1 (a) Find the number of different arrangements of the 8 letters in the word DECEIVED in which all three Es are together and the two Ds are together. [2]

DDEEECIV

EEE DD

one object

one object

$5! = \underline{120}$

(b) Find the number of different arrangements of the 8 letters in the word DECEIVED in which the three Es are not all together. [4]

With 3 Es together:

EEE

one object

$\frac{6!}{2!} = 360$

2 Ds

No restrictions:

$\frac{8!}{3! \times 2!} = 3360$

3 Es

2Ds

$3360 - 360 = \underline{3000}$

2 (a) Find the number of different arrangements that can be made from the 9 letters of the word JEWELLERY in which the three Es are together and the two Ls are together. [2]

JEEEWLLRY

one object → EEE

LL ← one object

$6! = \underline{720}$

(b) Find the number of different arrangements that can be made from the 9 letters of the word JEWELLERY in which the two Ls are not next to each other. [4]

Ls together:

one object → LL

$\frac{8!}{3!} = 6\ 720$

3 Es → 3!

No restrictions:

$\frac{9!}{3! \times 2!} = 30\ 240$

3 Es → 3!, 2 Ls → 2!

$30\ 240 - 6720 = \underline{23\ 520}$

3 (a) How many different arrangements are there of the 8 letters in the word RELEASED? [1]

REEELASD

$$\frac{8!}{3!} = \underline{6720}$$

3 Es

(b) How many different arrangements are there of the 8 letters in the word RELEASED in which the letters LED appear together in that order? [3]

LED

one object

$$\frac{6!}{2!} = \underline{360}$$

remaining 2 Es

(c) An arrangement of the 8 letters in the word RELEASED is chosen at random.

Find the probability that the letters A and D are not together. [4]

A and D together:

AD

one object

$$\frac{7!}{3!} \times 2! = 1680$$

3 Es

could be AD or DA

Without restrictions:

$$\frac{8!}{3!} = 6720$$

$\rightarrow$ $6720 - 1680 = 5040$

$$\text{probability} = \frac{5040}{6720}$$

$$= \frac{3}{4}$$

6 (a) Find the total number of different arrangements of the 11 letters in the word CATERPILLAR. [2]

CAATERRPILL

$$\frac{11!}{2! \times 2! \times 2!} = \underline{4\,989\,600}$$

2 As 2 Rs 2 Ls

(b) Find the total number of different arrangements of the 11 letters in the word CATERPILLAR in which there is an R at the beginning and an R at the end, and the two As are not together. [4]

With As together:

R _ _ _ _ _ _ _ _ R

fixed (AA) fixed

one object

$$\frac{8!}{2!} = 20\,160$$

2 Ls

With no restrictions (apart from Rs at ends):

R _ _ _ _ _ _ _ _ _ R

fixed fixed

$$\frac{9!}{2! \times 2!} = 90\,720$$

2 As 2 Ls

$90\,270 - 20\,160 = \underline{70\,560}$

(c) Find the total number of different selections of 6 letters from the 11 letters of the word CATERPILLAR that contain both Rs and at least one A and at least one L. [4]

1A, 1L: R R A L _ _ $^5C_2 = 10$

pick two letters from CTEPI

2A, 1L: R R A A L _ $^5C_1 = 5$

1A, 2L: R R A L L _ $^5C_1 = 5$

2A, 2L: R R A A L L $^5C_0 = 1$

$10 + 5 + 5 + 1 = \underline{\underline{26}}$

3 (a) Find the number of different arrangements of the 8 letters in the word COCOONED. [1]

CCOOONED

$$\frac{8!}{2! \times 3!} = \underline{3360}$$

2Cs → ← 3Os

(b) Find the number of different arrangements of the 8 letters in the word COCOONED in which the first letter is O and the last letter is N. [2]

O _ _ _ _ _ _ N

fixed fixed

$$\frac{6!}{2! \times 2!} = \underline{180}$$

2Cs → ← 2Os in the middle

(c) Find the probability that a randomly chosen arrangement of the 8 letters in the word COCOONED has all three Os together given that the two Cs are next to each other. [3]

$$P(O \cap C) = P(O|C) \times P(C)$$

$$P(O|C) = \frac{P(O \cap C)}{P(C)}$$

Probability of Cs together:

one object → CC

$$\frac{7!}{3!} = 840$$

3Os → 3!

