**5** Marco has four boxes labelled $K$, $L$, $M$ and $N$. He places them in a straight line in the order $K$, $L$, $M$, $N$ with $K$ on the left. Marco also has four coloured marbles: one is red, one is green, one is white and one is yellow. He places a single marble in each box, at random. Events $A$ and $B$ are defined as follows.

$A$: The white marble is in either box $L$ or box $M$.

$B$: The red marble is to the left of both the green marble and the yellow marble.

Determine whether or not events $A$ and $B$ are independent. [3]

A:

White marble has $\frac{1}{4}$ chance of being in each box:

$$P(A) = \frac{1}{4} + \frac{1}{4}$$

$$= \underline{\frac{1}{2}}$$

B:

| K | L | M | N |
|---|---|---|---|
| R | | | |

fixed

$\underbrace{\quad\quad\quad}_{\text{Permutate } G, W, Y = 3!}$

$= 1 \times 3!$

$= 6$ ways

OR:

| K | L | M | N |
|---|---|---|---|
| W | R | | |

$\underbrace{\quad\quad}_{\text{Permutate } G, Y = 2!}$

$= 1 \times 1 \times 2!$

$= 2$ ways

Without restrictions $= 4!$ (permutate $R, G, W, Y$)

$= 24$

so $P(B) = \frac{6+2}{24} = \underline{\frac{1}{3}}$

continued

$A \cap B$:

| K | L | M | N |
|---|---|---|---|
| R | W | | |

$= 1 \times 1 \times 2!$

Permutate G, Y $= 2!$ $= 2$ ways

| K | L | M | N |
|---|---|---|---|
| R | | W | |

$= 1 \times 1 \times 2!$

Permutate G, Y $= 2!$ $= 2$ ways

$$P(A \cap B) = \frac{2+2}{24} = \underline{\frac{1}{6}}$$

If independent, $P(A) \times P(B) = P(A \cap B)$

$$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$

<u>So they are independent</u>

7 A children's wildlife magazine is published every Monday. For the next 12 weeks it will include a model animal as a free gift. There are five different models: tiger, leopard, rhinoceros, elephant and buffalo, each with the same probability of being included in the magazine.

Sahim buys one copy of the magazine every Monday.

E

**(a)** Find the probability that the first time that the free gift is an elephant is before the 6th Monday. [2]

$P(E) = 0.2$

$E \sim Geo(0.2)$

$P(E < 6) = P(E \leqslant 5)$

$= 1 - q^5$ probability of 5 failures

$= 1 - 0.8^5$

$= \underline{0.67232}$

**(b)** Find the probability that Sahim will get more than two leopards in the 12 magazines. [3]

$P(L) = 0.2$

$L \sim B(12, 0.2)$

$P(L > 2) = 1 - (P(0) + P(1) + P(2))$

$= 1 - ({}^{12}C_0 \times 0.2^0 \times 0.8^{12} + {}^{12}C_1 \times 0.2^1 \times 0.8^{11} + {}^{12}C_2 \times 0.2^2 \times 0.8^{10})$

$= 1 - (0.5583)$

$= \underline{0.442}$

(c) Find the probability that after 5 weeks Sahim has exactly one of each animal. [3]

Probability of getting all five animals in order:
Tiger, Leopard, Rhino, Elephant, Buffalo:

$0.2 \times 0.2 \times 0.2 \times 0.2 \times 0.2 = 0.00032$

But order doesn't matter, so permutate these five objects by multiplying by 5! :

$0.00032 \times 5! = \underline{0.0384}$

7 Hanna buys 12 hollow chocolate eggs that each contain a sweet. The eggs look identical but Hanna knows that 3 contain a red sweet, 4 contain an orange sweet and 5 contain a yellow sweet. Each of Hanna's three children in turn randomly chooses and eats one of the eggs, keeping the sweet it contained.

