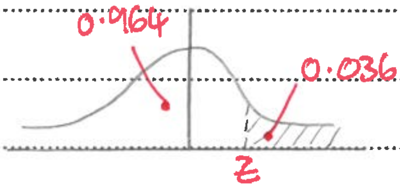
2 The weights of bags of sugar are normally distributed with mean 1.04 kg and standard deviation σ kg. In a random sample of 2000 bags of sugar, 72 weighed more than 1.10 kg.

Find the value of σ. [4]

$$P(W > 1.10) = \frac{72}{2000}$$

$$P(W > 1.10) = 0.036$$

$$P\left(Z > \frac{1.10 - 1.04}{\sigma}\right) = 0.036$$



$$0.964 = \Phi(1.798)$$

$$\therefore z = 1.798$$

$$\frac{1.10 - 1.04}{\sigma} = 1.798$$

$$0.06 = 1.798\sigma$$

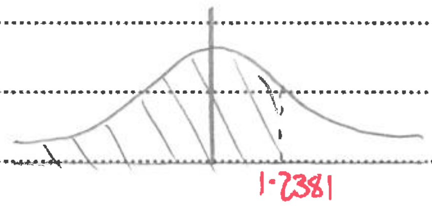
$$\sigma = \underline{\underline{0.0334}}$$

3 In a certain town, the time, X hours, for which people watch television in a week has a normal distribution with mean 15.8 hours and standard deviation 4.2 hours.

(a) Find the probability that a randomly chosen person from this town watches television for less than 21 hours in a week. [2]

$P(X < 21) = P\left(Z < \frac{21 - 15.8}{4.2}\right)$

$= P(Z < 1.2381)$



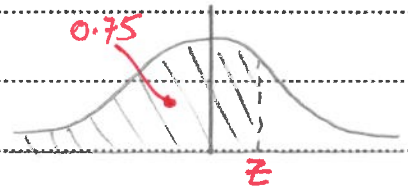
$= \Phi(1.238)$

$= \underline{0.8922}$

(b) Find the value of k such that $P(X < k) = 0.75$. [3]

$P(X < k) = 0.75$

$P\left(Z < \frac{k - 15.8}{4.2}\right) = 0.75$



$\underline{0.75} = \Phi(0.674)$

critical value

$\therefore z = 0.674$

$\frac{k - 15.8}{4.2} = 0.674$

$k - 15.8 = 2.8308$

$k = \underline{18.6}$ hrs

2 A company produces a particular type of metal rod. The lengths of these rods are normally distributed with mean 25.2 cm and standard deviation 0.4 cm. A random sample of 500 of these rods is chosen.

How many rods in this sample would you expect to have a length that is within 0.5 cm of the mean length? [5]

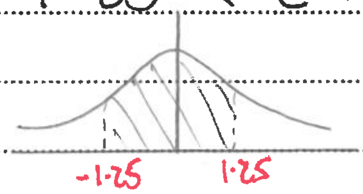
$25.2 + 0.5 = 25.7$

$25.2 - 0.5 = 24.7$

$P(24.7 < L < 25.7)$

$P\left(\frac{24.7 - 25.2}{0.4} < Z < \frac{25.7 - 25.2}{0.4}\right)$

$P(-1.25 < Z < 1.25)$



$= \Phi(1.25) - (1 - \Phi(1.25))$

$= 0.8944 - (1 - 0.8944)$

$= 0.7888$

Number $= 0.7888 \times 500$

$= 394.4$

$\simeq 394$ rods

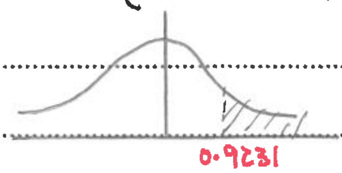
  [Turn over

3 Pia runs 2 km every day and her times in minutes are normally distributed with mean 10.1 and standard deviation 1.3.

(a) Find the probability that on a randomly chosen day Pia takes longer than 11.3 minutes to run 2 km. [3]

