1 A crane is used to raise a block of mass 600 kg vertically upwards at a constant speed through a height of 15 m. There is a resistance to the motion of the block, which the crane does 10 000 J of work to overcome.

**(a)** Find the total work done by the crane. [2]

$$\text{Work}_{IN} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + \text{Work}_{out}$$

$$W + \tfrac{1}{2}(600)v^2 + 0 = \tfrac{1}{2}(600)v^2 + 600 \times 10 \times 15 + 10\,000$$

$$W + \cancel{300v^2} = \cancel{300v^2} + 90\,000 + 10\,000$$

$$W = \underline{\underline{100\,000\,J}}$$

**(b)** Given that the average power exerted by the crane is 12.5 kW, find the total time for which the block is in motion. [2]

$$\text{Power} = \frac{\text{Work done}}{\text{time}}$$

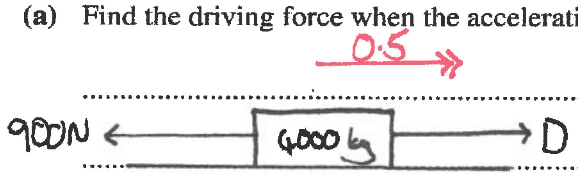
$$12\,500 = \frac{100\,000}{t}$$

$$12\,500t = 100\,000$$

$$t = \underline{8s}$$

2 A minibus of mass 4000 kg is travelling along a straight horizontal road. The resistance to motion is 900 N.

(a) Find the driving force when the acceleration of the minibus is $0.5\,\mathrm{m\,s^{-2}}$. [2]




$R(\rightarrow)$: $F = ma$

$D - 900 = 4000 \times 0.5$

$D - 900 = 2000$

$D = 2900\ N$

(b) Find the power required for the minibus to maintain a constant speed of $25\,\mathrm{m\,s^{-1}}$. [2]

At constant speed, $a = 0$:

$R(\rightarrow)$: $D - 900 = 0$

$D = 900N$

Power $= D \times V$

$= 900 \times 25$

$= 22\,500\ W$

**1** A lorry of mass 16 000 kg is travelling along a straight horizontal road. The engine of the lorry is working at constant power. The work done by the driving force in 10 s is 750 000 J.

**(a)** Find the power of the lorry's engine. [1]

$$\text{Power} = \frac{\text{work done}}{\text{time}}$$

$$= \frac{750\,000}{10}$$

$$= \underline{75\,000\,\text{W}}$$

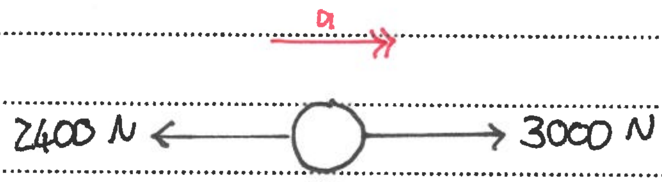
**(b)** There is a constant resistance force acting on the lorry of magnitude 2400 N.

Find the acceleration of the lorry at an instant when its speed is 25 m s$^{-1}$. [3]

$\text{Power} = Dv$

$75\,000 = D \times 25$

$D = 3000\,\text{N}$



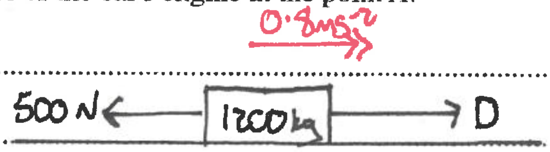

$R(\rightarrow)$: $3000 - 2400 = ma$

$600 = 16000a$

$\underline{a = 0.0375\,\text{ms}^{-2}}$

4 A car of mass 1200 kg is travelling along a straight horizontal road $AB$. There is a constant resistance force of magnitude 500 N. When the car passes point $A$, it has a speed of $15\,\mathrm{m\,s^{-1}}$ and an acceleration of $0.8\,\mathrm{m\,s^{-2}}$.

**(a)** Find the power of the car's engine at the point $A$. [3]




$R(\rightarrow):\quad F = ma$

$D - 500 = 1200 \times 0.8$

$D - 500 = 960$

$D = 1460\,\mathrm{N}$

$\text{Power} = Dv$

$= 1460 \times 15$

$= \underline{21900\,\mathrm{W}}$

The car continues to work with this power as it travels from $A$ to $B$. The car takes 53 seconds to travel from $A$ to $B$ and the speed of the car at $B$ is $32\,\mathrm{m\,s^{-1}}$.

