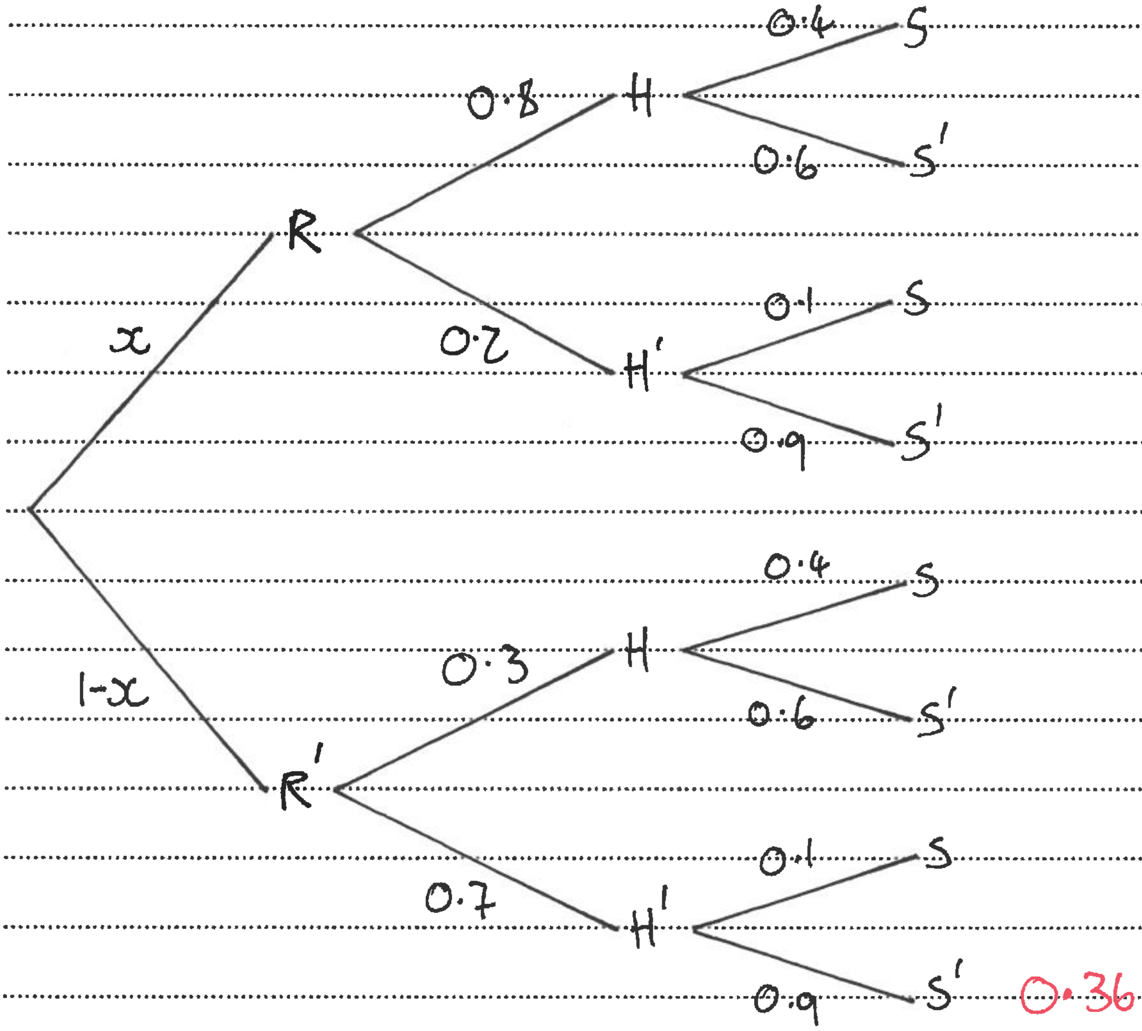
4 The probability that it will rain on any given day is x. If it is raining, the probability that Aran wears a hat is 0.8 and if it is not raining, the probability that he wears a hat is 0.3. Whether it is raining or not, if Aran wears a hat, the probability that he wears a scarf is 0.4. If he does not wear a hat, the probability that he wears a scarf is 0.1. The probability that on a randomly chosen day it is not raining and Aran is not wearing a hat or a scarf is 0.36.