$P(T > 11.3) = P\left(Z > \dfrac{11.3 - 10.1}{1.3}\right)$

$= P(Z > 0.9231)$



$= 1 - \Phi(0.923)$

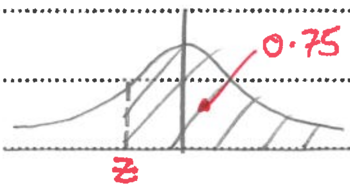
$= 1 - 0.8220$

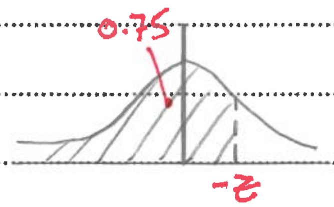
$= \underline{0.178}$

(b) On 75% of days, Pia takes longer than t minutes to run 2 km. Find the value of t. [3]

$P(T > t) = 0.75$

$P\left(Z > \dfrac{t - 10.1}{1.3}\right) = 0.75$





$\underline{0.75} = \Phi(0.674)$

↑ critical value

$\therefore \ z = -0.674$

$\dfrac{t - 10.1}{1.3} = -0.674$

$t - 10.1 = -0.8762$

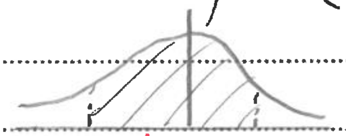
$t = \underline{9.2238}$

(c) On how many days in a period of 90 days would you expect Pia to take between 8.9 and 11.3 minutes to run 2 km? [3]

$P(8.9 < T < 11.3) = P\left(\dfrac{8.9 - 10.1}{1.3} < Z < \dfrac{11.3 - 10.1}{1.3}\right)$

$= P(-0.9231 < Z < 0.9231)$

$= P(Z < 0.9231) - P(Z < -0.9231)$



$= \Phi(0.923) - (1 - \Phi(0.923))$

$= 0.822 - (1 - 0.822)$

$= \underline{0.644}$

Number of days $= 0.644 \times 90$

$= 57.96$

$\simeq \underline{58 \text{ days}}$ *

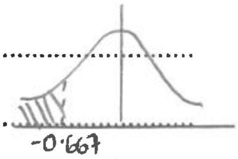
*N.B: I disagree with the mark scheme's 57. As this is a probability, the result of 58 is more likely than 57.

2 The lengths of the rods produced by a company are normally distributed with mean 55.6 mm and standard deviation 1.2 mm.

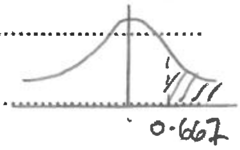
(a) In a random sample of 400 of these rods, how many would you expect to have length less than 54.8 mm? [4]

$P(L < 54.8) = P\left(Z < \frac{54.8 - 55.6}{1.2}\right)$

$= P(Z < -0.667)$



$= 1 - \Phi(0.667)$



$= 1 - 0.7477$

$= 0.2523$

Expected number $= 0.2523 \times 400$

$= 100.92$

$= \underline{101}$

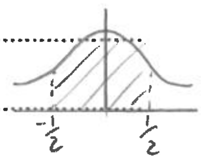
(b) Find the probability that a randomly chosen rod produced by this company has a length that is within half a standard deviation of the mean. [3]

$55.6 + 0.5(1.2) = 56.2$

$55.6 - 0.5(1.2) = 55$

$P(55 < L < 56.2) = P\left(\frac{55 - 55.6}{1.2} < Z < \frac{56.2 - 55.6}{1.2}\right)$

can skip straight to here → $= P\left(-\frac{1}{2} < Z < \frac{1}{2}\right)$



$= P(Z < \frac{1}{2}) - P(Z < -\frac{1}{2})$

$= \Phi(\frac{1}{2}) - (1 - \Phi(\frac{1}{2}))$

$= 0.6915 - (1 - 0.6915)$

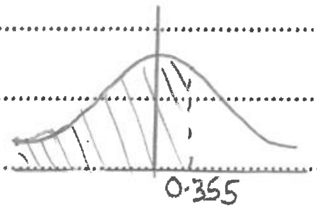
$= \underline{0.383}$

4 In a large population, the systolic blood pressure (SBP) of adults is normally distributed with mean 125.4 and standard deviation 18.6.