Probability of Os together and Cs together:

one object → OOO   CC one object

$$5! = 120$$

Without restrictions = 3360 (from part (a))

$$P(C) = \frac{840}{3360} \qquad P(O \cap C) = \frac{120}{3360}$$

$$P(O|C) = \frac{\frac{120}{3360}}{\frac{840}{3360}}$$

$$= \frac{1}{7}$$

7 (a) Find the number of different arrangements of the 9 letters in the word ANDROMEDA in which no consonant is next to another consonant. (The letters D, M, N and R are consonants and the letters A, E and O are **not** consonants.) [3]

AANDDROME

Arrange vowels:

$$\frac{4!}{2!} = 12$$

2 As $\rightarrow$ 2!

Arrange consonants in spaces between vowels:

↑ V ↑ V ↑ V ↑ V ↑

5 spaces $\rightarrow$ ${}^5P_5$ $\leftarrow$ 5 consonants

$$\frac{{}^5P_5}{2!} = 60$$

2 Ds $\rightarrow$ 2!

$12 \times 60 = \underline{720}$

(b) Find the number of different arrangements of the 9 letters in the word ANDROMEDA in which there is an A at each end and the Ds are **not** together. [3]

With Ds together:

A (fixed) DD _ _ _ _ _ A (fixed)

one object $\rightarrow$ DD

$6! = 720$

With no restrictions (except for As at ends):

A (fixed) _ _ _ _ _ _ _ A (fixed)

$$\frac{7!}{2!} = 2520$$

2 Ds $\rightarrow$ 2!

$2520 - 720 = \underline{1800}$

Four letters are selected at random from the 9 letters in the word ANDROMEDA.

(c) Find the probability that this selection contains at least one D and exactly one A. [4]

We are picking letters at random, not looking for different selections, so the As and Ds are treated as distinguishable.

1 D from 2

A D __ __ $\quad {}^2C_1 \times {}^2C_1 \times {}^5C_2 = 40$

1 A from 2 $\quad$ 2 from NROME

A D D __ $\quad {}^2C_1 \times {}^2C_2 \times {}^5C_1 = 10$

$40 + 10 = 50$

Number of selections without restrictions $= {}^9C_4$

$= 126$

Probability $= \dfrac{50}{126}$

$= \dfrac{25}{63}$

7 (a) Find the number of different possible arrangements of the 9 letters in the word CELESTIAL. [1]

CEELLSTIA

$$\frac{9!}{\underset{\uparrow\ 2\text{ Es}}{2!} \times \underset{\uparrow\ 2\text{ Ls}}{2!}} = \underline{90\,720}$$

(b) Find the number of different arrangements of the 9 letters in the word CELESTIAL in which the first letter is C, the fifth letter is T and the last letter is E. [2]

$\underset{\text{fixed}}{\text{C}}$ _ _ _ $\underset{\text{fixed}}{\text{T}}$ _ _ _ $\underset{\text{fixed}}{\text{E}}$

$$\frac{6!}{\underset{\uparrow\ 2\text{ Ls}}{2!}} = \underline{360}$$

(c) Find the probability that a randomly chosen arrangement of the 9 letters in the word CELESTIAL does not have the two Es together. [4]

Two Es together:

EE _ _ _ _ _ _ _

one object → EE

$$\frac{8!}{\underset{\uparrow\ 2\text{ Ls}}{2!}} = 20\,160$$

No restrictions: 90 720 (from part (a))

Es not together: 90720 − 20160 = 70560

probability $= \frac{70560}{90\,720} = \underline{\frac{7}{9}}$

5 letters are selected at random from the 9 letters in the word CELESTIAL.

(d) Find the number of different selections if the 5 letters include at least one E and at most one L. [3]

1 E, 0L: E _ _ _ _ $\quad {}^5C_4 = 5$

↑pick 4 from CSTIA

2E, 0L: E E _ _ _ $\quad {}^5C_3 = 10$

1E, 1L: E L _ _ _ $\quad {}^5C_3 = 10$

2E, 1L: E E L _ _ $\quad {}^5C_2 = 10$

$5 + 10 + 10 + 10 = \underline{\underline{35}}$

5 The 8 letters in the word RESERVED are arranged in a random order.

(a) Find the probability that the arrangement has V as the first letter and E as the last letter. [3]

RREEESVD

fixed fixed

V _ _ _ _ _ _ E

$\frac{6!}{2! \times 2!} = 180$

two Rs two Es

Without restrictions:

$\frac{8!}{2! \times 3!} = 3360$

two Rs three Es

probability $= \frac{180}{3360} = \frac{3}{56}$

(b) Find the probability that the arrangement has both Rs together given that all three Es are together. [4]

P(R) P(E)

$P(R \cap E) = P(R|E) \times P(E)$

$P(R|E) = \frac{P(R \cap E)}{P(E)}$

P(E): All Es together

P(R∩E): Es together and Rs together

one object → EEE

$\frac{6!}{2!} = 360$ ← 2 Rs

EEE RR

one object one object

$5! = 120$

$P(R|E) = \frac{\frac{120}{3360}}{\frac{360}{3360}} = \frac{1}{3}$

6 (a) Find the number of different ways in which the 10 letters of the word SUMMERTIME can be arranged so that there is an E at the beginning and an E at the end. [2]

SUMMMEERTI

E _ _ _ _ _ _ _ _ E

fixed fixed

$$\frac{8!}{3!} = \underline{6720}$$

3 Ms →

(b) Find the number of different ways in which the 10 letters of the word SUMMERTIME can be arranged so that the Es are not together. [4]

Es together:

_ _ _ _ _ _ _ _ _

One object → EE

$$\frac{9!}{3!} = 60\,480$$

3 Ms →

No restrictions:

$$\frac{10!}{3! \times 2!} = 302\,400$$

3 Ms → ← 2 Es

$302\,400 - 60\,480 = \underline{241\,920}$

(c) Four letters are selected from the 10 letters of the word SUMMERTIME. Find the number of different selections if the four letters include at least one M and exactly one E. [3]

1M, 1E: M E __ __ $^5C_2 = 10$

pick 2 from SURTI

2M, 1E: M M E __ $^5C_1 = 5$

3M, 1E: M M M E $^5C_0 = 1$

$10 + 5 + 1 = \underline{\underline{16}}$

6 **(a)** Find the total number of different arrangements of the 8 letters in the word TOMORROW. [2]

TOOOMRRW

$$\frac{8!}{2! \times 3!} = \underline{3\,360}$$

2Rs 3Os

(b) Find the total number of different arrangements of the 8 letters in the word TOMORROW that have an R at the beginning and an R at the end, and in which the three Os are not all together. [3]

R _ _ _ _ _ _ R

With three Os together:

R _ _ _ _ R

3 Os are one object → OOO T M W

4 objects arranged in 4 spaces: 4!

With no restrictions:

R _ _ _ _ _ _ R

O O O T M W

$$\frac{6!}{3!}$$

3Os

ANS: $\frac{6!}{3!} - 4! = \underline{96}$

Four letters are selected at random from the 8 letters of the word TOMORROW.

(c) Find the probability that the selection contains at least one O and at least one R. [5]

O R _ _ $\quad {}^3C_1 \times {}^2C_1 \times {}^3C_2 = 18$

1 O from 3 $\quad$ 1 R from 2 $\quad$ 2 letters from T, M, W

O O R _ $\quad {}^3C_2 \times {}^2C_1 \times {}^3C_1 = 18$

O O O R $\quad {}^3C_3 \times {}^2C_1 = 2$

O R R _ $\quad {}^3C_1 \times {}^2C_2 \times {}^3C_1 = 9$

O O R R $\quad {}^3C_2 \times {}^2C_2 = 3$

$18 + 18 + 2 + 9 + 3 = 50$

Number of selections without restrictions $= {}^8C_4$
$= 70$

Probability $= \frac{50}{70}$
$= \frac{5}{7}$

7 (a) Find the number of different ways in which the 10 letters of the word SHOPKEEPER can be arranged so that all 3 Es are together. [2]

SHOPPKEEER

EEE

one object

$$\frac{8!}{2!} = \underline{20\,160}$$

2 Ps

(b) Find the number of different ways in which the 10 letters of the word SHOPKEEPER can be arranged so that the Ps are not next to each other. [4]

Ps next to each other:

PP

one object

$$\frac{9!}{3!} = 60\,480$$

3 Es

No restrictions:

$$\frac{10!}{3! \times 2!} = 302\,400$$

3 Es 2 Ps

$$302\,400 - 60\,480 = \underline{241\,920}$$

(c) Find the probability that a randomly chosen arrangement of the 10 letters of the word SHOPKEEPER has an E at the beginning and an E at the end. [2]

E _ _ _ _ _ _ _ _ E
fixed fixed

$$\frac{8!}{2!} = 20\,160$$

2 Ps → 2!

$$\text{Probability} = \frac{20160}{302\,400} \leftarrow \text{from part (b)}$$

$$= \frac{1}{15}$$

Four letters are selected from the 10 letters of the word SHOPKEEPER.