Find the value of x. [3]



$$(1-x) \times 0.7 \times 0.9 = 0.36$$

$$(1-x) \times 0.63 = 0.36$$

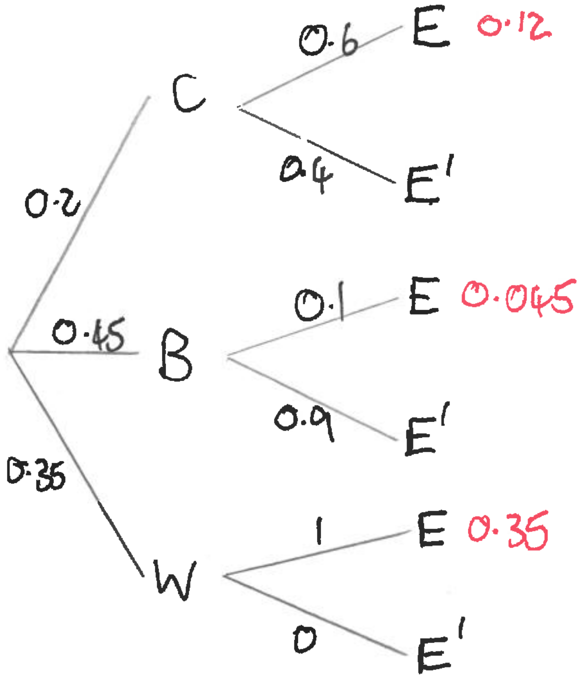
$$1 - x = \frac{4}{7}$$

$$-x = \frac{-3}{7}$$

$$x = \frac{3}{7}$$

1 Juan goes to college each day by any one of car or bus or walking. The probability that he goes by car is 0.2, the probability that he goes by bus is 0.45 and the probability that he walks is 0.35. When Juan goes by car, the probability that he arrives early is 0.6. When he goes by bus, the probability that he arrives early is 0.1. When he walks he always arrives early.

(a) Draw a fully labelled tree diagram to represent this information. [2]



(b) Find the probability that Juan goes to college by car given that he arrives early. [4]

$P(C \cap E) = P(C|E) \times P(E)$

$P(C|E) = \dfrac{P(C \cap E)}{P(E)}$

$P(C \cap E) = 0.12$

$P(E) = 0.12 + 0.045 + 0.35$

$= 0.515$

$P(C|E) = \dfrac{0.12}{0.515} = \dfrac{24}{103}$

N.B: Error in mark scheme: $\frac{12}{515}$ is incorrect, it should be $\frac{120}{515}$.

1 Two ordinary fair dice, one red and the other blue, are thrown.

Event A is 'the score on the red die is divisible by 3'.

Event B is 'the sum of the two scores is at least 9'.

(a) Find $P(A \cap B)$. [2]

Red

Blue	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$P(A \cap B)$:

$$P(A \cap B) = \frac{5}{36}$$

(b) Hence determine whether or not the events A and B are independent. [2]

If independent, $P(A) \times P(B) = P(A \cap B)$

$$P(A) = \frac{12}{36} \qquad P(B) = \frac{10}{36}$$

$$P(A) \times P(B) = \frac{12}{36} \times \frac{10}{36}$$

$$= \frac{5}{54}$$

$\frac{5}{54} \neq \frac{5}{36}$ so not independent.

2 A total of 500 students were asked which one of four colleges they attended and whether they preferred soccer or hockey. The numbers of students in each category are shown in the following table.

	Soccer	Hockey	Total
Amos	54	32	86
Benn	84	72	156
Canton	22	56	78
Devar	120	60	180
Total	280	220	500

(a) Find the probability that a randomly chosen student is at Canton college and prefers hockey. [1]

$\frac{56}{500} = \frac{14}{125}$

(b) Find the probability that a randomly chosen student is at Devar college given that he prefers soccer. [2]

P(S) P(D)

$P(D \cap S) = P(D|S) \times P(S)$

$P(D|S) = \frac{P(D \cap S)}{P(S)}$

$P(D \cap S) = \frac{120}{500}$

$P(S) = \frac{280}{500}$

$P(D|S) = \frac{120/500}{280/500}$

$= \frac{3}{7}$

(c) One of the students is chosen at random. Determine whether the events 'the student prefers hockey' and 'the student is at Amos college or Benn college' are independent, justifying your answer. [2]

