

- 5 Marco has four boxes labelled K , L , M and N . He places them in a straight line in the order K , L , M , N with K on the left. Marco also has four coloured marbles: one is red, one is green, one is white and one is yellow. He places a single marble in each box, at random. Events A and B are defined as follows.

A : The white marble is in either box L or box M .

B : The red marble is to the left of both the green marble and the yellow marble.

Determine whether or not events A and B are independent.

[3]

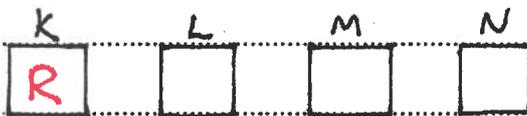
A :

White marble has $\frac{1}{4}$ chance of being in each box:

$$P(A) = \frac{1}{4} + \frac{1}{4}$$

$$= \underline{\underline{\frac{1}{2}}}$$

B :



fixed

Permute $G, W, Y = 3!$

$$= 1 \times 3!$$

$$= 6 \text{ ways}$$

OR:



Permute $G, Y = 2!$

$$= 1 \times 1 \times 2!$$

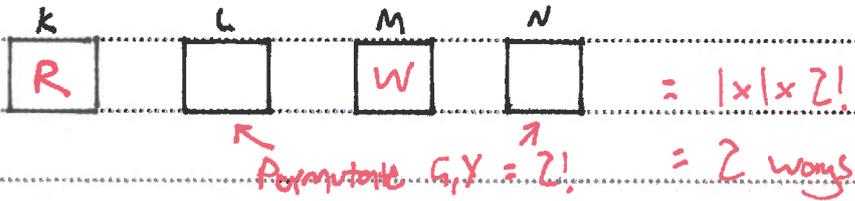
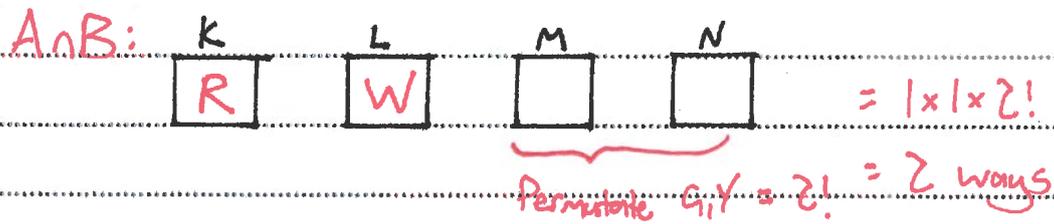
$$= 2 \text{ ways}$$

Without restrictions = $4!$ (permute R, G, W, Y)

$$= 24$$

$$\text{so } P(B) = \frac{6+2}{24} = \underline{\underline{\frac{1}{3}}}$$

continued



$$P(A \cap B) = \frac{2+2}{24} = \underline{\underline{\frac{1}{6}}}$$

If independent, $P(A) \times P(B) = P(A \cap B)$

$$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$

So they are independent

- 7 A children's wildlife magazine is published every Monday. For the next 12 weeks it will include a model animal as a free gift. There are five different models: tiger, leopard, rhinoceros, elephant and buffalo, each with the same probability of being included in the magazine.

Sahim buys one copy of the magazine every Monday.

- (a) Find the probability that the first time that the free gift is an elephant is before the 6th Monday. [2]

$$P(E) = 0.2$$

$$E \sim \text{Geo}(0.2)$$

$$P(E < 6) = P(E \leq 5)$$

$$= 1 - 0.8^5$$

← probability of 5 failures

$$= 1 - 0.8^5$$

$$= \underline{0.67232}$$

- (b) Find the probability that Sahim will get more than two leopards in the 12 magazines. [3]

$$P(L) = 0.2$$

$$L \sim B(12, 0.2)$$

$$P(L > 2) = 1 - (P(0) + P(1) + P(2))$$

$$= 1 - \left({}^{12}C_0 \times 0.2^0 \times 0.8^{12} + {}^{12}C_1 \times 0.2^1 \times 0.8^{11} + {}^{12}C_2 \times 0.2^2 \times 0.8^{10} \right)$$

$$= 1 - (0.5583)$$

$$= \underline{0.442}$$

- (c) Find the probability that after 5 weeks Sahim has exactly one of each animal. [3]

Probability of getting all five animals in order:
Tiger, Leopard, Rhino, Elephant, Buffalo:

$$0.2 \times 0.2 \times 0.2 \times 0.2 \times 0.2 = 0.00032$$

But order doesn't matter, so permute these five objects by multiplying by $5!$:

$$0.00032 \times 5! = \underline{0.0384}$$

- 7 Hanna buys 12 hollow chocolate eggs that each contain a sweet. The eggs look identical but Hanna knows that 3 contain a red sweet, 4 contain an orange sweet and 5 contain a yellow sweet. Each of Hanna's three children in turn randomly chooses and eats one of the eggs, keeping the sweet it contained.