(d) Find the number of different selections if the four letters include exactly one P. [3]

1P, No Es: P _ _ _ $^5C_3 = 10$

↑ pick 3 from S, H, O, K, R

1P, 1E : P E _ _ $^5C_2 = 10$

1P, 2Es: P E E _ $^5C_1 = 5$

1P, 3Es: P E E E $^5C_0 = 1$

$10 + 10 + 5 + 1 = 26$

Note that we want different selections, so Ps and Es are indistinguishable, so no $^2C_1$ for Ps.

4 (a) In how many different ways can the 9 letters of the word TELESCOPE be arranged? [2]

TEEELSCOP

$$\frac{9!}{3!} = \underline{60\ 480}$$

3Es

(b) In how many different ways can the 9 letters of the word TELESCOPE be arranged so that there are exactly two letters between the T and the C? [4]

T __ __ C (could also be C __ __ T)

Pick two of the remaining letters to go between T and C and permutate: $^7P_2$

Treat T__C as one object and permutate with the remaining 5 letters:

__ __ __ __ __ __

6!

T__C or C__T

$$\rightarrow \quad {}^7P_2 \times \frac{6!}{3!} \times 2!$$

3Es

$$= \underline{10\ 080}$$

6 **(a)** How many different arrangements are there of the 11 letters in the word REQUIREMENT? [2]

RREEEQUIMNT

$$\frac{11!}{2! \times 3!} = 3\,326\,400$$

2 Rs → ← 3 Es

(b) How many different arrangements are there of the 11 letters in the word REQUIREMENT in which the two Rs are together and the three Es are together? [1]

RR EEE

one object → ← one object

$$8! = 40\,320$$

(c) How many different arrangements are there of the 11 letters in the word REQUIREMENT in which there are exactly three letters between the two Rs? [3]

R _ _ _ R

Pick three letters to go between the Rs and permutate: ${}^9P_3$

Treat R _ _ _ R as one object and permutate with the other 6 letters:

R___R

$7!$

$$\rightarrow {}^9P_3 \times \frac{7!}{3!} = 423\,360$$

← 3 Es

Five of the 11 letters in the word REQUIREMENT are selected.

(d) How many possible selections contain at least two Es and at least one R? [4]

2E, 1R: E E R _ _ $\quad {}^6C_2 = 15$

pick two from QUIMNT

3E, 1R: E E E R _ $\quad {}^6C_1 = 6$

2E, 2R: E E R R _ $\quad {}^6C_1 = 6$

3E, 2R: E E E R R $\quad {}^6C_0 = 1$

$15 + 6 + 6 + 1 = \underline{\underline{28}}$

7 (a) Find the number of different arrangements of the 10 letters in the word CASABLANCA in which the two Cs are **not** together. [3]

CCAAAASBLN

With Cs together:

__ __ __ __ __ __ __ __ __

one object → CC

4 As → $\frac{9!}{4!} = 15120$

No restrictions:

$$\frac{10!}{2! \times 4!} = 75600$$

2Cs → 2! 4! ← 4As

$75600 - 15120 = \underline{60480}$

(b) Find the number of different arrangements of the 10 letters in the word CASABLANCA which have an A at the beginning, an A at the end and exactly 3 letters between the 2 Cs. [3]

Group of letters with Cs:

C __ __ __ C

Choose 3 letters to go between the 2 Cs from the remaining letters (AASBLN) and permutate: $^6P_3$

Now treat this as one object:

A __ __ __ __ A

one object → C___C

$4! \times {}^6P_3 \div 2! = \underline{1440}$

4 spaces, 4 objects

for the 2As in the middle (not the ends)

Five letters are selected from the 10 letters in the word CASABLANCA.