(a) Find the probability that the SBP of a randomly chosen adult is less than 132. [2]

$P(S < 132) = P\left(Z < \frac{132 - 125.4}{18.6}\right)$

$= P(Z < 0.355)$



$= \Phi(0.355)$

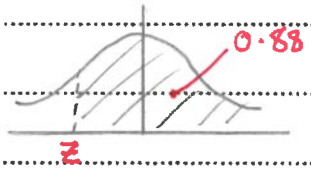
$= \underline{0.6387}$

The SBP of 12-year-old children in the same population is normally distributed with mean 117. Of these children 88% have SBP more than 108.

(b) Find the standard deviation of this distribution. [3]

$P(C > 108) = 0.88$

$P\left(Z > \frac{108 - 117}{\sigma}\right) = 0.88$



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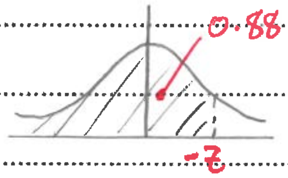


$0.88 = \Phi(1.175)$ from table

$\therefore z = -1.175$

$\frac{108 - 117}{\sigma} = -1.175$

$-9 = -1.175\sigma$

$\underline{\sigma = 7.66}$

Three adults are chosen at random from this population.

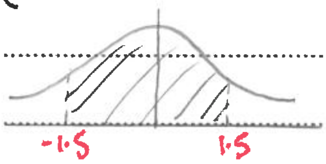
(c) Find the probability that each of these three adults has SBP within 1.5 standard deviations of the mean. [4]

$125.4 + 1.5(18.6) = 153.3$

$125.4 - 1.5(18.6) = 97.5$

$P\left(\frac{97.5 - 125.4}{18.6} < Z < \frac{153.3 - 125.4}{18.6}\right)$

$P(-1.5 < Z < 1.5)$ ← can skip straight to here



$= \Phi(1.5) - (1 - \Phi(1.5))$

$= 0.9332 - (1 - 0.9332)$

$= 0.8664$

Probability of 3 adults $= 0.8664^3$

$= 0.650$

5 Farmer Jones grows apples. The weights, in grams, of the apples grown this year are normally distributed with mean 170 and standard deviation 25. Apples that weigh between 142 grams and 205 grams are sold to a supermarket.

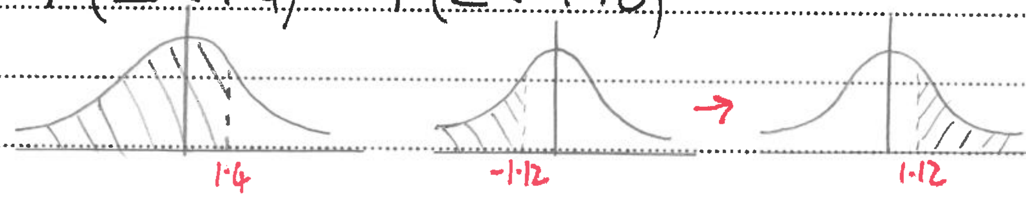
(a) Find the probability that a randomly chosen apple grown by Farmer Jones this year is sold to the supermarket. [4]

$P(142 < W < 205)$

$P\left(\frac{142-170}{25} < Z < \frac{205-170}{25}\right)$

$P(-1.12 < Z < 1.4)$

$= P(Z < 1.4) - P(Z < -1.12)$



$= \Phi(1.4) - (1 - \Phi(1.12))$

$= 0.9192 - (1 - 0.8686)$

$= \underline{0.7878}$

Farmer Jones sells the apples to the supermarket at \$0.24 each. He sells apples that weigh more than 205 grams to a local shop at \$0.30 each. He does not sell apples that weigh less than 142 grams.

The total number of apples grown by Farmer Jones this year is 20 000.

(b) Calculate an estimate for his total income from this year's apples. [3]

$P(W > 205) = P(Z > 1.4)$ (part (a))

$= 1 - \Phi(1.4)$

$= 1 - 0.9192$

$= 0.0808$

large apples: $0.0808 \times 20000 = 1616$

$1616 \times 0.30 = \$484.80$

normal apples: $0.7878 \times 20000 = 15756$

$15756 \times 0.24 = \$3781.44$

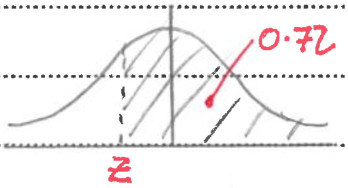
Total: $484.80 + 3781.44 = \$4266.24$

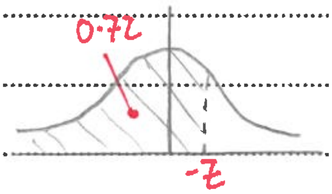
Farmer Tan also grows apples. The weights, in grams, of the apples grown this year follow the distribution $N(182, 20^2)$. 72% of these apples have a weight more than w grams.

