3 The equation of a curve is $y = (x - 3)\sqrt{x + 1} + 3$. The following points lie on the curve. Non-exact values are rounded to 4 decimal places.

$A\,(2,\ k)$ $B\,(2.9,\ 2.8025)$ $C\,(2.99,\ 2.9800)$ $D\,(2.999,\ 2.9980)$ $E\,(3,\ 3)$

(a) Find k, giving your answer correct to 4 decimal places. [1]

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(b) Find the gradient of AE, giving your answer correct to 4 decimal places. [1]

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The gradients of BE, CE and DE, rounded to 4 decimal places, are 1.9748, 1.9975 and 1.9997 respectively.

(c) State, giving a reason for your answer, what the values of the four gradients suggest about the gradient of the curve at the point E. [2]

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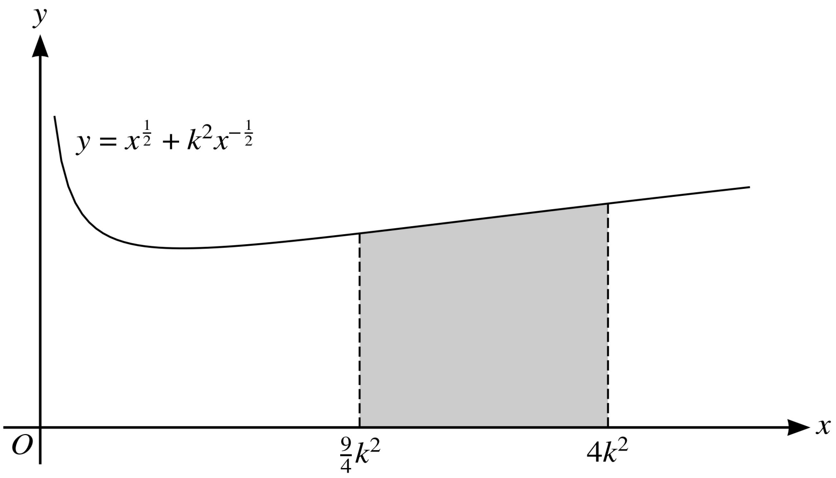
11 The gradient of a curve is given by $\frac{dy}{dx} = 6(3x - 5)^3 - kx^2$, where k is a constant. The curve has a stationary point at $(2, -3.5)$.

(a) Find the value of k. [2]

11



The diagram shows part of the curve with equation $y = x^{\frac{1}{2}} + k^2x^{-\frac{1}{2}}$, where k is a positive constant.

(a) Find the coordinates of the minimum point of the curve, giving your answer in terms of k. [4]

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10 At the point $(4, -1)$ on a curve, the gradient of the curve is $-\frac{3}{2}$. It is given that $\frac{dy}{dx} = x^{-\frac{1}{2}} + k$, where k is a constant.

(c) Find the coordinates of the stationary point. [3]

(d) Determine the nature of the stationary point. [2]

9 The equation of a curve is $y = (3 - 2x)^3 + 24x$.

(a) Find expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [4]

(b) Find the coordinates of each of the stationary points on the curve. [3]

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(c) Determine the nature of each stationary point. [2]

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9 The equation of a curve is $y = 3x + 1 - 4(3x + 1)^{\frac{1}{2}}$ for $x > -\frac{1}{3}$.

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [3]

(b) Find the coordinates of the stationary point of the curve and determine its nature. [4]

10 The gradient of a curve at the point (x, y) is given by $\frac{dy}{dx} = 2(x+3)^{\frac{1}{2}} - x$. The curve has a stationary point at $(a, 14)$, where a is a positive constant.

(a) Find the value of a. [3]

(b) Determine the nature of the stationary point. [3]

11 The equation of a curve is

$$y = k\sqrt{4x+1} - x + 5,$$

where k is a positive constant.

(a) Find $\dfrac{dy}{dx}$. [2]

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(b) Find the x-coordinate of the stationary point in terms of k. [2]

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8 The equation of a curve is such that $\frac{dy}{dx} = 3x^{\frac{1}{2}} - 3x^{-\frac{1}{2}}$. The curve passes through the point (3, 5).

(b) Find the x-coordinate of the stationary point. [2]

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(c) State the set of values of x for which y increases as x increases. [1]

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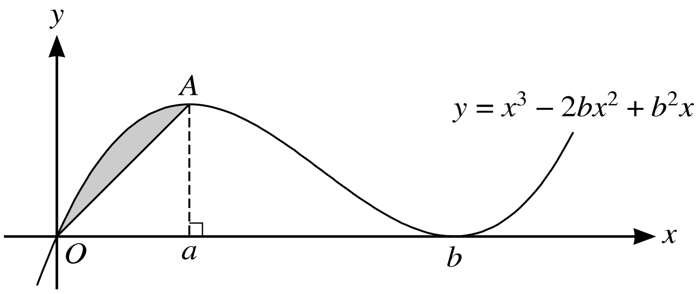

The diagram shows part of the curve with equation $y = x^3 - 2bx^2 + b^2x$ and the line OA, where A is the maximum point on the curve. The x-coordinate of A is a and the curve has a minimum point at $(b, 0)$, where a and b are positive constants.

(a) Show that $b = 3a$. [4]

11 It is given that a curve has equation $y = k(3x - k)^{-1} + 3x$, where k is a constant.

(a) Find, in terms of k, the values of x at which there is a stationary point. [4]

The function f has a stationary value at $x = a$ and is defined by

$$f(x) = 4(3x-4)^{-1} + 3x \quad \text{for } x \geqslant \tfrac{3}{2}.$$

(b) Find the value of a and determine the nature of the stationary value. [3]

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(c) The function g is defined by $g(x) = -(3x+1)^{-1} + 3x$ for $x \geqslant 0$.

Determine, making your reasoning clear, whether g is an increasing function, a decreasing function or neither. [2]

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