(c) Find the number of different selections in which the five letters include at least two As and at most one C. [3]

2 As A A ___ ___ ___ $^4C_3 = 4$

↑ pick 3 from SBLN

3 As A A A ___ ___ $^4C_2 = 6$

4 As A A A A ___ $^4C_1 = 4$

2As, 1C A A C ___ ___ $^4C_2 = 6$

3As, 1C A A A C ___ $^4C_1 = 4$

4As, 1C A A A A C $^4C_0 = 1$

$4 + 6 + 4 + 6 + 4 + 1 = \underline{\underline{25}}$

7 (a) Find the number of different arrangements of the 9 letters in the word DELIVERED in which the three Es are together and the two Ds are **not** next to each other. [4]

DDEEELIVR

With 3Es together:

one object → EEE

$$2Ds \rightarrow \frac{7!}{2!} = 2520$$

With 3Es together and 2Ds together:

one object → EEE DD ← one object

$$6! = 720$$

$$2520 - 720 = \underline{1800}$$

(b) Find the probability that a randomly chosen arrangement of the 9 letters in the word DELIVERED has exactly 4 letters between the two Ds. [5]

Picking letters at random, NOT looking for different selections, so the Ds and Es are treated as distinguishable:

Group of letters between Ds:

D_1 __ __ __ __ D_2

from 7 remaining letters (excluding Ds)

Choose 4 letters to go between the Ds and permutate: ${}^7P_4$

D_1 and D_2 could be the other way around so: ${}^7P_4 \times 2!$

continued..

Treat the group with Ds as one object:

__ __ __ __

one object → $D_1 - - - - D_2$

$4!$

So the number of arrangement is: ${}^7P_4 \times 2! \times 4! = 40\,320$

The number of arrangements without restrictions is: $9! = 362\,880$
(remember that the Ds and Es are distinguishable)

So probability $= \dfrac{40\,320}{362\,880} = \dfrac{1}{9}$

Five letters are selected from the 9 letters in the word DELIVERED.

(c) Find the number of different selections if the 5 letters include at least one D and at least one E. [3]

1D, 1E D E _ _ _ ${}^4C_3 = 4$
pick 3 from LIVR

2Ds, 1E D D E _ _ ${}^4C_2 = 6$

1D, 2Es D E E _ _ ${}^4C_2 = 6$

2Ds, 2Es D D E E _ ${}^4C_1 = 4$

1D, 3Es D E E E _ ${}^4C_1 = 4$

2Ds, 3Es D D E E E ${}^4C_0 = 1$

$4 + 6 + 6 + 4 + 4 + 1 = 25$

6 **(a)** Find the number of different arrangements of the 9 letters in the word ACTIVATED. [2]

AACTTIVED

$\frac{9!}{2! \times 2!} = \underline{90720}$

2 As → ← 2 Ts

(b) Find the number of different arrangements of the 9 letters in the word ACTIVATED in which there are at least 5 letters between the two As. [3]

5 letters between: A _ _ _ _ _ A _ _ $= \frac{7!}{2!} = 2520$

2Ts

or: _ A _ _ _ _ _ A _ $= \frac{7!}{2!} = 2520$

or: _ _ A _ _ _ _ _ A $= \frac{7!}{2!} = 2520$

6 letters between:

A _ _ _ _ _ _ A _ $= \frac{7!}{2!} = 2520$

or: _ A _ _ _ _ _ _ A $= \frac{7!}{2!} = 2520$

7 letters between:

A _ _ _ _ _ _ _ A $= \frac{7!}{2!} = 2520$

$6 \times 2520 = \underline{15120}$

Five letters are selected at random from the 9 letters in the word ACTIVATED.

(c) Find the probability that the selection does **not** contain more Ts than As. [5]

We are picking letters at random and not looking for different selections, so the As and Ts are distinguishable.

A __ __ __ __ $\quad {}^5C_4 \times {}^2C_1 = 10$

pick 4 from C, I, V, E, D $\quad$ pick one A from the two.

A A __ __ __ $\quad {}^5C_3 \times {}^2C_2 = 10$

3 from CIVED $\quad$ pick both As

A T __ __ __ $\quad {}^5C_3 \times {}^2C_1 \times {}^2C_1 = 40$

one A $\quad$ one T

A A T __ __ $\quad {}^5C_2 \times {}^2C_2 \times {}^2C_1 = 20$

A A T T __ $\quad {}^5C_1 \times {}^2C_2 \times {}^2C_2 = 5$

__ __ __ __ __ $\quad {}^5C_5 = 1$

no As or Ts

$10 + 10 + 40 + 20 + 5 + 1 = 86$

Total number of selections without restrictions $= {}^9C_5$
$= 126$

Probability $= \dfrac{86}{126}$

$= \dfrac{43}{63}$

7 (a) Find the number of different arrangements of the 9 letters in the word ALLIGATOR in which the two As are together and the two Ls are together. [2]

AALLIGTOR

AA LL IGTOR

one object

one object

7 objects in 7 spaces: $7! = 5040$

(b) The 9 letters in the word ALLIGATOR are arranged in a random order.