If independent, $P(H) \times P(A\,or\,B) = P(H \cap A\,or\,B)$

$P(H) = \frac{220}{500}$

$P(A\,or\,B) = \frac{86}{500} + \frac{156}{500}$

$= \frac{242}{500}$

$P(H \cap A\,or\,B) = \frac{32}{500} + \frac{72}{500}$

$= \frac{104}{500}$

$= \frac{26}{125}$

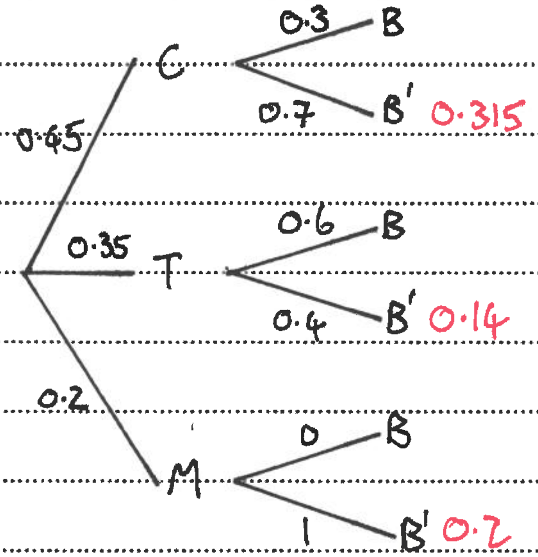
$P(H) \times P(A\,or\,B) = \frac{220}{500} \times \frac{242}{500}$

$= \frac{1331}{6250}$

$\frac{1331}{6250} \neq \frac{26}{125}$ so not independent.

3 For her bedtime drink, Suki has either chocolate, tea or milk with probabilities 0.45, 0.35 and 0.2 respectively. When she has chocolate, the probability that she has a biscuit is 0.3. When she has tea, the probability that she has a biscuit is 0.6. When she has milk, she never has a biscuit.

Find the probability that Suki has tea given that she does not have a biscuit. [5]



$$P(T \cap B') = P(T|B') \times P(B')$$

$$P(T|B') = \frac{P(T \cap B')}{P(B')}$$

$$= \frac{0.14}{0.315 + 0.14 + 0.2}$$

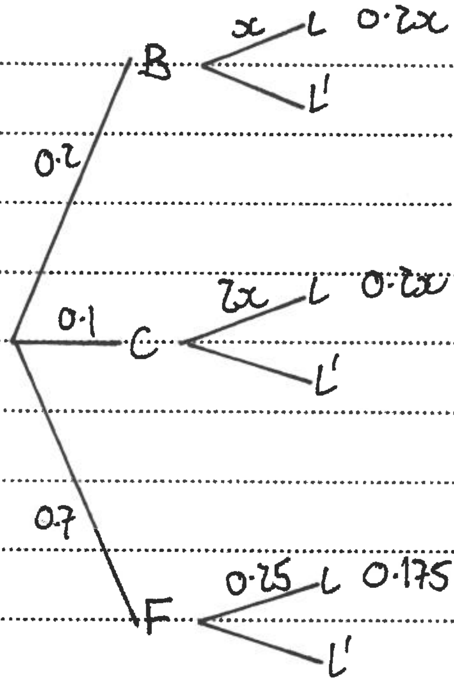
$$= \frac{0.14}{0.655}$$

$$= \frac{28}{131}$$

1 On any day, Kino travels to school by bus, by car or on foot with probabilities 0.2, 0.1 and 0.7 respectively. The probability that he is late when he travels by bus is x. The probability that he is late when he travels by car is $2x$ and the probability that he is late when he travels on foot is 0.25.

The probability that, on a randomly chosen day, Kino is late is 0.235.

(a) Find the value of x. [3]



$0{\cdot}2x + 0{\cdot}2x + 0{\cdot}175 = 0{\cdot}235$

$0{\cdot}4x + 0{\cdot}175 = 0{\cdot}235$

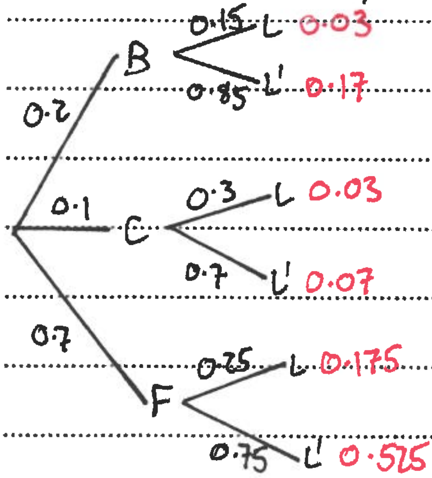
$0{\cdot}4x = 0{\cdot}06$

$\underline{x = 0{\cdot}15}$

(b) Find the probability that, on a randomly chosen day, Kino travels to school by car given that he is not late. [2]