**(b)** Show that the distance $AB$ is 1362.6 m. [3]

Work done by engine $=$ power $\times$ time

$= 21900 \times 53$

$= 1160700\,\mathrm{J}$

$\text{Work}_{IN} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + \text{Work}_{OUT}$

$1160700 + \frac{1}{2}\times 1200 \times 15^2 + 0 = \frac{1}{2}\times 1200 \times 32^2 + 0 + 500d$

$F \times d$

$1160700 + 135000 = 614400 + 500d$

$1295700 = 614400 + 500d$

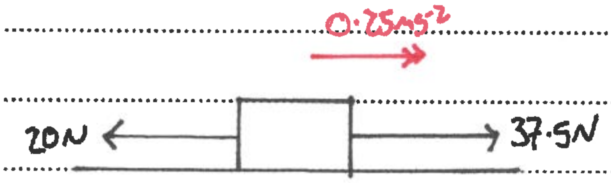
$500d = 681300$

$d = \underline{1362.6\,\mathrm{m}}$ QED

2 A cyclist is travelling along a straight horizontal road. She is working at a constant rate of 150 W. At an instant when her speed is $4\,\text{m}\,\text{s}^{-1}$, her acceleration is $0.25\,\text{m}\,\text{s}^{-2}$. The resistance to motion is 20 N.

**(a)** Find the total mass of the cyclist and her bicycle. [3]

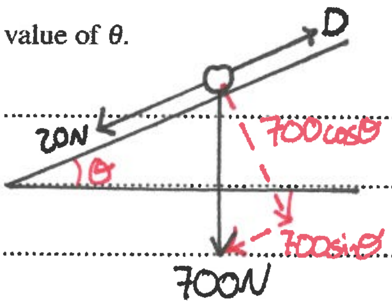
Power = D × V
150 = D × 4
D = 37.5 N




R(→): F = ma
37.5 − 20 = m × 0.25
17.5 = m × 0.25
m = 70 kg

The cyclist comes to a straight hill inclined at an angle $\theta$ above the horizontal. She ascends the hill at constant speed $3\,\text{m}\,\text{s}^{-1}$. She continues to work at the same rate as before and the resistance force is unchanged.

**(b)** Find the value of $\theta$. [2]




Power = D × V
150 = D × 3
D = 50 N

R(↗): $50 - 700\sin\theta - 20 = ma$ ← a = 0 (constant speed)
$30 - 700\sin\theta = 0$
$700\sin\theta = 30$
$\sin\theta = \frac{30}{700}$
$\theta = 2.5°$ (1dp)

**4** An athlete of mass 84 kg is running along a straight road.

**(a)** Initially the road is horizontal and he runs at a constant speed of $3\,\mathrm{m\,s^{-1}}$. The athlete produces a constant power of 60 W.

Find the resistive force which acts on the athlete. [1]

Power = $D \times V$

$60 = D \times 3$

$\underline{D = 20\,\mathrm{N}}$

Constant speed, so $a = 0$, so

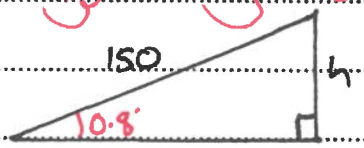
$D$ = resistive force:

resistive force = $\underline{20\,\mathrm{N}}$

**(b)** The athlete then runs up a 150 m section of the road which is inclined at 0.8° to the horizontal. The speed of the athlete at the start of this section of road is $3\,\mathrm{m\,s^{-1}}$ and he now produces a constant driving force of 24 N. The total resistive force which acts on the athlete along this section of road has constant magnitude 13 N.

Use an energy method to find the speed of the athlete at the end of the 150 m section of road. [6]

Change in height over 150m:




$\sin 0.8 = \frac{h}{150}$

$\underline{h = 150\sin 0.8}$

$\text{Work}_{IN} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + \text{Work}_{OUT}$

$24 \times 150 + \frac{1}{2} \times 84 \times 3^2 + 0 = \frac{1}{2} \times 84 \times V^2 + 84 \times 10 \times 150\sin 0.8 + 13 \times 150$

Fxd

Fxd

$3600 + 378 = 42V^2 + 1759.2 + 1950$

$3978 = 42V^2 + 3709.2$

$268.77 = 42V^2$

$V^2 = 6.399$

$\underline{V = 2.53\,\mathrm{ms^{-1}}}$

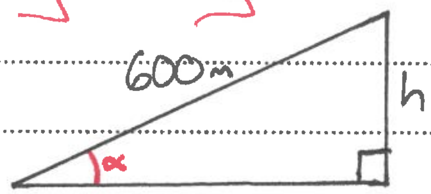
$\sin\alpha = 0.12$

5 A car of mass 1600 kg travels at constant speed $20\,\mathrm{m\,s^{-1}}$ up a straight road inclined at an angle of $\sin^{-1} 0.12$ to the horizontal.