(c) Find the value of w. [3]

$P(W > w) = 0.72$

$P\left(Z > \frac{w - 182}{20}\right) = 0.72$





$0.72 = \Phi(0.583)$

$\therefore z = -0.583$

$\frac{w - 182}{20} = -0.583 \rightarrow w - 182 = -11.66$

$w = 170.34g$

5 Company *A* produces bags of sugar. An inspector finds that on average 10% of the bags are underweight.

10 of the bags are chosen at random.

(a) Find the probability that fewer than 3 of these bags are underweight. [3]

$X \sim B(10, 0.1)$

$P(X<3) = P(0) + P(1) + P(2)$

$= {}^{10}C_0 \times 0.1^0 \times 0.9^{10} + {}^{10}C_1 \times 0.1^1 \times 0.9^9 + {}^{10}C_2 \times 0.1^2 \times 0.9^8$

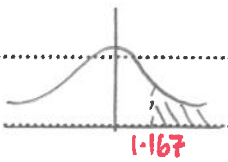
$= \underline{0.930}$

The weights of the bags of sugar produced by company *B* are normally distributed with mean 1.04 kg and standard deviation 0.06 kg.

(b) Find the probability that a randomly chosen bag produced by company *B* weighs more than 1.11 kg. [3]

$P(B > 1.11) = P\left(Z > \frac{1.11 - 1.04}{0.06}\right)$

$= P(Z > 1.167)$



$= 1 - \Phi(1.167)$

$= 1 - 0.8784$

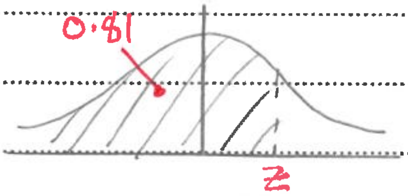
$= \underline{0.122}$

81% of the bags of sugar produced by company B weigh less than w kg.

(c) Find the value of w. [3]

$P(B < w) = 0.81$

$P\left(Z < \frac{w - 1.04}{0.06}\right) = 0.81$



$0.81 = \Phi(0.878)$ from table

$\frac{w - 1.04}{0.06} = 0.878$

$w - 1.04 = 0.05268$

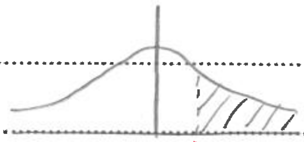
$\underline{w = 1.09268}$

7 The times, in minutes, that Karli spends each day on social media are normally distributed with mean 125 and standard deviation 24.

(a) (i) On how many days of the year (365 days) would you expect Karli to spend more than 142 minutes on social media? [5]

$P(K > 142) = P\left(Z > \frac{142-125}{24}\right)$

$= P(Z > 0.7083)$



0.7083

$= 1 - \Phi(0.7083)$

$= 1 - 0.7604$

$= 0.2396$

number of days $= 0.2396 \times 365$

$= 87.454$

≈ 87 days

(ii) Find the probability that Karli spends more than 142 minutes on social media on fewer than 2 of 10 randomly chosen days. [3]

$T \sim B(10, 0.2396)$

$P(T < 2) = P(T=0) + P(T=1)$

$= {}^{10}C_0 \times 0.2396^0 \times 0.7604^{10} + {}^{10}C_1 \times 0.2396^1 \times 0.7604^9$

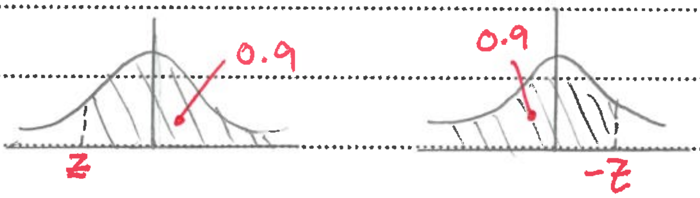
$= 0.268$

(b) On 90% of days, Karli spends more than t minutes on social media.

