1 The 40 members of a club include Ranuf and Saed. All 40 members will travel to a concert. 35 members will travel in a coach and the other 5 will travel in a car. Ranuf will be in the coach and Saed will be in the car.

In how many ways can the members who will travel in the coach be chosen? [3]

38 people left to choose from, 34 places left on coach:

$^{38}C_{34} = \underline{73\,815}$

1 The 26 members of the local sports club include Mr and Mrs Khan and their son Abad. The club is holding a party to celebrate Abad's birthday, but there is only room for 20 people to attend.

In how many ways can the 20 people be chosen from the 26 members of the club, given that Mr and Mrs Khan and Abad must be included? [2]

$${}^{3}C_{3} \times {}^{23}C_{17} = \underline{100\,947}$$

pick Mr & Ms Khan and Abad (3 from 3)

pick 17 more people from the remaining 23

1 A bag contains 12 marbles, each of a different size. 8 of the marbles are red and 4 of the marbles are blue.

How many different selections of 5 marbles contain at least 4 marbles of the same colour? [4]

4 Red, 1 Blue:

$${}^{8}C_4 \times {}^{4}C_1 = 280$$

pick 4 reds from 8. pick 1 blue from 4

5 Red, no Blue:

$${}^{8}C_5 = 56$$

4 Blue, 1 Red:

$${}^{4}C_4 \times {}^{8}C_1 = 8$$

5 Blue not possible.

$$280 + 56 + 8 = \underline{344}$$

2 A group of 6 people is to be chosen from 4 men and 11 women.

(a) In how many different ways can a group of 6 be chosen if it must contain exactly 1 man? [2]

$^{4}C_1 \times {}^{11}C_5 = 1848$

pick one man from four

pick five women from eleven

Two of the 11 women are sisters Jane and Kate.

(b) In how many different ways can a group of 6 be chosen if Jane and Kate cannot both be in the group? [3]

Only one of Jane or Kate:

$^{2}C_1 \times {}^{13}C_5 = 2574$

pick one of Jane/Kate

pick five more people from the remaining 13

Neither of Jane or Kate:

$^{13}C_6 = 1716$

pick six people from the thirteen people, excluding Jane + Kate.

$2574 + 1716 = 4290$

2 (a) Find the number of ways in which a committee of 6 people can be chosen from 6 men and 8 women if it must include 3 men and 3 women. [2]

$${}^{6}C_{3} \times {}^{8}C_{3} = \underline{1120}$$

3 men from 6

3 women from 8

A different committee of 6 people is to be chosen from 6 men and 8 women. Three of the 6 men are brothers.

(b) Find the number of ways in which this committee can be chosen if there are no restrictions on the numbers of men and women, but it must include no more than two of the brothers. [3]

No Brothers:

$${}^{3}C_{0} \times {}^{11}C_{6} = 462$$

0 brothers from 3

6 from remaining 11 people

One Brother:

$${}^{3}C_{1} \times {}^{11}C_{5} = 1386$$

Two Brothers:

$${}^{3}C_{2} \times {}^{11}C_{4} = 990$$

$$462 + 1386 + 990 = \underline{2838}$$

4 In a music competition, there are 8 pianists, 4 guitarists and 6 violinists. 7 of these musicians will be selected to go through to the final.

How many different selections of 7 finalists can be made if there must be at least 2 pianists, at least 1 guitarist and more violinists than guitarists? [4]

2P, 1G, 4V: $^8C_2 \times {}^4C_1 \times {}^6C_4 = 1680$

2 pianists from 8 1 guitarist from 4 4 violinists from 6

2P, 2G, 3V: $^8C_2 \times {}^4C_2 \times {}^6C_3 = 3360$

3P, 1G, 3V: $^8C_3 \times {}^4C_1 \times {}^6C_3 = 4480$

4P, 1G, 2V: $^8C_4 \times {}^4C_1 \times {}^6C_2 = 4200$

$1680 + 3360 + 4480 + 4200 = \underline{\underline{13720}}$

4 Richard has 3 blue candles, 2 red candles and 6 green candles. The candles are identical apart from their colours. He arranges the 11 candles in a line.