$P(C \cap L') = P(C|L') \times P(L')$

$P(C|L') = \dfrac{P(C \cap L')}{P(L')}$



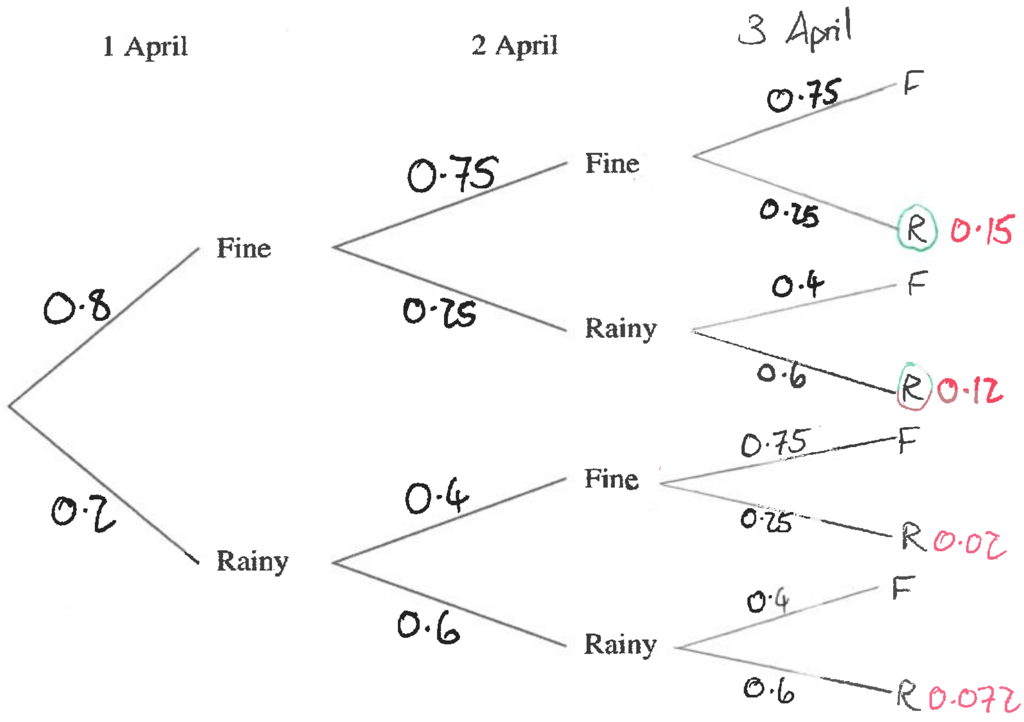
$P(C|L') = \dfrac{0{\cdot}07}{0{\cdot}17 + 0{\cdot}07 + 0{\cdot}525}$ or $1 - 0{\cdot}235$

$= \dfrac{0{\cdot}07}{0{\cdot}765}$

$= \underline{0{\cdot}0915}$

4 In a certain country, the weather each day is classified as fine or rainy. The probability that a fine day is followed by a fine day is 0.75 and the probability that a rainy day is followed by a fine day is 0.4. The probability that it is fine on 1 April is 0.8. The tree diagram below shows the possibilities for the weather on 1 April and 2 April.

(a) Complete the tree diagram to show the probabilities. [1]



(b) Find the probability that 2 April is fine. [2]

$0.8 \times 0.75 + 0.2 \times 0.4 = 0.68$

Let X be the event that 1 April is fine and Y be the event that 3 April is rainy.

(c) Find the value of $\mathrm{P}(X \cap Y)$. [3]

see diagram

$$P(X \cap Y) = 0.15 + 0.12$$
$$= 0.27$$

(d) Find the probability that 1 April is fine given that 3 April is rainy. [3]

$$P(X \cap Y) = P(X|Y) \times P(Y)$$

$$P(X|Y) = \frac{P(X \cap Y)}{P(Y)}$$

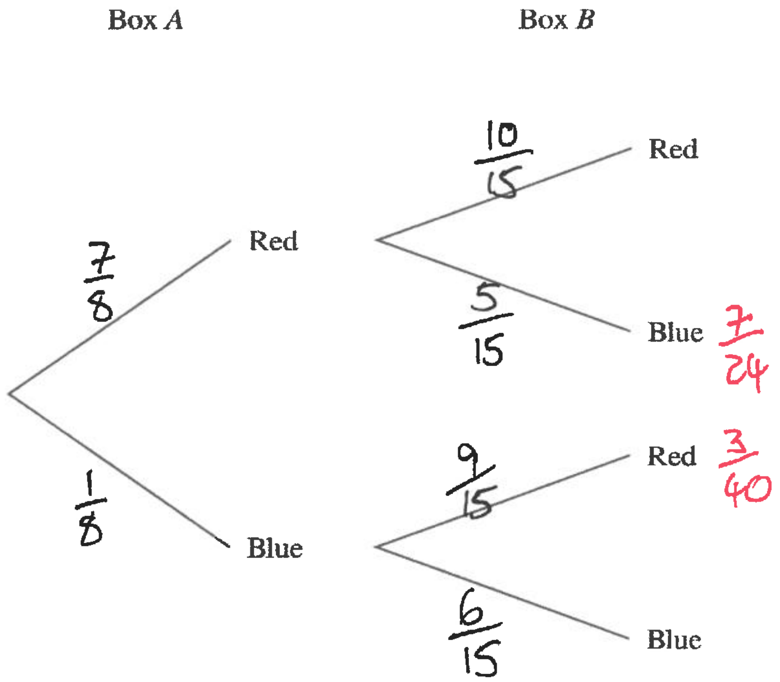
$$= \frac{0.27}{0.15 + 0.12 + 0.02 + 0.072}$$