(a) Find the probability that all 3 eggs chosen contain the same colour sweet.

[4]

$$P(RRR) = \frac{{}^3C_3}{{}^{12}C_3} = \frac{1}{220}$$

$$P(OOO) = \frac{{}^4C_3}{{}^{12}C_3} = \frac{1}{55}$$

$$P(YYY) = \frac{{}^5C_3}{{}^{12}C_3} = \frac{1}{22}$$

$$P(RRR) + P(OOO) + P(YYY) = \frac{3}{44}$$

- (b) Find the probability that all 3 eggs chosen contain a yellow sweet, given that all three children have the same colour sweet. [2]

$$P(YYY) = \frac{1}{22}$$

$$P(\text{all same}) = \frac{3}{44}$$

$$P(YYY \cap \text{all same}) = P(YYY | \text{all same}) \times P(\text{all same})$$

$$P(YYY | \text{all same}) = \frac{P(YYY \cap \text{all same})}{P(\text{all same})}$$

$$= \frac{\frac{1}{22}}{\frac{3}{44}} = \frac{2}{3}$$

- (c) Find the probability that at least one of Hanna's three children chooses an egg that contains an orange sweet. [3]

$$P(\text{one orange}) = \frac{{}^4C_1 \times {}^8C_2}{{}^{12}C_3} = \frac{28}{55}$$

$$P(\text{two orange}) = \frac{{}^4C_2 \times {}^8C_1}{{}^{12}C_3} = \frac{12}{55}$$

$$P(\text{three orange}) = \frac{{}^4C_3}{{}^{12}C_3} = \frac{1}{55}$$

$$P(\text{at least one orange}) = \frac{28}{55} + \frac{12}{55} + \frac{1}{55}$$

$$= \frac{41}{55}$$

- 6 A factory produces chocolates in three flavours: lemon, orange and strawberry in the ratio 3 : 5 : 7 respectively. Nell checks the chocolates on the production line by choosing chocolates randomly one at a time.

- (a) Find the probability that the first chocolate with lemon flavour that Nell chooses is the 7th chocolate that she checks. [1]

$$P(\text{lemon}) = \frac{3}{15} = \frac{1}{5}$$

$$L \sim \text{Geo}\left(\frac{1}{5}\right)$$

$$P(L=7) = \left(\frac{1}{5}\right) \times \left(\frac{4}{5}\right)^6$$

$$= \underline{\underline{0.0524}}$$

- (b) Find the probability that the first chocolate with lemon flavour that Nell chooses is after she has checked at least 6 chocolates. [2]

$$\text{After 6 chocolates} = P(L > 6)$$

$$= \left(\frac{4}{5}\right)^6 \leftarrow \text{probability of 6 failures}$$

$$= \underline{\underline{0.262}}$$

'Surprise' boxes of chocolates each contain 15 chocolates: 3 are lemon, 5 are orange and 7 are strawberry.

Petra has a box of Surprise chocolates. She chooses 3 chocolates at random from the box. She eats each chocolate before choosing the next one.

- (c) Find the probability that none of Petra's 3 chocolates has orange flavour. [2]

Probability is not constant, so can't use binomial or geometric distributions.

$$\text{pick 3 from 10 (lemon and strawberry)} \rightarrow \frac{{}^{10}C_3 \times {}^5C_0}{{}^{15}C_3} \leftarrow \begin{array}{l} \text{no oranges} \\ \text{total number of possibilities} \end{array}$$

$$= \underline{\underline{\frac{24}{91}}}$$

- (d) Find the probability that each of Petra's 3 chocolates has a different flavour. [3]

$$\frac{{}^3C_1 \times {}^5C_1 \times {}^7C_1}{{}^{15}C_3}$$

$\xrightarrow{1 \text{ lemon}} \quad \xrightarrow{1 \text{ orange}} \quad \xrightarrow{1 \text{ strawberry}}$

$$= \frac{3}{13}$$

- (e) Find the probability that at least 2 of Petra's 3 chocolates have strawberry flavour given that none of them has orange flavour. [4]

$P(O')$

$P(S)$

[4]

$$P(S \cap O') = P(S|O') \times P(O')$$

$$P(S|O') = \frac{P(S \cap O')}{P(O')}$$

$P(S \cap O')$:

$$S S S = \frac{{}^7C_3}{{}^{15}C_3} = \frac{1}{13}$$

S S L

S L S

L S S

$$\left. \begin{array}{l} S S L \\ S L S \\ L S S \end{array} \right\} + = \frac{{}^7C_2 \times {}^3C_1}{{}^{15}C_3} = \frac{9}{65}$$

$$P(S|O') = \frac{\frac{14}{65}}{\frac{24}{91}}$$

$$= \frac{49}{60}$$

from part (c)

$$P(S \cap O') = \frac{1}{13} + \frac{9}{65}$$

$$= \frac{14}{65}$$