(a) Find the number of different arrangements of the 11 candles if there is a red candle at each end. [2]

R _ _ _ _ _ _ _ _ _ R

$$\frac{9!}{3! \times 6!} = \underline{84}$$

3 Bs → 3!   6 Gs → 6!

(b) Find the number of different arrangements of the 11 candles if all the blue candles are together and the red candles are not together. [4]

Blue candles together:

_ _ _ _ _ _ _ _ _

One object → BBB

$$\frac{9!}{2! \times 6!} = 252$$

2 Rs → 2!   6 Gs → 6!

Blues and Reds together:

_ _ _ _ _ _ _ _

BBB RR

$$\frac{8!}{6!} = 56$$

6 Gs → 6!

$$252 - 56 = \underline{196}$$

2 There are 6 men and 8 women in a Book Club. The committee of the club consists of five of its members. Mr Lan and Mrs Lan are members of the club.

(a) In how many different ways can the committee be selected if exactly one of Mr Lan and Mrs Lan must be on the committee? [2]

$^{2}C_1 \times {}^{12}C_4 = \underline{990}$

pick one of Mr/Mrs Lan

pick 4 members from the remaining 12 (excluding Mr & Mrs Lan)

(b) In how many different ways can the committee be selected if Mrs Lan must be on the committee and there must be more women than men on the committee? [4]

3 Women (inc. Mrs Lan), 2 Men:

Mrs Lan W W M M

$^{7}C_2 \times {}^{6}C_2 = 315$

pick 2 more women from 7

pick 2 men from 6

4 Women (inc. Mrs Lan), 1 Man:

Mrs Lan W W W M

$^{7}C_3 \times {}^{6}C_1 = 210$

5 Women (inc. Mrs Lan):

Mrs Lan W W W W

$^{7}C_4 = 35$

$315 + 210 + 35 = \underline{560}$

3 A committee of 6 people is to be chosen from 9 women and 5 men.

(a) Find the number of ways in which the 6 people can be chosen if there must be more women than men on the committee. [3]

4W, 2M:

$${}^{9}C_{4} \times {}^{5}C_{2} = 1260$$

pick 4 women from 9 pick 2 men from 5

5W, 1M:

$${}^{9}C_{5} \times {}^{5}C_{1} = 630$$

6W, 0M:

$${}^{9}C_{6} \times {}^{5}C_{0} = 84$$

= 1

$$1260 + 630 + 84 = \underline{1974}$$

The 9 women and 5 men include a sister and brother.

(b) Find the number of ways in which the committee can be chosen if the sister and brother cannot both be on the committee. [3]

With both sister and brother on the committee:

$${}^{2}C_{2} \times {}^{12}C_{4} = 495$$

pick sister and brother pick 4 more from the remaining 12 people

Without restrictions:

$${}^{14}C_{6} = 3003$$

$$3003 - 495 = \underline{2508}$$

5 A group of 12 people consists of 3 boys, 4 girls and 5 adults.

(a) In how many ways can a team of 5 people be chosen from the group if exactly one adult is included? [2]

$^5C_1 \times {}^7C_4 = 175$

pick one adult from 5

pick 4 children from 7

(b) In how many ways can a team of 5 people be chosen from the group if the team includes at least 2 boys and at least 1 girl? [4]

2B, 1G: B B G _ _ $^3C_2 \times {}^4C_1 \times {}^5C_2 = 120$

pick 2 boys from 3 pick one girl pick 2 adults

3B, 1G: B B B G _ $^3C_3 \times {}^4C_1 \times {}^5C_1 = 20$

2B, 2G: B B G G _ $^3C_2 \times {}^4C_2 \times {}^5C_1 = 90$

3B, 2G: B B B G G $^3C_3 \times {}^4C_2 = 6$

2B, 3G: B B G G G $^3C_2 \times {}^4C_3 = 12$

$120 + 20 + 90 + 6 + 12 = 248$

The same group of 12 people stand in a line.

(c) How many different arrangements are there in which the 3 boys stand together and an adult is at each end of the line? [4]

Pick two adults for the ends and permutate: $^5P_2$

Treat the three boys as one object and permutate with remaining people:

A _ _ _ _ _ _ _ _ A

fixed BBB fixed

one object

8!