$$= \frac{0.27}{0.362}$$

$$= \frac{135}{181}$$

6 Box A contains 7 red balls and 1 blue ball. Box B contains 9 red balls and 5 blue balls. A ball is chosen at random from box A and placed in box B. A ball is then chosen at random from box B. The tree diagram below shows the possibilities for the colours of the balls chosen.

(a) Complete the tree diagram to show the probabilities. [3]



Initially: Box A: $P(R) = \frac{7}{8}$ Box B: $P(R) = \frac{9}{14}$

$P(B) = \frac{1}{8}$ $P(B) = \frac{5}{14}$

(b) Find the probability that the two balls chosen are not the same colour. [2]

$P(RB) + P(BR) = \frac{7}{24} + \frac{3}{40}$

$= \frac{11}{30}$

(c) Find the probability that the ball chosen from box A is blue given that the ball chosen from box B is blue. [4]

$P(B_A)$ $P(B_B)$

$P(B_A \cap B_B) = P(B_A | B_B) \times P(B_B)$

$P(B_A | B_B) = \frac{P(B_A \cap B_B)}{P(B_B)}$

$P(B_A \cap B_B) = \frac{1}{8} \times \frac{6}{15}$

$= \frac{1}{20}$

$P(B_B) = \frac{7}{24} + \frac{1}{20}$

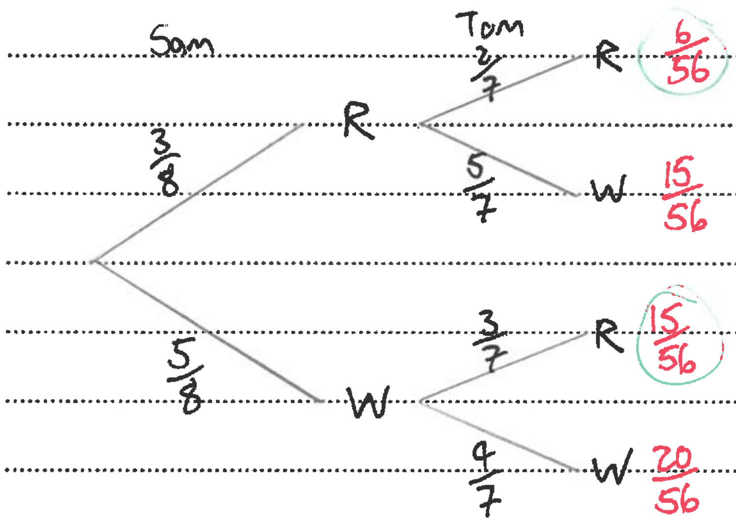
$= \frac{41}{120}$

$P(B_A | B_B) = \frac{\frac{1}{20}}{\frac{41}{120}} = \frac{6}{41}$

7 Sam and Tom are playing a game which involves a bag containing 5 white discs and 3 red discs. They take turns to remove one disc from the bag at random. Discs that are removed are not replaced into the bag. The game ends as soon as one player has removed two red discs from the bag. That player wins the game.