Find the value of t. [3]

$P(K > t) = 0.9$

$P\left(Z > \frac{t-125}{24}\right) = 0.9$



$0.9 = \Phi(1.282)$

critical value

$\therefore z = -1.282$

$\frac{t-125}{24} = -1.282$

$t - 125 = -30.768$

$t = 94.232$

$t \simeq 94.2$ days

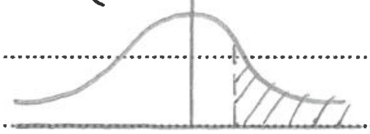
4 A mathematical puzzle is given to a large number of students. The times taken to complete the puzzle are normally distributed with mean 14.6 minutes and standard deviation 5.2 minutes.

(a) In a random sample of 250 of the students, how many would you expect to have taken more than 20 minutes to complete the puzzle? [4]

$P(T > 20)$

$P\left(Z > \frac{20 - 14.6}{5.2}\right)$

$P(Z > 1.038)$



$= 1 - \Phi(1.038)$

$= 1 - 0.8504$

$= 0.1496$

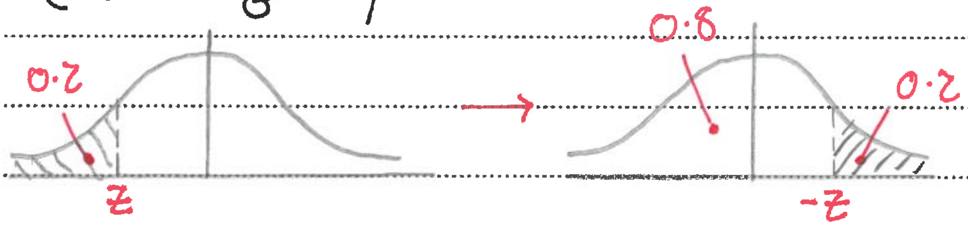
$250 \times 0.1496 = 37$ students

All the students are given a second puzzle to complete. Their times, in minutes, are normally distributed with mean μ and standard deviation σ. It is found that 20% of the students have times less than 14.5 minutes and 67% of the students have times greater than 18.5 minutes.

(b) Find the value of μ and the value of σ. [5]

$P(T < 14.5) = 0.2$

$P\left(Z < \frac{14.5 - \mu}{\sigma}\right) = 0.2$



$0.8 = \Phi(0.842)$

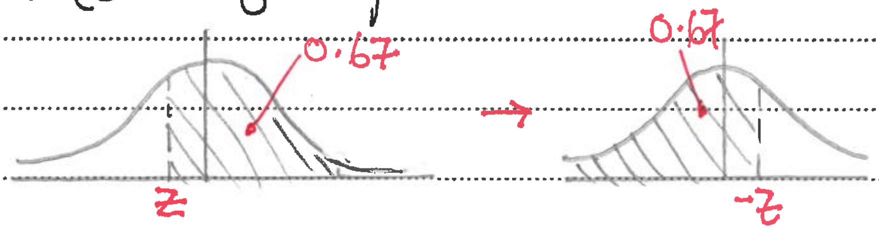
$z = -0.842$

$\frac{14.5 - \mu}{\sigma} = -0.842$

$14.5 - \mu = -0.842\sigma$ ①

$P(T > 18.5) = 0.67$

$P\left(Z > \frac{18.5 - \mu}{\sigma}\right) = 0.67$



$0.67 = \Phi(0.44)$

$z = -0.44$

$\frac{18.5 - \mu}{\sigma} = -0.44$

$18.5 - \mu = -0.44\sigma$ ②

② − ①: $4 = 0.402\sigma$

$\sigma = 9.95$ STO

②: $18.5 - \mu = -0.44 \times 9.95$

$18.5 - \mu = -4.378$

$-\mu = -22.88$

$\mu = 22.9$

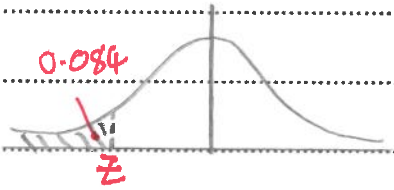
5 The lengths of the leaves of a particular type of tree are modelled by a normal distribution. A scientist measures the lengths of a random sample of 500 leaves from this type of tree and finds that 42 are less than 4 cm long and 100 are more than 10 cm long.