$\rightarrow {}^5P_2 \times 8! \times 3! = \underline{\underline{4\,838\,400}}$

permutate the three boys

6 In a group of 25 people there are 6 swimmers, 8 cyclists and 11 runners. Each person competes in only one of these sports. A team of 7 people is selected from these 25 people to take part in a competition.

(a) Find the number of different ways in which the team of 7 can be selected if it consists of exactly 1 swimmer, at least 4 cyclists and at most 2 runners. [4]

pick 1 swimmer from 6

S	C	R	
1	4	2	${}^6C_1 \times {}^8C_4 \times {}^{11}C_2 = 23100$
1	5	1	${}^6C_1 \times {}^8C_5 \times {}^{11}C_1 = 3696$
1	6	0	${}^6C_1 \times {}^8C_6 \times {}^{11}C_0 = 168$

$23100 + 3696 + 168 = \underline{26964}$

For another competition, a team of 9 people consists of 2 swimmers, 3 cyclists and 4 runners. The team members stand in a line for a photograph.

(b) How many different arrangements are there of the 9 people if the swimmers stand together, the cyclists stand together and the runners stand together? [2]

swimmers cyclists runners

$2! \times 3! \times 4!$

but the swimmers could be in the middle, or the end etc. so groups can be in any order, so $\times 3!$

$\rightarrow 2! \times 3! \times 4! \times 3! = \underline{1728}$

(c) How many different arrangements are there of the 9 people if none of the cyclists stand next to each other? [4]

Arrange the swimmers and runners:

__ __ __ __ __ __

$6!$

Arrange the cyclists in gaps:

↑ __ ↑ __ ↑ __ ↑ __ ↑ __ ↑ __ ↑

7 spaces → $^7P_3$ ← 3 cyclists

$6! \times {}^7P_3 = \underline{151\,200}$

6 Jai and his wife Kaz are having a party. Jai has invited five friends and each friend will bring his wife.

(a) At the beginning of the party, the 12 people will stand in a line for a photograph.

(i) How many different arrangements are there of the 12 people if Jai stands next to Kaz and each friend stands next to his own wife? **[3]**

Treat each couple as one object:

M_1W_1 M_2W_2 M_3W_3 M_4W_4 M_5W_5 M_6W_6

$6! = 720$

But in each couple, they could be standing MW or WM, so:

$6! \times 2! \times 2! \times 2! \times 2! \times 2! \times 2! = \underline{46\,080}$

(ii) How many different arrangements are there of the 12 people if Jai and Kaz occupy the two middle positions in the line, with Jai's five friends on one side and the five wives of the friends on the other side? **[2]**

2! (JK or KJ)

5! (friends in any order) 5! (wives in any order)

But the friends could be either on the left, with the wives on the right, or vice-versa, so $\times 2!$

$\rightarrow 2! \times 5! \times 5! \times 2! = \underline{57\,600}$

(b) For a competition during the party, the 12 people are divided at random into a group of 5, a group of 4 and a group of 3.

Find the probability that Jai and Kaz are in the same group as each other. [5]

J and K in group of 5:

$$\underbrace{{}^{2}C_{2}}_{\text{J and K}} \times \underbrace{{}^{10}C_{3}}_{\text{3 more people for group of 5}} \times \underbrace{{}^{7}C_{4}}_{\text{pick 4 people from remaining 7 for group of 4}} \times \underbrace{{}^{3}C_{3}}_{\text{group of 3}} = 4200$$

J and K in group of 4:

$$\underbrace{{}^{2}C_{2} \times {}^{10}C_{2}}_{\text{group of 4}} \times \underbrace{{}^{8}C_{5}}_{\text{group of 5}} \times \underbrace{{}^{3}C_{3}}_{\text{group of 3}} = 2520$$

J and K in group of 3:

$$\underbrace{{}^{2}C_{2} \times {}^{10}C_{1}}_{\text{group of 3}} \times \underbrace{{}^{9}C_{5}}_{\text{group of 5}} \times \underbrace{{}^{4}C_{4}}_{\text{group of 4}} = 1260$$