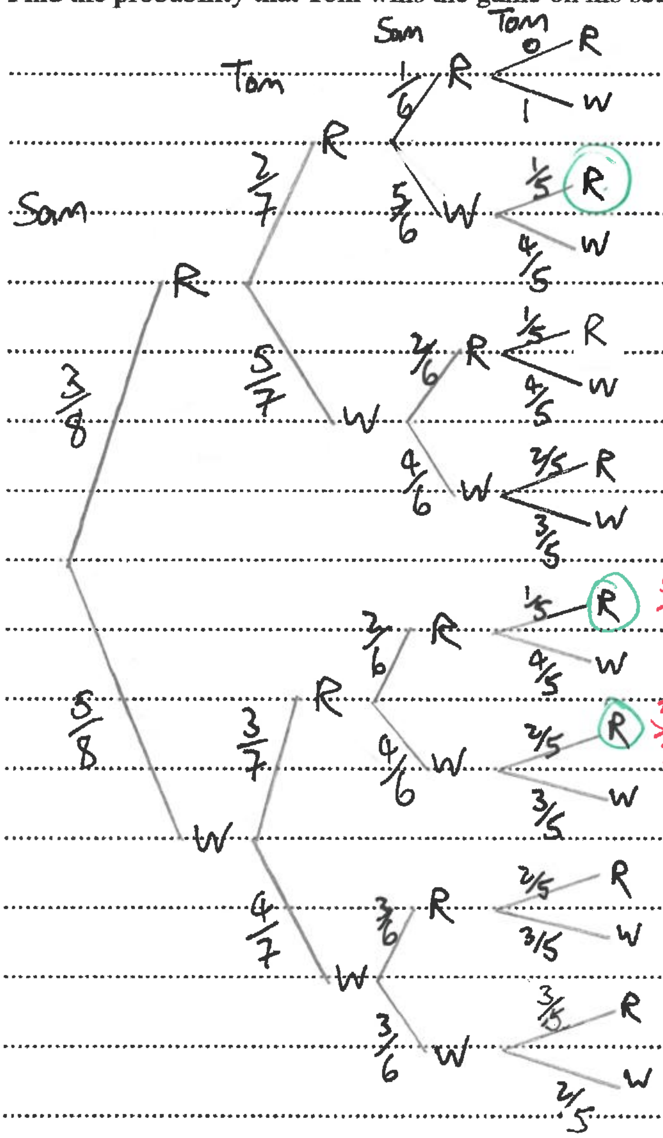
Sam removes the first disc.

(a) Find the probability that Tom removes a red disc on his first turn. [2]



$$\frac{6}{56} + \frac{15}{56} = \frac{21}{56}$$

(b) Find the probability that Tom wins the game on his second turn. [4]



$\frac{3}{8} \times \frac{2}{7} \times \frac{5}{6} \times \frac{1}{5} = \frac{1}{56}$

$\frac{5}{8} \times \frac{3}{7} \times \frac{2}{6} \times \frac{1}{5} = \frac{1}{56}$

$\frac{5}{8} \times \frac{3}{7} \times \frac{4}{6} \times \frac{2}{5} = \frac{4}{56}$

$\frac{1}{56} + \frac{1}{56} + \frac{4}{56} = \frac{6}{56}$

$= \frac{3}{28}$

(c) Find the probability that Sam removes a red disc on his first turn given that Tom wins the game on his second turn. [2]

P(X)

P(Y)

$P(X \cap Y) = P(X|Y) \times P(Y)$

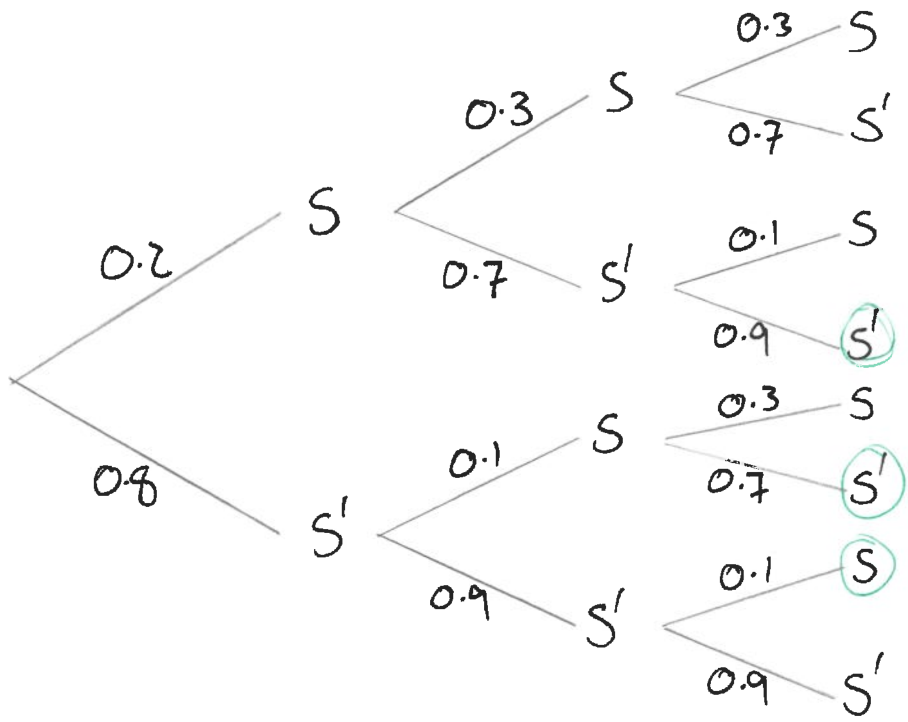
$P(X|Y) = \frac{P(X \cap Y)}{P(Y)}$

$= \frac{\frac{1}{56}}{3/28}$

$= \frac{1}{6}$

6 Sajid is practising for a long jump competition. He counts any jump that is longer than 6 m as a success. On any day, the probability that he has a success with his first jump is 0.2. For any subsequent jump, the probability of a success is 0.3 if the previous jump was a success and 0.1 otherwise. Sajid makes three jumps.

