 10$$

$$x(\ln 8 + \ln 3) = \ln 10$$

$$x(\ln 24) = \ln 10$$

$$x = \frac{\ln 10}{\ln 24}$$

1 Find the value of x for which $3(2^{1-x}) = 7^x$. Give your answer in the form $\frac{\ln a}{\ln b}$, where a and b are integers. [4]

$$3(2^{1-x}) = 7^x$$

$$\ln\left[3(2^{1-x})\right] = \ln 7^x$$

$$\ln 3 + \ln 2^{1-x} = x \ln 7$$

$$\ln 3 + (1-x)\ln 2 = x\ln 7$$

$$\ln 3 + \ln 2 - x\ln 2 = x\ln 7$$

$$\ln 3 + \ln 2 = x\ln 7 + x\ln 2$$

$$\ln 6 = x(\ln 7 + \ln 2)$$

$$\ln 6 = x\ln 14$$

$$x = \frac{\ln 6}{\ln 14}$$

1 Find the set of values of x for which $2(3^{1-2x}) < 5^x$. Give your answer in a simplified exact form. [4]

$$2(3^{1-2x}) < 5^x$$

$$\ln[2(3^{1-2x})] < \ln 5^x$$

$$\ln 2 + \ln(3^{1-2x}) < x\ln 5$$

$$\ln 2 + (1-2x)\ln 3 < x\ln 5$$

$$\ln 2 + \ln 3 - 2x\ln 3 < x\ln 5$$

$$\ln 2 + \ln 3 < x\ln 5 + 2x\ln 3$$

$$\ln 2 + \ln 3 < x(\ln 5 + 2\ln 3)$$

$$x > \frac{\ln 2 + \ln 3}{\ln 5 + 2\ln 3}$$

$$x > \frac{\ln 6}{\ln 5 + \ln 3^2}$$

$$x > \frac{\ln 6}{\ln 5 + \ln 9}$$

$$x > \frac{\ln 6}{\ln 45}$$

3 Solve the equation $4^{x-2} = 4^x - 4^2$, giving your answer correct to 3 decimal places. [4]

$$4^{x-2} = 4^x - 4^2$$

$$4^x \times 4^{-2} = 4^x - 4^2$$

$$\frac{1}{4^2} \times 4^x = 4^x - 4^2$$

$$\frac{1}{16} \times 4^x = 4^x - 16$$

$\times 16$ $\times 16$ $\times 16$

$$4^x = 16 \times 4^x - 256$$

-4^x -4^x

$$0 = 15 \times 4^x - 256$$

$$15 \times 4^x = 256$$

$$4^x = \frac{256}{15}$$

$$\log_4 4^x = \log_4\left(\frac{256}{15}\right)$$

$$x = \log_4\left(\frac{256}{15}\right)$$

$$\underline{x = 2.047}$$