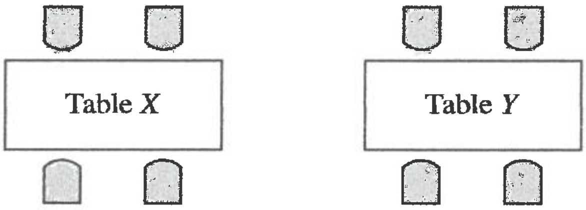
Number of ways with J and K together $= 4200 + 2520 + 1260$
$= \underline{7980}$

Without any restrictions:

$$\underbrace{{}^{12}C_{5}}_{\text{group of 5}} \times \underbrace{{}^{7}C_{4}}_{\text{group of 4}} \times \underbrace{{}^{3}C_{3}}_{\text{group of 3}} = 27\,720$$

probability $= \dfrac{7980}{27\,720} = \underline{\dfrac{19}{66}}$

6



In a restaurant, the tables are rectangular. Each table seats four people: two along each of the longer sides of the table (see diagram). Eight friends have booked two tables, X and Y. Rajid, Sue and Tan are three of these friends.

(a) The eight friends will be divided into two groups of 4, one group for table X and one group for table Y.

Find the number of ways in which this can be done if Rajid and Sue must sit at the same table as each other and Tan must sit at the other table. [3]

$\underbrace{{}^2C_2 \times {}^5C_2}_{\text{group 1}} \times \underbrace{{}^1C_1 \times {}^3C_3}_{\text{group 2}}$

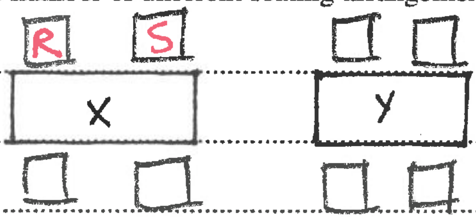
R and S — 2 more to sit with R and S, but excluding Tan. — Tan — remaining 3 to sit with Tan.

Group 1 could be sitting at X or at Y so:

${}^2C_2 \times {}^5C_2 \times {}^1C_1 \times {}^3C_3 \times 2! = 20$ ways

When the friends arrive at the restaurant, Rajid and Sue now decide to sit at table X on the same side as each other. Tan decides that he does not mind at which table he sits.

(b) Find the number of different seating arrangements for the 8 friends. [3]



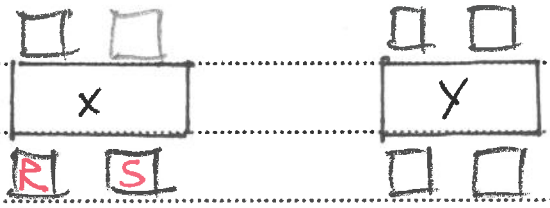
R and S in these places, but could be RS or SR: $2!$

remaining 6 to be placed in any of the 6 spaces: $6!$

$\rightarrow 2! \times 6! = 1440$ (not finished....)

cont

But R and S could be on the other side:



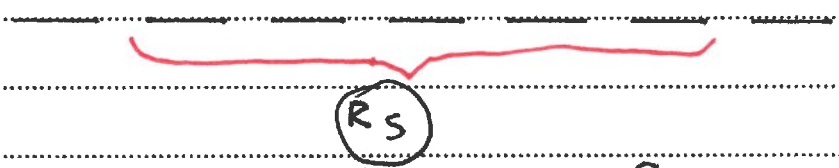
$2! \times 6! = 1440$

$1440 + 1440 = \underline{2880 \text{ ways}}$

As they leave the restaurant, the 8 friends stand in a line for a photograph.

(c) Find the number of different arrangements if Rajid and Sue stand next to each other, but neither is at an end of the line. [4]

Treat R and S as one object and pick a space in the middle for them:



Choose one of the five spaces: ${}^5C_1$

Could be RS or SR : ${}^5C_1 \times 2!$

Arrange 6 remaining people in 6 spaces:

${}^5C_1 \times 2! \times 6! = \underline{7200}$

5 A security code consists of 2 letters followed by a 4-digit number. The letters are chosen from {A, B, C, D, E} and the digits are chosen from {1, 2, 3, 4, 5, 6, 7}. No letter or digit may appear more than once. An example of a code is BE3216.