(a) Draw a tree diagram to illustrate this information, showing all the probabilities. [2]



(b) Find the probability that Sajid has exactly one success given that he has at least one success. [5]

$P(X)$ $P(Y)$

$P(X)$: see ◯ on diagram

$P(X) = 0.2 \times 0.7 \times 0.9 + 0.8 \times 0.1 \times 0.7 + 0.8 \times 0.9 \times 0.1$

$= 0.126 + 0.056 + 0.072$

$= \underline{0.254}$

$P(Y) = 1 - P(S'S'S')$

$= 1 - 0.8 \times 0.9 \times 0.9$

$= 1 - 0.648$

$= \underline{0.352}$

$P(X|Y) = \dfrac{P(X \cap Y)}{P(Y)}$

$= \dfrac{0.254}{0.352}$

$= \underline{0.722}$

$P(X \cap Y) = P(X|Y) \times P(Y)$

On another day, Sajid makes six jumps.

(c) Find the probability that only his first three jumps are successes or only his last three jumps are successes. [3]

$P(SSSS'S'S') = 0.2 \times 0.3 \times 0.3 \times 0.7 \times 0.9 \times 0.9 = 0.010206$

$P(S'S'S'SSS) = 0.8 \times 0.9 \times 0.9 \times 0.1 \times 0.3 \times 0.3 = 0.005832$

$P(SSSS'S'S') + P(S'S'S'SSS) = \underline{0.016038}$

3 Tim has two bags of marbles, A and B.

Bag A contains 8 white, 4 red and 3 yellow marbles.

Bag B contains 6 white, 7 red and 2 yellow marbles.

Tim also has an ordinary fair 6-sided dice. He rolls the dice. If he obtains a 1 or 2, he chooses two marbles at random from bag A, without replacement. If he obtains a 3, 4, 5 or 6, he chooses two marbles at random from bag B, without replacement.

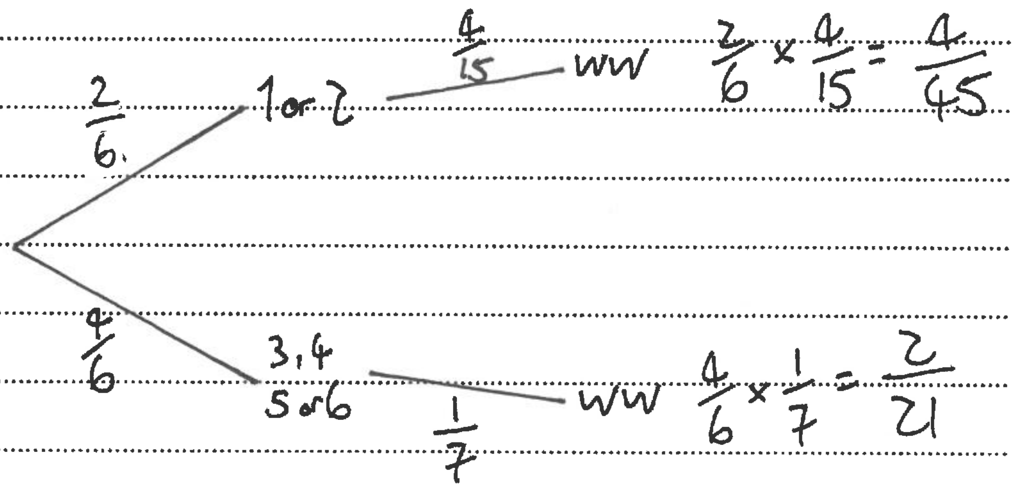
(a) Find the probability that both marbles are white. [3]

Bag A:

$$P(WW) = \frac{8}{15} \times \frac{7}{14} = \frac{4}{15}$$

Bag B:

$$P(WW) = \frac{6}{15} \times \frac{5}{14} = \frac{1}{7}$$



$$\frac{4}{45} + \frac{2}{21} = \frac{58}{315}$$

(b) Find the probability that the two marbles come from bag B given that one is white and one is red. [4]