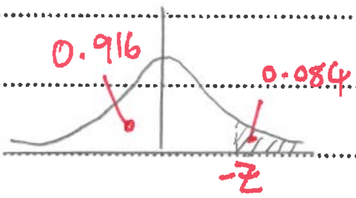
(a) Find estimates for the mean and standard deviation of the lengths of leaves from this type of tree. [5]

$P(L < 4) = \frac{42}{500}$

$P(L < 4) = 0.084$

$P\left(Z < \frac{4-\mu}{\sigma}\right) = 0.084$





$0.916 = \Phi(1.378)$

$\therefore z = -1.378$

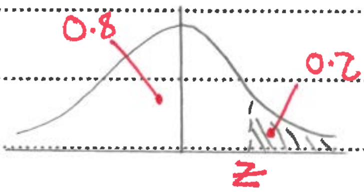
$\frac{4-\mu}{\sigma} = -1.378$

$4-\mu = -1.378$ ①

$P(L > 10) = \frac{100}{500}$

$P(L > 10) = 0.2$

$P\left(Z > \frac{10-\mu}{\sigma}\right) = 0.2$



$0.8 = \Phi(0.842)$

$\therefore z = 0.842$

$\frac{10-\mu}{\sigma} = 0.842$

$10-\mu = 0.842\sigma$ ②

② − ①:

$6 = 2.220\sigma$

$\sigma = 2.70$ ← store

→ ②:

$10-\mu = 0.842 \times 2.702...$

$\mu = 10 - 2.275..$

$= 7.72$

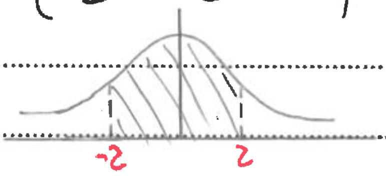
The lengths, in cm, of the leaves of a different type of tree have the distribution $N(\mu, \sigma^2)$. The scientist takes a random sample of 800 leaves from this type of tree.

(b) Find how many of these leaves the scientist would expect to have lengths, in cm, between $\mu - 2\sigma$ and $\mu + 2\sigma$. [4]

$P\left(\frac{\mu - 2\sigma - \mu}{\sigma} < Z < \frac{\mu + 2\sigma - \mu}{\sigma}\right)$

$P\left(\frac{-2\sigma}{\sigma} < Z < \frac{2\sigma}{\sigma}\right)$

$P(-2 < Z < 2)$



$= \Phi(2) - (1 - \Phi(2))$

$= 0.9772 - (1 - 0.9772)$

$= \underline{0.9544}$

Number of leaves $= 0.9544 \times 800$

$= 763.52$

$\simeq \underline{764}$

6 The lengths of female snakes of a particular species are normally distributed with mean 54 cm and standard deviation 6.1 cm.

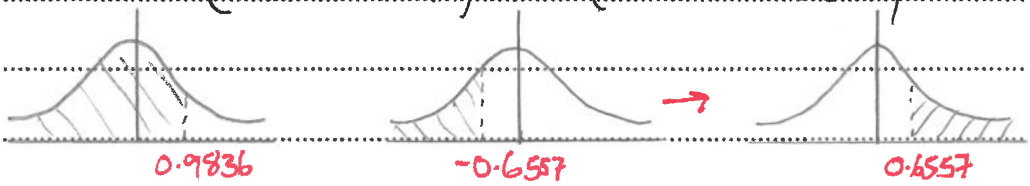
(a) Find the probability that a randomly chosen female snake of this species has length between 50 cm and 60 cm. [4]

$P(50 < L < 60)$

$= P\left(\frac{50-54}{6.1} < Z < \frac{60-54}{6.1}\right)$

$= P(-0.6557 < Z < 0.9836)$

$= P(Z < 0.9836) - P(Z < -0.6557)$



$= \Phi(0.984) - (1 - \Phi(0.656))$

$= 0.8375 - (1 - 0.7441)$

$= \underline{0.5816}$

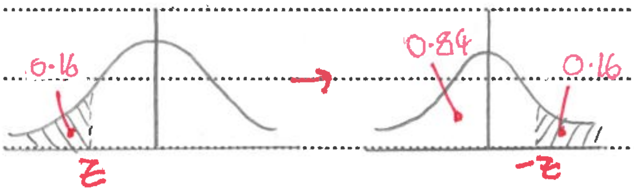
The lengths of male snakes of this species also have a normal distribution. A scientist measures the lengths of a random sample of 200 male snakes of this species. He finds that 32 have lengths less than 45 cm and 17 have lengths more than 56 cm.