(a) How many different codes can be formed? [2]

letters numbers

$$^5P_2 \times {}^7P_4 = 16\,800$$

Pick two letters from the five and permutate them

Pick four numbers from the seven and permutate them

(b) Find the number of different codes that include the letter A or the digit 5 or both. [3]

A only:

A letter

pick a letter from B,C,D,E

$$^4C_1 \times 2! \times {}^6P_4 = 2880$$

permutate the two letters

pick four from the six remaining numbers (exclude 5)

5 only:

$$^4P_2 \times {}^6C_3 \times 4! = 5760$$

pick two letters from BCDE and permutate

pick three numbers to go with the 5.

permutate the numbers

A and 5:

$$^4C_1 \times 2! \times {}^6C_3 \times 4! = 3840$$

$$2880 + 5760 + 3840 = 12480$$

A security code is formed at random.

(c) Find the probability that the code is DE followed by a number between 4500 and 5000. [3]

DE 4 5 __ __ $= {}^5P_2 = 20$

pick two numbers from the remaining five and permutate

DE 4 6 __ __ $= {}^5P_2 = 20$

DE 4 7 __ __ $= {}^5P_2 = 20$

$20 + 20 + 20 = \underline{60}$

Number of codes without restrictions = 16 800 (from (a))

$$\text{Probability} = \frac{60}{16800}$$

$$= \frac{1}{280}$$

5 Raman and Sanjay are members of a quiz team which has 9 members in total. Two photographs of the quiz team are to be taken.

For the first photograph, the 9 members will stand in a line.

(a) How many different arrangements of the 9 members are possible in which Raman will be at the centre of the line? [1]

fixed

_ _ _ _ R _ _ _ _

permutate remaining 8 people into 8 spaces:

$8! = 40\,320$

(b) How many different arrangements of the 9 members are possible in which Raman and Sanjay are not next to each other? [3]

Number of arrangements without any restrictions:

$9! = 362\,880$

Arrangements with R and S together:

_ _ _ _ _ _ _ _

RS

one object

$8! \times 2! = 80\,640$

permutate R and S

$362\,880 - 80\,640 = 282\,240$

For the second photograph, the members will stand in two rows, with 5 in the back row and 4 in the front row.

(c) In how many different ways can the 9 members be divided into a group of 5 and a group of 4? [2]

Not permutating, just picking people for groups.

$^{9}C_5 \times {^{4}C_4} = \underline{126}$

pick five people for the first group

pick four from remaining four (= 1)

(d) For a random division into a group of 5 and a group of 4, find the probability that Raman and Sanjay are in the same group as each other. [4]

R and S in back row:

$^{2}C_2 \times {^{7}C_3} \times {^{4}C_4} = 35$

pick both R and S for back row

pick 3 more people for the back row

four people for front row

R and S in front row:

$^{7}C_5 \times {^{2}C_2} \times {^{2}C_2} = 21$

pick 5 for back row from 7 (not including R and S)

pick R and S for front

pick two more for front

$35 + 21 = \underline{56} \rightarrow$ Probability $= \frac{56}{126} = \underline{\frac{4}{9}}$

6 A Social Club has 15 members, of whom 8 are men and 7 are women. The committee of the club consists of 5 of its members.

(a) Find the number of different ways in which the committee can be formed from the 15 members if it must include more men than women. [4]

5 Men : ${}^8C_5 = 56$

pick 5 men from 8

4 Men, 1 Woman: ${}^8C_4 \times {}^7C_1 = 490$

4 men from 8 1 woman from 7

3 Men, 2 Women: ${}^8C_3 \times {}^7C_2 = 1176$

$56 + 490 + 1176 = \underline{\underline{1722}}$

The 15 members are having their photograph taken. They stand in three rows, with 3 people in the front row, 5 people in the middle row and 7 people in the back row.

(b) In how many different ways can the 15 members of the club be divided into a group of 3, a group of 5 and a group of 7? [3]

Not permutating - we are splitting into groups, not arranging.

$${}^{15}C_3 \times {}^{12}C_5 \times {}^{7}C_7 = \underline{360\,360}$$

pick 3 people from all 15 for the front

pick 5 from the 12 who are left for the middle

pick 7 from the remaing 7 for the back.

In one photograph Abel, Betty, Cally, Doug, Eve, Freya and Gino are the 7 members in the back row.