$P(B \cap WR) = P(B|WR) \times P(WR)$

or RW

$P(B|WR) = \dfrac{P(B \cap WR)}{P(WR)}$

Bag A:

$P(WR)$ in any order $= P(WR) + P(RW)$

$= \frac{8}{15} \times \frac{4}{14} + \frac{4}{15} \times \frac{8}{14}$

$= \frac{32}{105}$

or RW

$P(A \cap WR) = \frac{2}{6} \times \frac{32}{105}$

$= \frac{32}{315}$

Bag B:

$P(WR)$ in any order $= P(WR) + P(RW)$

$= \frac{6}{15} \times \frac{7}{14} + \frac{7}{15} \times \frac{6}{14}$

$= \frac{2}{5}$

or RW

$P(B \cap WR) = \frac{4}{6} \times \frac{2}{5}$

$= 4/15$

$P(B|WR) = \dfrac{\frac{4}{15}}{\frac{32}{315} + \frac{4}{15}}$

or RW

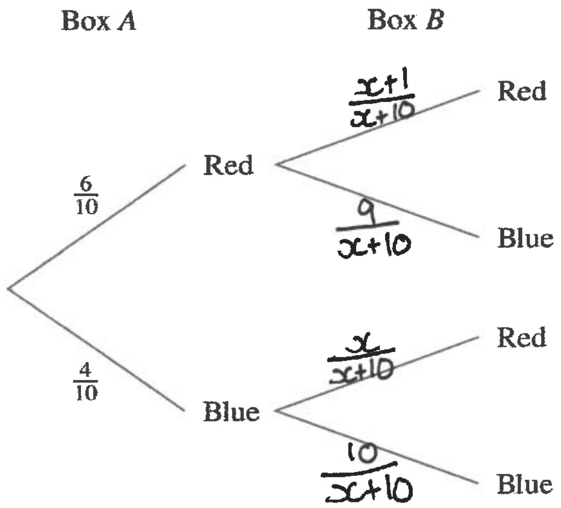
$= \frac{21}{29}$

7 Box A contains 6 red balls and 4 blue balls. Box B contains x red balls and 9 blue balls. A ball is chosen at random from box A and placed in box B. A ball is then chosen at random from box B.

(a) Complete the tree diagram below, giving the remaining four probabilities in terms of x. [3]

Box B: $\frac{x}{x+9}$ red

$\frac{9}{x+9}$ blue



(b) Show that the probability that both balls chosen are blue is $\frac{4}{x+10}$. [2]

$$P(B \cap B) = \frac{4}{\cancel{10}} \times \frac{\cancel{10}}{x+10}$$

$$= \underline{\underline{\frac{4}{x+10}}} \quad \text{QED}$$

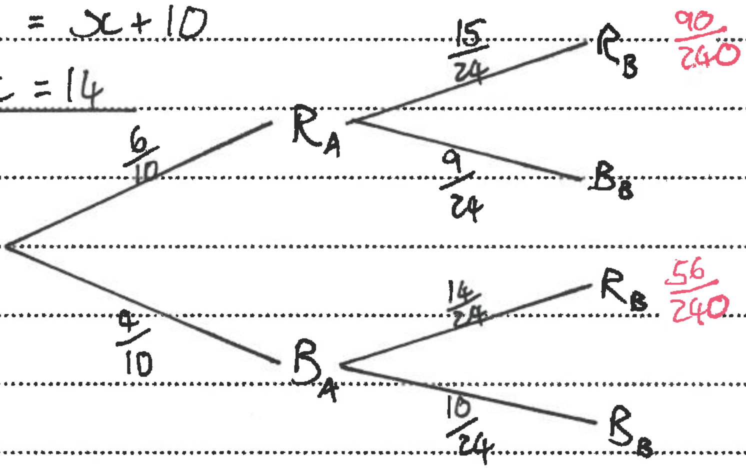
It is given that the probability that both balls chosen are blue is $\frac{1}{6}$.

(c) Find the probability, correct to 3 significant figures, that the ball chosen from box A is red given that the ball chosen from box B is red. [5]

$$\frac{4}{x+10} = \frac{1}{6}$$

$$24 = x + 10$$

$$x = 14$$



$$P(R_A \cap R_B) = P(R_A | R_B) \times P(R_B)$$

$$P(R_A | R_B) = \frac{P(R_A \cap R_B)}{P(R_B)}$$

$$= \frac{90/240}{\frac{90}{240} + \frac{56}{240}}$$

$$= \frac{90}{146}$$

$$= \frac{45}{73}$$

$$= \underline{0.616}$$