(b) Find estimates for the mean and standard deviation of the lengths of male snakes of this species. [5]

$P(L<45) = \frac{32}{200}$

$P(L<45) = 0.16$

$P\left(Z < \frac{45-\mu}{\sigma}\right) = 0.16$



$0.84 = \Phi(0.994)$

$\therefore z = -0.994$

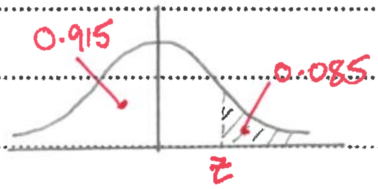
$\frac{45-\mu}{\sigma} = -0.994$

$45-\mu = -0.994\sigma$ (1)

$P(L>56) = \frac{17}{200}$

$P(L>56) = 0.085$

$P\left(Z > \frac{56-\mu}{\sigma}\right) = 0.085$



$0.915 = \Phi(1.372)$

$\therefore z = 1.372$

$\frac{56-\mu}{\sigma} = 1.372$

$56-\mu = 1.372\sigma$ (2)

(2) − (1):

$11 = 2.366\sigma$

$\sigma = \underline{4.65}$ ⇐ store

sub → (2):

$56-\mu = 1.372 \times 4.65\ldots$

$\mu = 56 - 6.379$

$\mu = \underline{49.6}$

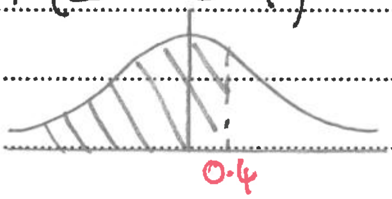
5 **(a)** The heights of the members of a club are normally distributed with mean 166 cm and standard deviation 10 cm.

(i) Find the probability that a randomly chosen member of the club has height less than 170 cm. [2]

$P(H < 170)$

$P\left(Z < \dfrac{170 - 166}{10}\right)$

$P(Z < 0.4)$



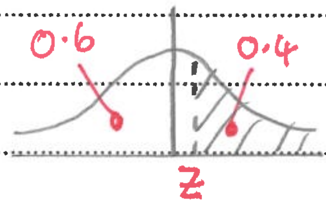
$= \Phi(0.4)$

$= \underline{0.6554}$

(ii) Given that 40% of the members have heights greater than h cm, find the value of h correct to 2 decimal places. [3]

$P(H > h) = 0.4$

$P\left(Z > \dfrac{h - 166}{10}\right) = 0.4$



$0.6 = \Phi(0.253)$ from table (nearest value)

$z = 0.253$

$\dfrac{h - 166}{10} = 0.253$

$h - 166 = 2.53$

$\underline{h = 168.53 \text{ cm}}$

(b) The random variable X is normally distributed with mean μ and standard deviation σ.

Given that $\sigma = \frac{2}{3}\mu$, find the probability that a randomly chosen value of X is positive. [3]

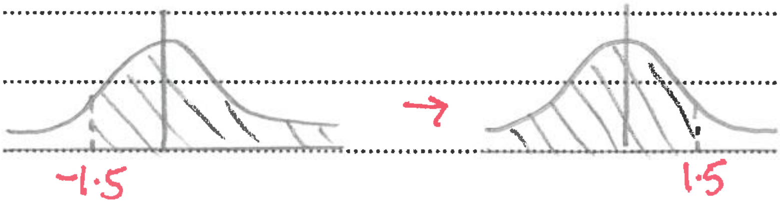
$P(X > 0)$

$P\left(Z > \dfrac{0 - \mu}{\frac{2\mu}{3}}\right)$

$P\left(Z > \dfrac{-\mu \;{\times 3}}{\frac{2\mu}{3} \;{\times 3}}\right)$

$P\left(Z > \dfrac{-3\cancel{\mu}}{2\cancel{\mu}}\right)$

$P(Z > -1.5)$



$P(Z > -1.5) = P(Z < 1.5)$

$= \Phi(1.5)$

$= \underline{0.9332}$