(c) In how many different ways can these 7 members be arranged so that Abel and Betty are next to each other and Freya and Gino are not next to each other? [3]

With A and B together:

AB C D E F G = $6! \times 2! = 1440$

one object

permutate 6 objects

permutate A and B

With A and B together and F and G together:

AB FG C D E = $5! \times 2! \times 2! = 480$

5 objects

A and B

F and G

$1440 - 480 = \underline{960}$

6 Mr and Mrs Ahmed with their two children, and Mr and Mrs Baker with their three children, are visiting an activity centre together. They will divide into groups for some of the activities.

(a) In how many ways can the 9 people be divided into a group of 6 and a group of 3? [2]

$${}^{9}C_6 \times {}^{3}C_3 = 84$$

pick 6 from 9 for first group

pick 3 from remaining 3 for second group

5 of the 9 people are selected at random for a particular activity.

(b) Find the probability that this group of 5 people contains all 3 of the Baker children. [3]

All 3 Baker children in one group:

$${}^{3}C_3 \times {}^{6}C_2 = 15$$

pick 3 from 3 Baker children

pick 2 more from the other 6 people

No restrictions:

$${}^{9}C_5 = 126$$

$$\text{probability} = \frac{15}{126}$$

$$= \frac{5}{42}$$

All 9 people stand in a line.

(c) Find the number of different arrangements in which Mr Ahmed is not standing next to Mr Baker. [3]

With Mr Ahmed next to Mr Baker:

one object → AB

$8! \times 2! = 80\,640$

permutate Mr Ahmed and Mr Baker: AB or BA

No restrictions:

$9! = 362\,880$

$362\,880 - 80\,640 = \underline{282\,240}$

(d) Find the number of different arrangements in which there is exactly one person between Mr Ahmed and Mr Baker. [3]

A __ B (could also be B __ A)

Pick one person to go between A and B: $^7C_1$

Treat A _ B as one object and permutate with the other 6 people:

$7!$

A _ B or B _ A

$\rightarrow {}^7C_1 \times 7! \times 2! = \underline{70\,560}$

7 A group of 15 friends visit an adventure park. The group consists of four families.

- Mr and Mrs Kenny and their four children
- Mr and Mrs Lizo and their three children
- Mrs Martin and her child
- Mr and Mrs Nantes

The group travel to the park in three cars, one containing 6 people, one containing 5 people and one containing 4 people. The cars are driven by Mr Lizo, Mrs Martin and Mr Nantes respectively.

(a) In how many different ways can the remaining 12 members of the group be divided between the three cars? [3]

ie. one group of 5, one of 4 and one of 3.

$^{12}C_5 \times {^7C_4} \times {^3C_3} = \underline{27720}$

pick 5 people from 12 for first car.

pick 4 from the remaining 7

pick 3 from 3

The group enter the park by walking through a gate one at a time.

(b) In how many different orders can the 15 friends go through the gate if Mr Lizo goes first and each family stays together? [3]

Mr Lizo fixed

$4!$

Kenny family $6!$ Martins $2!$ Nantes $2!$

in any order, so $\times 3!$

$4! \times 6! \times 2! \times 2! \times 3! = \underline{414720}$

In the park, the group enter a competition which requires a team of 4 adults and 3 children.

(c) In how many ways can the team be chosen from the group of 15 so that the 3 children are all from different families? [2]

$${}^{7}C_{4} \times {}^{4}C_{1} \times {}^{3}C_{1} \times {}^{1}C_{1} = \underline{420}$$

pick 4 adults from 7

1 child from Kenny family

1 from Lizos

1 from Martins

(d) In how many ways can the team be chosen so that at least one of Mr Kenny or Mr Lizo is included? [3]

Only one of Mr Kenny or Mr Lizo:

$${}^{2}C_{1} \times {}^{5}C_{3} \times {}^{8}C_{3} = 1120$$

pick one from Mr Kenny & Mr Lizo

3 more adults from

pick 3 children from all 8.

Both Mr Kenny and Mr Lizo:

$${}^{2}C_{2} \times {}^{5}C_{2} \times {}^{8}C_{3} = 560$$

both Mr Kenny + Mr Lizo

2 more adults

children

$$1120 + 560 = \underline{1680}$$