**(a)** Find the change in potential energy of the car in 30 s. [3]

Constant speed, so distance = speed × time: $d = 20 \times 30$
$= 600\,\mathrm{m}$

Change in height when travelling 600m:




$\sin\alpha = \frac{h}{600}$

$h = 600\sin\alpha$
$= 600 \times 0.12$
$= 72\,\mathrm{m}$

Change in PE $= mg\Delta h$
$= 1600 \times 10 \times 72$
$= 1\,152\,000\,\mathrm{J}$ (increase)

**(b)** Given that the total work done by the engine of the car in this time is 1960 kJ, find the constant force resisting the motion. [3]

$\text{Work}_{IN} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + \text{Work}_{OUT}$

$1960000 + \frac{1}{2}\times1600\times20^2 + 0 = \frac{1}{2}\times1600\times20^2 + 1152000 + 600F$

$F \times d$

$1960000 + \cancel{320000} = \cancel{320000} + 1152000 + 600F$

$1960000 = 1152000 + 600F$

$808000 = 600F$

$F = 1350\,\mathrm{N}$ (3sf)

STO

**(c)** Calculate, in kW, the power developed by the engine of the car. [2]

$$\text{Power} = \frac{\text{Work done}}{\text{time}}$$

$$= \frac{1\,960\,000}{30}$$

$$= 65333 \text{ W}$$

STO

$$= 65.3 \text{ kW (3sf)}$$

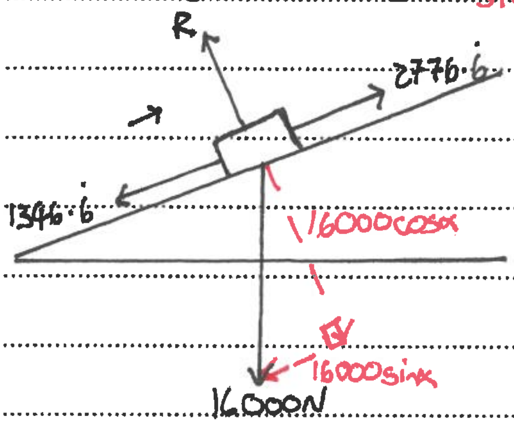
**(d)** Given that this power is suddenly decreased by 15%, find the instantaneous deceleration of the car. [3]

$65333 \times 0.85 = 55533$

$\text{Power} = Dv$

$55533 = D \times 20$

$D = 2776.\dot{6}$ STO




$R(\nearrow)$: $F = ma$

$2776.\dot{6} - 1346.\dot{6} - 16000\sin\alpha = 1600a$

$1430 - 16000\sin\alpha = 1600a$

$1430 - 16000 \times 0.12 = 1600a$

$1430 - 1920 = 1600a$

$-490 = 1600a$

$a = -0.306 \text{ ms}^{-2}$

$\rightarrow$ deceleration $= 0.306 \text{ ms}^{-2}$

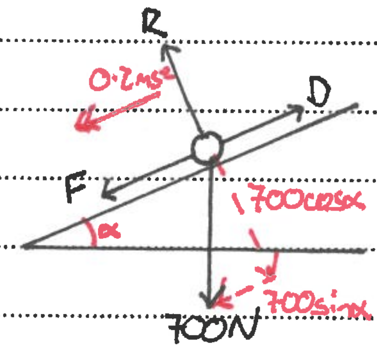
4 The total mass of a cyclist and her bicycle is 70 kg. The cyclist is riding with constant power of 180 W up a straight hill inclined at an angle $\alpha$ to the horizontal, where $\sin\alpha = 0.05$. At an instant when the cyclist's speed is $6\,\text{m}\,\text{s}^{-1}$, her acceleration is $-0.2\,\text{m}\,\text{s}^{-2}$. There is a constant resistance to motion of magnitude $F$ N.

**(a)** Find the value of $F$. [4]

Driving Force of cyclist: Power = $Dv$

$180 = D \times 6$

$D = 30N$




$R(\nearrow)$: $30 - F - 700\sin\alpha = ma$

$30 - F - 700 \times 0.05 = 70 \times -0.2$

$30 - F - 35 = -14$

$-F - 5 = -14$

$-F = -9$

$F = 9N$

**(b)** Find the steady speed that the cyclist could maintain up the hill when working at this power. [2]

At steady speed, acceleration = 0:

$R(\nearrow)$: $D - F - 700\sin\alpha = ma$

$D - 9 - 35 = 0$

$D - 44 = 0$

$D = 44\text{ N}$

Power = $Dv$

$180 = 44v$

$v = 4.09\text{ ms}^{-1}$

5 Two racing cars $A$ and $B$ are at rest alongside each other at a point $O$ on a straight horizontal test track. The mass of $A$ is 1200 kg. The engine of $A$ produces a constant driving force of 4500 N. When $A$ arrives at a point $P$ its speed is $25\,\mathrm{m\,s^{-1}}$. The distance $OP$ is $d$ m. The work done against the resistance force experienced by $A$ between $O$ and $P$ is 75 000 J.