**(a)** Find the probability that all 3 eggs chosen contain the same colour sweet. [4]

$P(RRR) = \frac{^3C_3}{^{12}C_3} = \frac{1}{220}$

$P(OOO) = \frac{^4C_3}{^{12}C_3} = \frac{1}{55}$

$P(YYY) = \frac{^5C_3}{^{12}C_3} = \frac{1}{22}$

$P(RRR) + P(OOO) + P(YYY) = \frac{3}{44}$

**(b)** Find the probability that all 3 eggs chosen contain a yellow sweet, given that all three children have the same colour sweet. [2]

$P(YYY) = \frac{1}{22}$

$P(\text{all same}) = \frac{3}{44}$

$P(YYY \cap \text{all same}) = P(YYY \mid \text{all same}) \times P(\text{all same})$

$P(YYY \mid \text{all same} = \frac{P(YYY \cap \text{all same})}{P(\text{all same})}$

$= \frac{\frac{1}{22}}{3/44} = \frac{2}{3}$

**(c)** Find the probability that at least one of Hanna's three children chooses an egg that contains an orange sweet. [3]

1 orange 2 others

$P(\text{one orange}) = \frac{{}^{4}C_{1} \times {}^{8}C_{2}}{{}^{12}C_{3}} = \frac{28}{55}$

$P(\text{two orange}) = \frac{{}^{4}C_{2} \times {}^{8}C_{1}}{{}^{12}C_{3}} = \frac{12}{55}$

$P(\text{three orange}) = \frac{{}^{4}C_{3}}{{}^{12}C_{3}} = \frac{1}{55}$

$P(\text{at least one orange}) = \frac{28}{55} + \frac{12}{55} + \frac{1}{55}$

$= \frac{41}{55}$

6 A factory produces chocolates in three flavours: lemon, orange and strawberry in the ratio 3 : 5 : 7 respectively. Nell checks the chocolates on the production line by choosing chocolates randomly one at a time.

(a) Find the probability that the first chocolate with lemon flavour that Nell chooses is the 7th chocolate that she checks. [1]

$P(\text{lemon}) = \frac{3}{15} = \frac{1}{5}$

$L \sim \text{Geo}\left(\frac{1}{5}\right)$

$P(L=7) = \left(\frac{1}{5}\right) \times \left(\frac{4}{5}\right)^6$

$= 0.0524$

(b) Find the probability that the first chocolate with lemon flavour that Nell chooses is after she has checked at least 6 chocolates. [2]

After 6 chocolates $= P(L>6)$

$= q^6$ ← probability of 6 failures

$= \left(\frac{4}{5}\right)^6$

$= 0.262$

'Surprise' boxes of chocolates each contain 15 chocolates: 3 are lemon, 5 are orange and 7 are strawberry.

Petra has a box of Surprise chocolates. She chooses 3 chocolates at random from the box. She eats each chocolate before choosing the next one.

(c) Find the probability that none of Petra's 3 chocolates has orange flavour. [2]

Probability is not constant, so can't use binomial or geometric distributions.

pick 3 from 10 (lemon and strawberry) → $\frac{{}^{10}C_3 \times ({}^{5}C_0)}{{}^{15}C_3}$ ← no oranges ← total number of possibilities

$= \frac{24}{91}$

 

**(d)** Find the probability that each of Petra's 3 chocolates has a different flavour. [3]

1 lemon 1 orange 1 strawberry

$$\frac{{}^3C_1 \times {}^5C_1 \times {}^7C_1}{{}^{15}C_3}$$

$$= \frac{3}{13}$$

**(e)** Find the probability that at least 2 of Petra's 3 chocolates have strawberry flavour given that none of them has orange flavour. [4]

P(S) P(O')

$$P(S \cap O') = P(S|O') \times P(O')$$

$$P(S|O') = \frac{P(S \cap O')}{P(O')}$$

$P(S \cap O')$:

SSS $= \frac{{}^7C_3}{{}^{15}C_3} = \frac{1}{13}$

SSL, SLS, LSS $= \frac{{}^7C_2 \times {}^3C_1}{{}^{15}C_3} = \frac{9}{65}$

$$P(S \cap O') = \frac{1}{13} + \frac{9}{65} = \frac{14}{65}$$

$$P(S|O') = \frac{\frac{14}{65}}{\frac{24}{91}} = \frac{49}{60}$$

from part (c)