Find the probability that the two Ls are together and there are exactly 6 letters between the two As. [5]

LL AAIGTOR

one object

LL can't go here as that only leaves 5 letters to go between A and A.

① A _ _ _ _ _ A _

② _ A _ _ _ _ _ A

A _ _ _ _ _ A _

Pick one of IGTOR for this space: ${}^5C_1$

A __ __ __ __ __ A __

Now permutate the remaining 4 from IGTOR with the LL into the 5 spaces: 5!

$= {}^5C_1 \times 5!$

Same working for __ A __ __ __ __ __ A

$\rightarrow {}^5C_1 \times 5! \times 2 = 1200$

Without restrictions: $\frac{9!}{2! \times 2!} = 90720$

$\frac{1200}{90720} = \frac{5}{378}$

2Ls 2As

(c) Find the number of different selections of 5 letters from the 9 letters in the word ALLIGATOR which contain at least one A and at most one L. [3]

Number of different selections, so As and Ls are indistinguishable. Also not permutating as it says selections.

A __ __ __ __ $\quad {}^5C_4 = 5$

pick 4 letters from IGTOR

A A __ __ __ $\quad {}^5C_3 = 10$

A L __ __ __ $\quad {}^5C_3 = 10$

A A L __ __ $\quad {}^5C_2 = 10$

$10 + 10 + 10 + 5 = \underline{35}$

6 (a) Find the number of different arrangements of the 9 letters in the word CROCODILE. [1]

CCROODILE

$$\frac{9!}{2! \times 2!} = \underline{90\,720}$$

2Cs 2Os

(b) Find the number of different arrangements of the 9 letters in the word CROCODILE in which there is a C at each end and the two Os are not together. [3]

With Os together:

C _ _ _ _ _ _ C

fixed (OO) fixed

one object

$6! = 720$

No restrictions (but with Cs at ends):

C _ _ _ _ _ _ _ C

fixed fixed

$$\frac{7!}{2!} = 2520$$

2Os

$2520 - 720 = \underline{1800}$

(c) Four letters are selected from the 9 letters in the word CROCODILE.

Find the number of selections in which the number of Cs is not the same as the number of Os. [3]

1C, No Os C _ _ _ ${}^5C_3 = 10$

↑ pick 3 letters from R, D, I, L, E

2Cs, No Os C C _ _ ${}^5C_2 = 10$

2Cs, 1O C C O _ ${}^5C_1 = 5$

1O O _ _ _ ${}^5C_3 = 10$

$10+10+5+10+10+5$

2Os O O _ _ ${}^5C_2 = 10$ $= \underline{\underline{50}}$

2Os, 1C O O C _ ${}^5C_1 = 5$

(d) Find the number of ways in which the 9 letters in the word CROCODILE can be divided into three groups, each containing three letters, if the two Cs must be in different groups. [3] difficult!

1C with 2Os: C O O C _ _ _ _ _

pick 2 from RDILE → ${}^5C_2 \times {}^3C_3 = \underline{10}$

1C with O, one without C O _ C _ _ O _ _

${}^5C_1 \times {}^4C_2 \times {}^2C_2 = \underline{30}$

each C with O: C O _ C O _ _ _ _

${}^5C_1 \times {}^4C_1 \times {}^3C_3 = 20$

BUT: Imagine we picked R for the first group and D for the second:

C O R C O D _ _ _

We could also have picked D in the first group and R in the second:

C O D C O R _ _ _

Ending up with the same groups!

So we have to account for the fact that the first two groups can give identical results by dividing by 2! (Like if we had two identical letters in a word).

$$\rightarrow \frac{20}{2!} = \underline{10}$$

Two Os without C:

$\underline{C}$ __ __ $\underline{C}$ __ __ $\underline{O}$ $\underline{O}$ __

$${}^{5}C_{2} \times {}^{3}C_{2} \times {}^{1}C_{1} = 30$$

Once again, the first two groups can give identical results, so:

$$\frac{30}{2!} = \underline{15}$$

$$\rightarrow 10 + 30 + 10 + 15 = \underline{65}$$