(a) Show that $d = 100$. [3]

$$\text{Work}_{IN} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + \text{Work}_{out}$$

$$4500d + 0 + 0 = \tfrac{1}{2}\times 1200 \times 25^2 + 0 + 75000$$

$$4500d = 375000 + 75000$$

$$4500d = 450000$$

$$\underline{d = 100\text{m}}$$

Car $B$ starts off at the same instant as car $A$. The two cars arrive at $P$ simultaneously and with the same speed. The engine of $B$ produces a driving force of 3200 N and the car experiences a constant resistance to motion of 1200 N.

**(b)** Find the mass of $B$. [3]

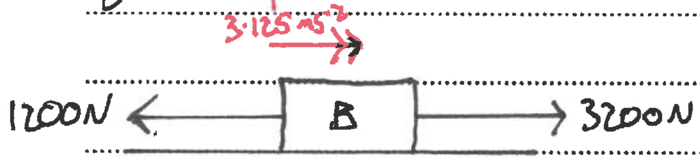
$s = 100$ | $v^2 = u^2 + 2as$

$u = 0$ | $25^2 = 0^2 + 2 \times a \times 100$

$v = 25$ | $625 = 200a$

$a =$ | $a = 3.125\ \text{ms}^{-2}$

$t =$




$R(\rightarrow): 3200 - 1200 = ma$

$2000 = m \times 3.125$

$m = 640\ \text{kg}$

**(c)** Find the steady speed which $B$ can maintain when its engine is working at the same rate as it is at $P$. [3]

$\text{Power} = D \times V$

$= 3200 \times 25$

$= 80\,000\ \text{W}$

At steady speed, acceleration = 0, so driving force equals resistance to motion = 1200 N:

$\text{Power} = D \times V$

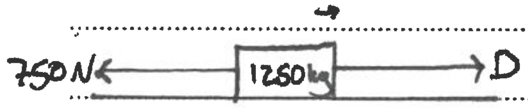
$80\,000 = 1200 \times V$

$V = 66.7\ \text{ms}^{-1}$

**5** A car of mass 1250 kg is moving on a straight road.

**(a)** On a horizontal section of the road, the car has a constant speed of 32 m s$^{-1}$ and there is a constant force of 750 N resisting the motion.

**(i)** Calculate, in kW, the power developed by the engine of the car. [2]




$R(\rightarrow): F = ma$ $=0$ (constant speed)

$D - 750 = 0$

$D = 750N$

$\text{Power} = D \times V$

$= 750 \times 32$

$= 24\,000W$

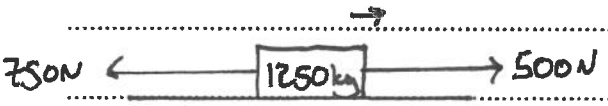
$= \underline{24kW}$

**(ii)** Given that this power is suddenly decreased by 8 kW, find the instantaneous deceleration of the car. [3]

$\text{Power} = D \times v$

$16000 = D \times 32$

$D = 500N$




$R(\rightarrow): F = ma$

$500 - 750 = 1250a$

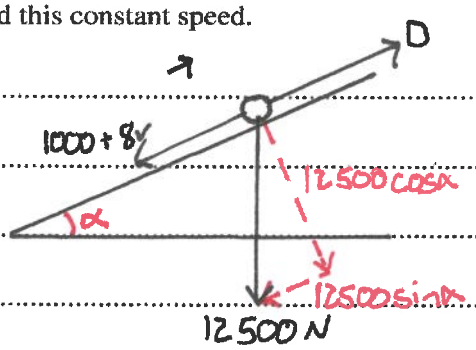
$-250 = 1250a$

$a = \underline{-0.2ms^{-2}}$

sin α = 0.096

**(b)** On a section of the road inclined at $\sin^{-1} 0.096$ to the horizontal, the resistance to the motion of the car is $(1000 + 8v)$ N when the speed of the car is $v$ m s$^{-1}$. The car travels up this section of the road at constant speed with the engine working at 60 kW.

Find this constant speed. [5]




=0 (constant speed)

$R(\nearrow)$: $D - 12500\sin\alpha - (1000 + 8v) = ma$

$D - 12500 \times 0.096 - 1000 - 8v = 0$

$D - 1200 - 1000 - 8v = 0$

$D - 2200 - 8v = 0$

$D = 2200 + 8v$

Power $= D \times v$

$60\,000 = (2200 + 8v)v$

$60\,000 = 2200v + 8v^2$

$8v^2 + 2200v - 60\,000 = 0$

$v^2 + 275v - 7500 = 0$

$v = \dfrac{-275 \pm \sqrt{(275)^2 - 4(1)(-7500)}}{2(1)}$

$v = 25$ ms$^{-1}$ ✓ or $v = -300$ ms$^{-1}$ ✗

wrong direction